# Preprocessing and Inprocessing Techniques in SAT

Armin Biere

Institute for Formal Models and Verification
Johannes Kepler University
Linz, Austria

joint work with Marijn Heule and Matti Järvisalo

WorKer'11

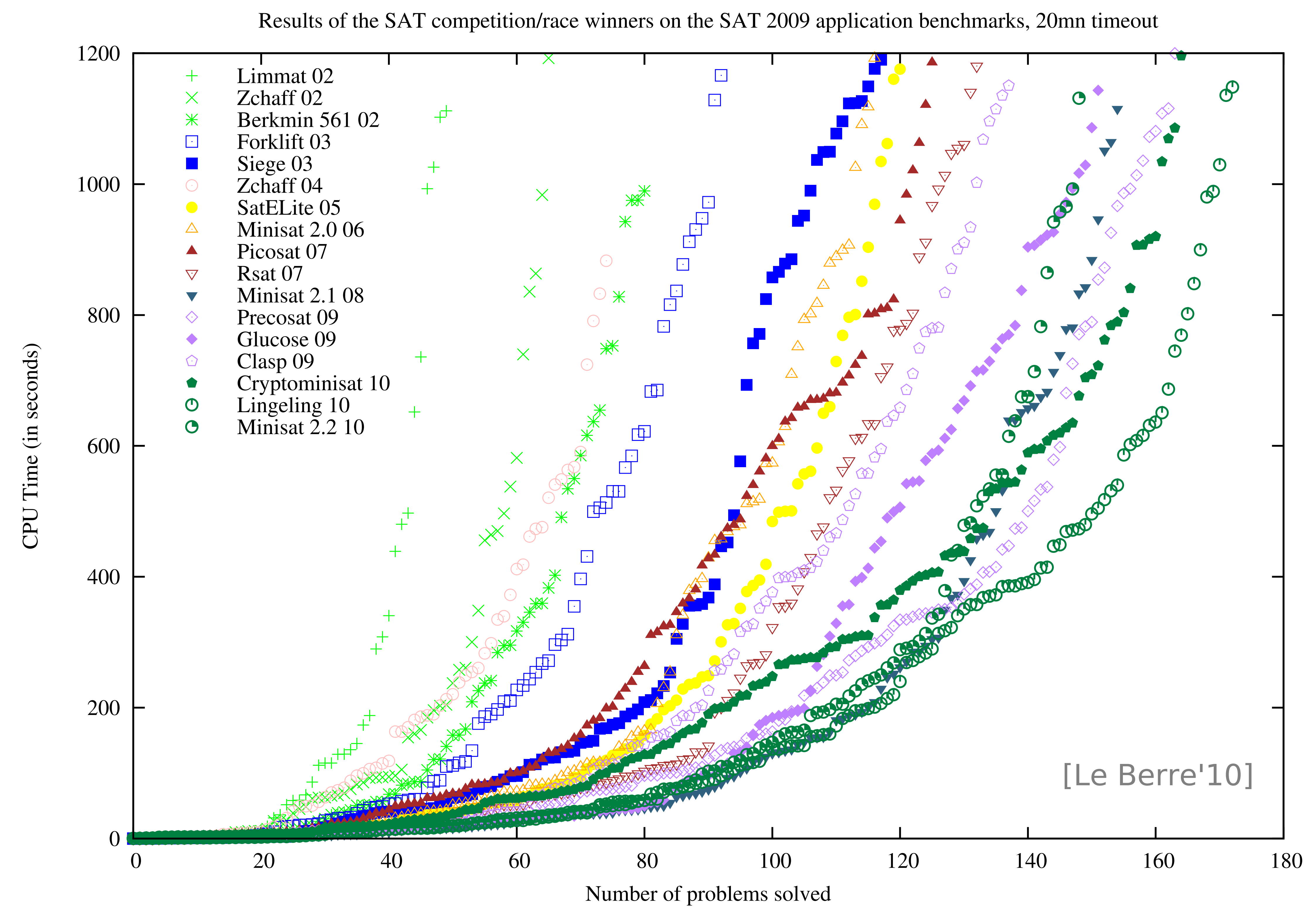
3rd Workshop on Kernelization
TU Vienna, Austria

Friday, September 2, 2011

- propositional logic:
  - variables **tie** **shirt**
  - negation $\neg$ (not)
  - disjunction $\vee$ disjunction (or)
  - conjunction $\wedge$ conjunction (and)

- three conditions / clauses:
  - clearly one should not wear a **tie** without a **shirt** $\neg\textbf{tie} \vee \textbf{shirt}$
  - not wearing a **tie** nor a **shirt** is impolite $\textbf{tie} \vee \textbf{shirt}$
  - wearing a **tie** and a **shirt** is overkill $\neg(\textbf{tie} \wedge \textbf{shirt}) \quad \equiv \quad \neg\textbf{tie} \vee \neg\textbf{shirt}$

- is the formula $(\neg\textbf{tie} \vee \textbf{shirt}) \wedge (\textbf{tie} \vee \textbf{shirt}) \wedge (\neg\textbf{tie} \vee \neg\textbf{shirt})$ satisfiable?



Results of the SAT competition/race winners on the SAT 2009 application benchmarks, 20mn timeout
CPU Time (in seconds)
Number of problems solved
Limmat 02
Zchaff 02
Berkmin 561 02
Forklift 03
Siege 03
Zchaff 04
SatELite 05
Minisat 2.0 06
Picosat 07
Rsat 07
Minisat 2.1 08
Precosat 09
Glucose 09
Clasp 09
Cryptominisat 10
Lingeling 10
Minisat 2.2 10
[Le Berre'10]
0
200
400
600
800
1000
1200
0
20
40
60
80
100
120
140
160
180

# Overview

- failed literal *probing*
- variable elimination (VE)
- inprocessing
- lazy hyper binary resolution
- blocked clause elimination (BCE)
- hidden tautologies elimination (HTE)
- unhiding

# Failed Literal Probing

*we are still working on tracking down the origin before* [Freeman'95] [LeBerre'01]

- key technique in look-ahead solvers such as Satz, OKSolver, March
  - failed literal probing at all search nodes
  - used to find the best decision variable and phase
- simple algorithm
  1. assume literal $l$, propagate (BCP), if this results in conflict, add unit clause $\neg l$
  2. continue with all literals $l$ until *saturation* (nothing changes)
- quadratic to cubic complexity
  - BCP linear in the size of the formula — 1st linear factor
  - each variable needs to be tried — 2nd linear factor
  - and tried again if some unit has been derived — 3rd linear factor

# Extensions



- lifting
  - complete case split: literals implied in all cases become units
  - similar to Stålmark's method and Recursive Learning [PradhamKunz'94]
- asymmetric branching
  - assume all but one literal of a clause to be false
  - if BCP leads to conflict remove originally remaining unassigned literal
  - implemented for a long time in MiniSAT but switched off by default
- generalizations:
  - vivification [PietteHamadiSais ECAI'08]
  - distillation [JinSomenzi'05][HanSomenzi DAC'07] probably most general (+ tries)

# Bounded Variable Elimination (VE)



[Biere'04][SubbarayanPradhan'04][EénBiere SAT'05]

- goes back to original Davis & Putnam algorithm [DP'60]
  - eliminate variable $x$ by adding all resolvents with $x$ as pivot …
  - … and removing all clauses in which $x$ or $\neg x$ occurs
  - eliminating one variable is in the worst case quadratic
- bounded = apply only if increment in size is small
  - Quantor [Biere'03,Biere'04] bound increase in terms of literals (priority queue)
  - NiVER [SubbarayanPradhan'04] do non increase number of clauses (round-robin)
  - SatELite [EénBiere'05] do not increase number of clauses (priority queue)

- fast subsumption and strengthening [Biere'04][EénBiere'05]
  - backward subsumption: traverse clauses of least occurring literal
  - forward subsumption: one-watched literal scheme [Zhang'05]
  - 1st and 2nd level signatures = Bloom-filters for faster checking
  - strengthen clauses through self-subsuming resolution (later again)

- functional substitution
  - if $x$ has a functional dependency, e.g. Tseitin translation of a gate
  - then only resolvents using exactly one "gate clause" need to be added

$$\overbrace{(\bar{x} \vee a)(\bar{x} \vee b)(x \vee \bar{a} \vee \bar{b})}^{x = a \wedge b}(x \vee c)(x \vee d)(\bar{x} \vee e)(\bar{x} \vee f)$$ 7 clauses

$$(a \vee c)(a \vee d)(b \vee c)(b \vee d)(\bar{a} \vee \bar{b} \vee e)(\bar{a} \vee \bar{b} \vee f)\cancel{(c \vee e)(c \vee f)(d \vee e)(d \vee f)}$$ 6 + 4 clauses

# Inprocessing: Interleaving Preprocessing and Search

- **preprocessing** can be extremely beneficial
  - most SAT competition solvers use variable elimination (VE) [EénBiere SAT'05]
  - equivalence & XOR reasoning beneficial
  - probing / failed literal preprocessing / hyper binary resolution useful
  - however, even though polynomial, can not be run until completion
- **inprocessing**: simple idea to benefit from full preprocessing without penalty
  - "preempt" preprocessors after some time
  - resume preprocessing between restarts
  - limit preprocessing time in relation to search time

# Benefits of Interleaving Preprocessing and Search



- allows to use costly preprocessors
  - without increasing run-time “much” in the worst-case
  - still useful for benchmarks where these costly techniques help
  - good examples: probing and distillation

    even VE can be costly

- additional benefit:
  - makes units / equivalences learned in search available to preprocessing
  - particularly interesting if preprocessing simulates encoding optimizations

- danger of hiding “bad” implementation though ...

- ... and hard(er) to debug

- one Hyper Binary Resolution step [Bacchus-AAAI02]

$$\frac{(l \vee l_1 \vee \cdots \vee l_n) \quad (\overline{l_1} \vee l') \quad \cdots \quad (\overline{l_n} \vee l')}{(l \vee l')}$$

  - combines multiple resolution steps into one
  - special case "hyper unary resolution" where $l = l'$
  - HBR stronger than unit propagation if it is repeated until (confluent) closure
  - equality reduction: if $(a \vee \overline{b}), (\overline{a} \vee b) \in f$ then replace $a$ by $b$ in $f$

- can be simulated by unit propagation [BacchusWinter-SAT03]

if $(l \vee l') \in \mathrm{HypBinRes}(f)$ then $l' \in \mathrm{UnitProp}(f \wedge \overline{l})$ or vice versa

- repeated probing, c.f. $\mathrm{HypBinResFast}$ [GershmanStrichman-SAT05]

# Previous Optimizations

[BacchusWinter-SAT03][GershmanStrichman-SAT05]

- maintain acyclic and transitively-reduced binary implication graph
  - acyclic: decomposition in *strongly connected components* (SCCs)

    $(\overline{a} \vee b)(\overline{b} \vee c)(\overline{c} \vee a) \wedge R$ equisatisfiable to $R[a/b, a/c]$

  - transitively-reduced: remove resp. do not add transitive edges

- not all literals have to be probed
  - if $l \in \mathrm{UnitProp}(r)$ and $\mathrm{UnitProp}(r)$ does not produce anything
    $\Rightarrow$ no need to probe $l$
  - at least with respect to units it is possible to focus on roots

- current algorithms too expensive to run until completion

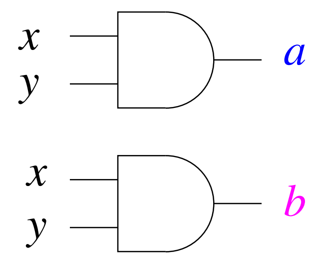
- **time** complexity: seems to be at least quadratic, unfortunately also in practice

- **space** complexity: unclear, at most quadratic, linear?

- hyper binary resolution simulates structural hashing for AND gates $a$ and $b$

$$F \quad \equiv \quad (\overline{a} \vee x)(\overline{a} \vee y)(a \vee \overline{x} \vee \overline{y}) \quad (\overline{b} \vee x)(\overline{b} \vee y)(b \vee \overline{x} \vee \overline{y}) \qquad \cdots$$

$$\frac{(\overline{a} \vee x)(\overline{a} \vee y)(b \vee \overline{x} \vee \overline{y})}{(\overline{a} \vee b)} \qquad \frac{(\overline{b} \vee x)(\overline{b} \vee y)(a \vee \overline{x} \vee \overline{y})}{(\overline{b} \vee a)}$$



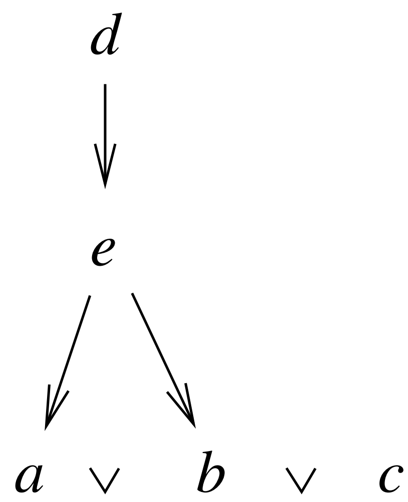

can also be seen by $b \in \mathrm{UnitProp}(F \wedge a)$ and $a \in \mathrm{UnitProp}(F \wedge b)$

- can not simulate structural hashing of XOR or ITE gates

- learn binary clauses lazily or on-the-fly
  - in the innermost (!) BCP loop
  - during BCP in search or during BCP in preprocessing (failed literal probing)

- whenever a **large** clause $(a_1 \vee \cdots \vee a_m \vee c)$ with $m \geq 2$ becomes a reason for $c$
  - partial assignment $\sigma$ with $\sigma(a_i) = 0$ and $\sigma(c) = 1$
  - check whether exists literal $d$ dominating all $\overline{a_i}$
  - in implication graph restricted to binary clauses
  - which is a tree!

- learn $(\overline{d} \vee c)$ if such a dominator exists better $(\bar{e} \vee c)$



# Avoid Transitive Edges by Prioritizing Binary Clause Propagation



1. trail contains assigned literals
2. set $n_2$ and $n_3$ to the trail level of those literals that still need to be propagated
3. while $0 \leq n_3 \leq n_2 < |\text{trail}|$ and there is no conflict
   (a) if $n_2 < |\text{trail}|$
      i. pick literal $l$ at position $n_2$, increment $n_2$ and visit binary clauses with $\bar{l}$
      ii. assign literals forced through these binary clauses first
   (b) otherwise (necessarily $n_3 < |\text{trail}|$)
      i. pick literal $l$ at position $n_3$, increment $n_3$ and visit large clauses with $\bar{l}$
      ii. assign literals forced through these large clauses

# How-To Check Existence of Dominators

- for each assigned literal $l$ keep **one** dominator $\mathsf{bindom}(l)$

  (in the implication graph restricted to binary clauses)

- thus $\mathsf{bindom}(l)$ is the root of binary implication tree of $l$

- decisions $l$ set $\mathsf{bindom}(l) = l$

- binary implications $(a_1 \vee c)$ with $\sigma(a_1) = 0$, $\sigma(c) = 1$ set $\mathsf{bindom}(c) = \mathsf{bindom}(\overline{a_1})$

- necessary & sufficient for $\overline{a_i}$ in large ($m \geq 2$) reasons to have common dominator:

$$(a_1 \vee \cdots \vee a_m \vee c) \qquad \mathsf{bindom}(\overline{a_1}) = \cdots = \mathsf{bindom}(\overline{a_m})$$

- if condition triggers, actually use least common ancestor (closest dominator)

- use $(\overline{d} \vee c)$ as **new reason** instead of $(a_1 \vee \cdots \vee a_m \vee c)$

# PreCoSAT Integration



- interleave search and preprocessing
  - bound time spent in search to roughly 80%
  - measured in number of propagations / resolutions

- during preprocessing / simplification on the top level
  - unit propagation on the top-level does LHBR — top-level (1)
  - failed literal probing learns most binary clauses with LHBR — probing (2)

- BCP during search learns binary clauses with LHBR — search (3)

- rerunning SAT'09 competition with competition version 236 of PrecoSAT
  - 900 seconds time out
  - roughly twice as fast machines
- PrecoSAT without LHBR solves 6 less instances
  - 171 instead of 177 out of 292
- statistics
  - LHBR learned 48 million binary clause
  - on 292 instances that is 181 thousand learned binary clauses on average
  - additionally 202 million learned clauses through conflict analysis
  - 19% learned (binary) clauses due to LHBR

- no measurable overhead doing LHBR during BCP
  - so at least not harmful — in contrast to many other “optimizations”
  - implementation in Lingeling became non-trivial
  - unfortunately does not simulate structural hashing in practice (!!)
- *not formally published* but implemented in PrecoSAT and Lingeling
  - source code of PrecoSAT available under MIT license
  - source code of Lingeling available under GPL license
  - extensions in [HanSomenzi-DATE’11]
- performs a limited version of on-the-fly strengthening / subsumption

  thus partially simulates distillation / vivification blocked clause $C \in F$ all clauses in $F$ with $\bar{l}$

$\bar{l} \vee \bar{a} \vee c$

fix a CNF $F$

$a \vee b \vee l$

$\bar{l} \vee \bar{b} \vee d$

since all resolvents of $C$ on $l$ are tautological $C$ can be removed

**Proof**

assignment $\sigma$ satisfying $F \backslash C$ but not $C$

can be extended to a satisfying assignment of $F$ by flipping value of $l$

**Definition** A literal $l$ in a clause $C$ of a CNF $F$ blocks $C$ w.r.t. $F$ if for every clause $C' \in F$ with $\bar{l} \in C'$, the resolvent $(C \setminus \{l\}) \cup (C' \setminus \{\bar{l}\})$ obtained from resolving $C$ and $C'$ on $l$ is a tautology.

**Definition** [Blocked Clause] A clause is blocked if has a literal that blocks it.

**Definition** [Blocked Literal] A literal is blocked if it blocks a clause.

**Example** $(a \vee b) \wedge (a \vee \bar{b} \vee \bar{c}) \wedge (\bar{a} \vee c)$

only first clause is not blocked.

second clause contains two blocked literals: $a$ and $\bar{c}$.

literal $c$ in the last clause is blocked.

after removing either $(a \vee \bar{b} \vee \bar{c})$ or $(\bar{a} \vee c)$, the clause $(a \vee b)$ becomes blocked
actually all clauses can be removed

[JärvisaloBiereHeule-TACAS10]

**COI** Cone-of-Influence reduction

**MIR** Monontone-Input-Reduction

**NSI** Non-Shared Inputs reduction

---

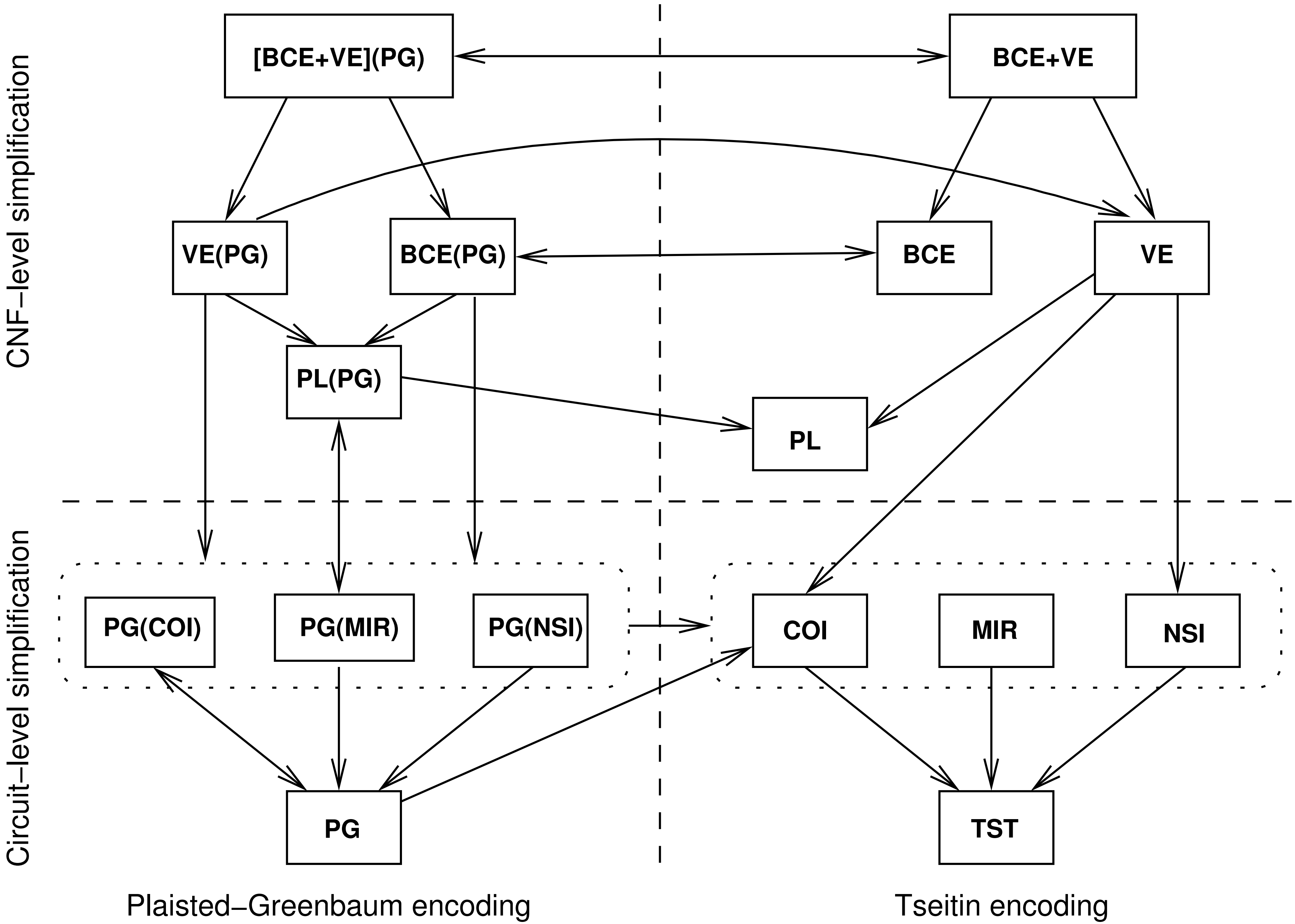
**PG** Plaisted-Greenbaum polarity based encoding

**TST** standard Tseitin encoding

---

**VE** Variable-Elimination as in DP / Quantor / SATeLite

**BCE** Blocked-Clause-Elimination



CNF−level simplification
Circuit−level simplification
[BCE+VE](PG)
BCE+VE
VE(PG)
BCE(PG)
BCE
VE
PL(PG)
PL
PG(COI)
PG(MIR)
PG(NSI)
COI
MIR
NSI
PG
TST
Plaisted−Greenbaum encoding
Tseitin encoding

| | encoding | | | b | | | be | | | beb | | | bebe | | | e | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | T | V | C | T | V | C | T | V | C | T | V | C | T | V | C | T | V | C |
| SU | 0 | 46 | 256 | 2303 | 29 | 178 | 1042 | 11 | 145 | 1188 | 11 | 145 | 569 | 11 | 144 | 2064 | 11 | 153 |
| AT | 12 | 9 | 27 | 116 | 7 | 18 | 1735 | 1 | 8 | 1835 | 1 | 6 | 34 | 1 | 6 | 244 | 1 | 9 |
| AP | 10 | 9 | 20 | 94 | 7 | 18 | 1900 | 1 | 6 | 36 | 1 | 6 | 34 | 1 | 6 | 1912 | 1 | 6 |
| AM | 190 | 1 | 8 | 42 | 1 | 7 | 178 | 1 | 7 | 675 | 1 | 7 | 68 | 1 | 7 | 48 | 1 | 8 |
| AN | 9 | 3 | 10 | 50 | 3 | 10 | 1855 | 1 | 6 | 36 | 1 | 6 | 34 | 1 | 6 | 1859 | 1 | 6 |
| HT | 147 | 121 | 347 | 1648 | 117 | 277 | 2641 | 18 | 118 | 567 | 18 | 118 | 594 | 18 | 116 | 3240 | 23 | 140 |
| HP | 130 | 121 | 286 | 1398 | 117 | 277 | 2630 | 18 | 118 | 567 | 18 | 118 | 595 | 18 | 116 | 2835 | 19 | 119 |
| HM | 6961 | 16 | 91 | 473 | 16 | 84 | 621 | 12 | 78 | 374 | 12 | 77 | 403 | 12 | 76 | 553 | 15 | 90 |
| HN | 134 | 34 | 124 | 573 | 34 | 122 | 1185 | 17 | 102 | 504 | 17 | 101 | 525 | 17 | 100 | 1246 | 17 | 103 |
| BT | 577 | 442 | 1253 | 5799 | 420 | 1119 | 7023 | 57 | 321 | 1410 | 56 | 310 | 1505 | 52 | 294 | 8076 | 64 | 363 |
| BP | 542 | 442 | 1153 | 5461 | 420 | 1119 | 7041 | 57 | 321 | 1413 | 56 | 310 | 1506 | 52 | 294 | 7642 | 57 | 322 |
| BM | 10024 | 59 | 311 | 1252 | 58 | 303 | 1351 | 53 | 287 | 1135 | 53 | 286 | 1211 | 52 | 280 | 1435 | 55 | 303 |
| BN | 13148 | 196 | 643 | 2902 | 193 | 635 | 4845 | 108 | 508 | 2444 | 107 | 504 | 2250 | 105 | 500 | 5076 | 114 | 518 |

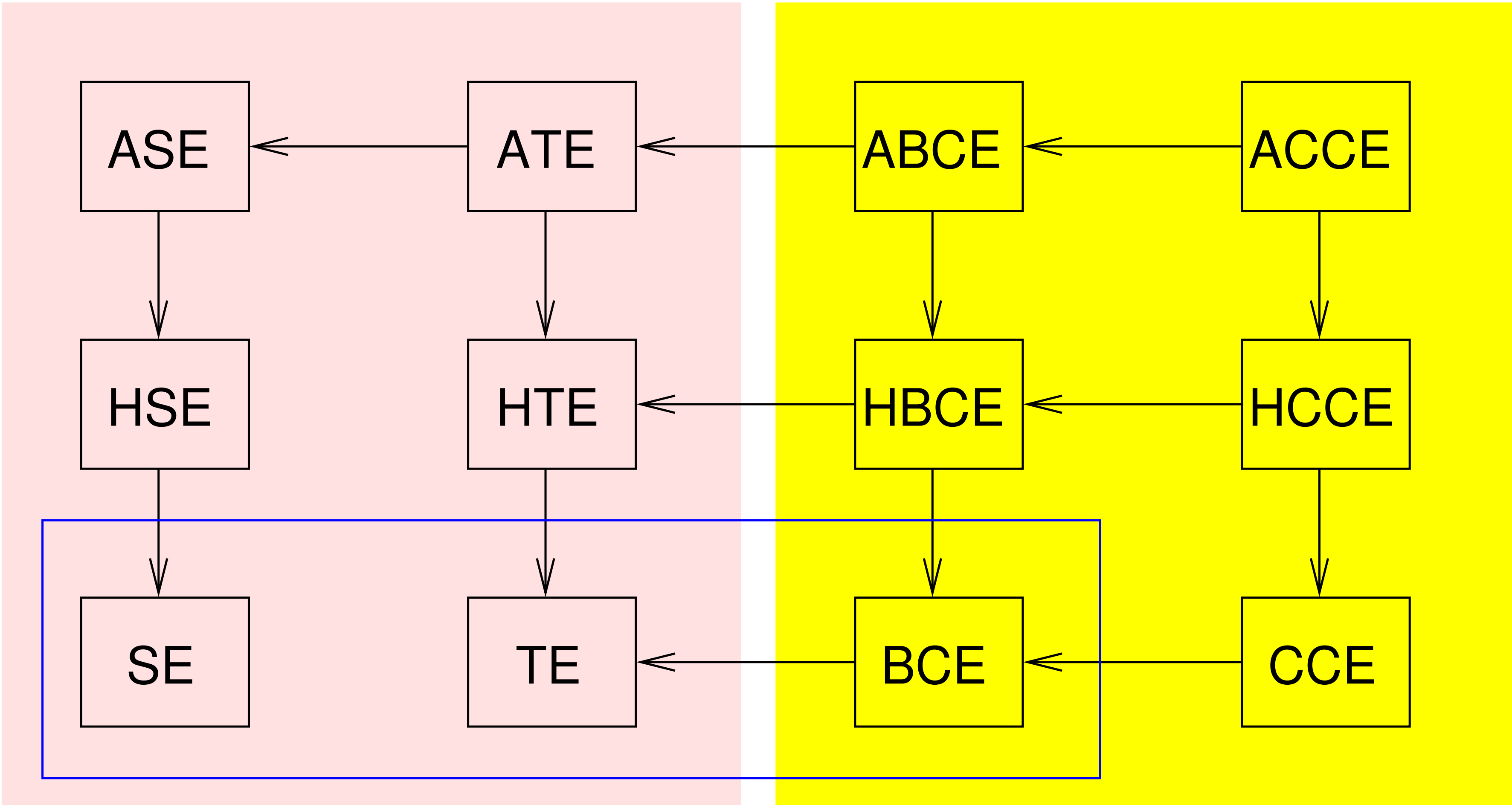
S = Sat competition
A = AIG competition
H = HW model checking competition
B = bit-vector SMT competition

T = plain Tseitin encoding
P = Plaisted Greenbaum
M = MiniCirc encoding
N = NiceDAGs

H = hidden, A = asymmetric,

SE = subsumption elimination, T = tautology elimination

BC = blocked clause elimination, CC = covered clause elimination




logically equivalent

satisfiability equivalent

**Definition** [self-subsuming resolution] [EénBiere SAT'05]

if $A \vee x$ and $B \vee \bar{x}$ are two clauses in a CNF and $\dfrac{A \vee x \quad \bar{x} \vee B}{A}$ a valid resolution, e.g. $B \subseteq A$, then replace the clause $A \vee x$ by $A$, in essence **removing** $x$ from $A \vee x$

example: if both $a \vee b \vee x$ and $b \vee \bar{x}$ are in a CNF remove $x$ from first clause

**Definition** [asymmetric literal addition] [HeuleJärvisaloBiere LPAR'10]

if $A$ and $B \vee \bar{x}$ are two clauses in a CNF and $\dfrac{A \vee x \quad \bar{x} \vee B}{A}$ a valid resolution, e.g. $B \subseteq A$, then replace the clause $A$ by $A \vee x$, in essence **adding** $x$ to $A$

example: if both $a \vee b$ and $b \vee \bar{x}$ are in a CNF add $x$ to first clause

# Asymmetric and Hidden Tautologies and Blocked Clauses



**Definition** [asymmetric tautology / blocked clause] [HeuleJärvisaloBiere LPAR'10]

apply asymmetric literal addition to a clause w.r.t. to fixed CNF as long as possible, if the result is a tautological / blocked then remove clause (otherwise keep original)

**Definition** hidden = only use binary side clauses $\bar{x} \vee B$

**Fact** AHTE can be simulated by asymmetric branching / distillation and BCP

**Fact** HTE can be implemented much faster by iterating over all literals instead of iterating over all clauses (partially implemented in Lingeling)

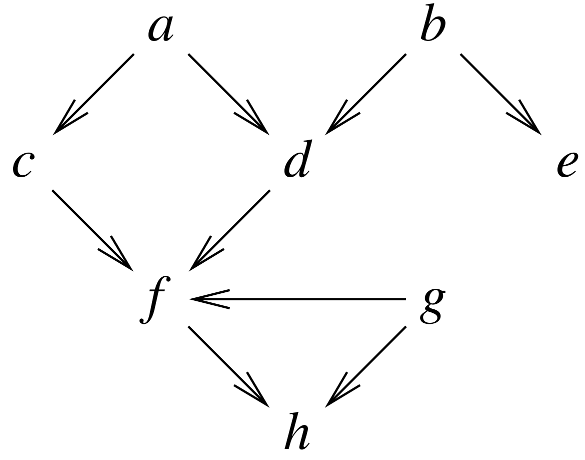
**Fact** HTE after equivalence reasoning and failed literal probing until completion on binary clauses only is confluent and BCP preserving

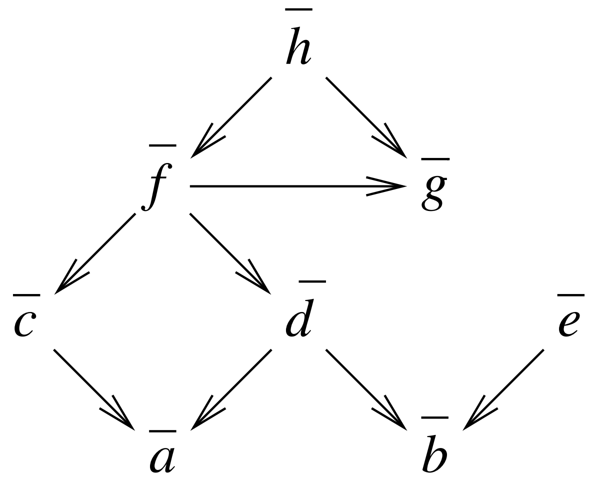
see our long and short LPAR'10 papers for more details

# Motivation for Unhiding

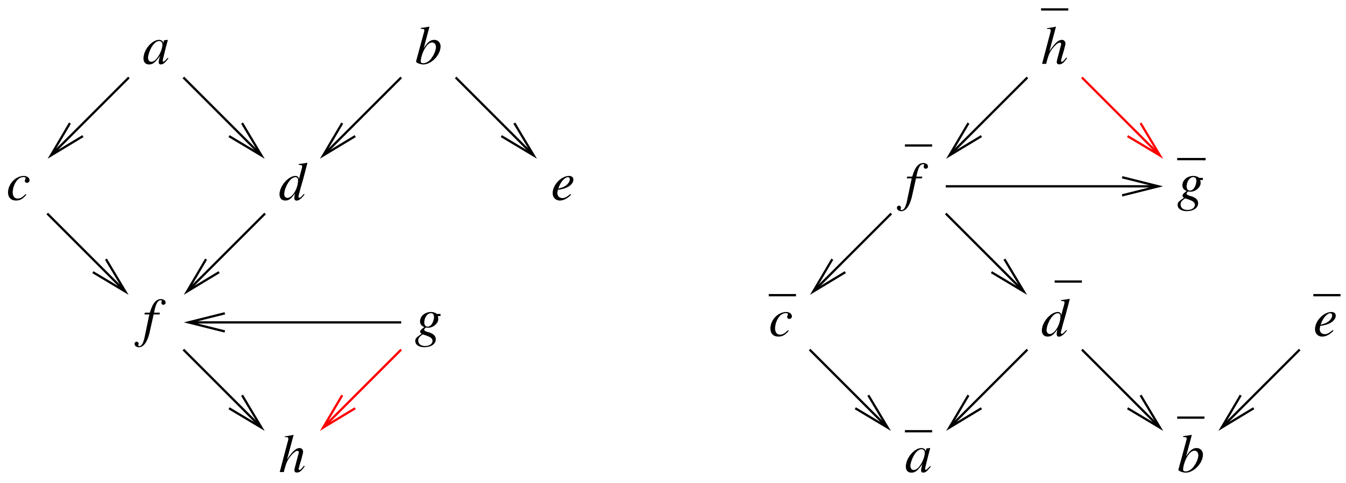


- SAT solvers applied to huge formulas
  - million of variables
  - fastests solvers use preprocessing/inprocessing
  - *need cheap and effective inprocessing techniques for millions of variables*
- this talk:
  - **unhiding** redundancy in large formulas
  - almost linear randomized algorithm
  - using the binary implication graph
  - fast enough to be applied to learned clauses
- see our SAT’11 paper for more details




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$
$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$
$$(\bar{g} \vee h) \wedge \underbrace{(\bar{a} \vee \bar{e} \vee h) \wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)}_{\text{non binary clauses}}$$




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$

$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$

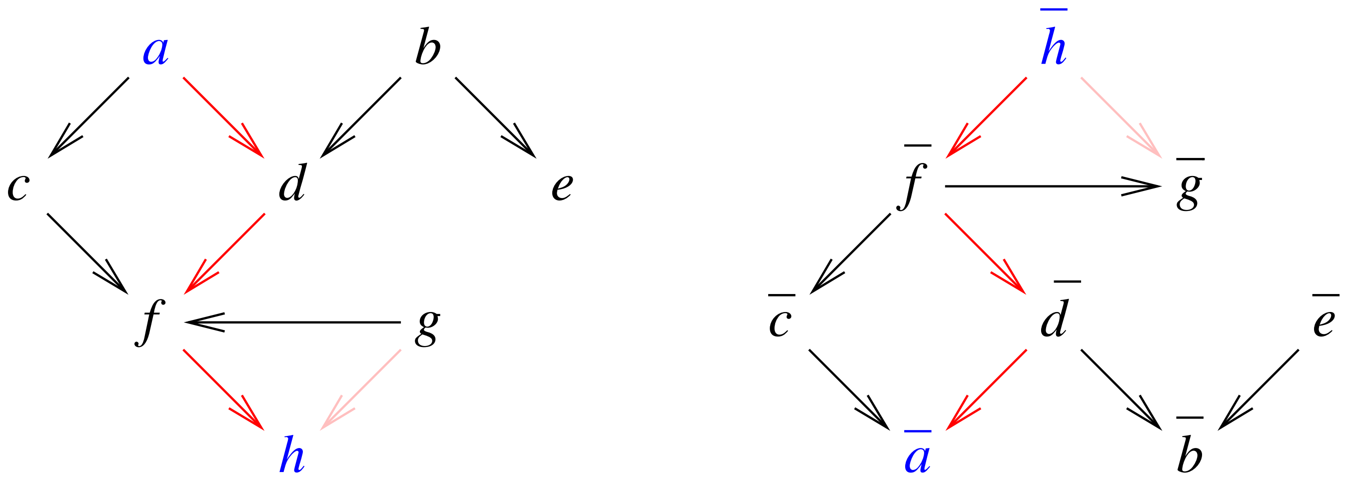
$$\cancel{(\bar{g} \vee h)} \wedge (\bar{a} \vee \bar{e} \vee h) \wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$$

TRD

$g \rightarrow f \rightarrow h$

# Hidden Tautology Elimination HTE

[HeuleJärvisaloBiere LPAR'2010]




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$

$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$

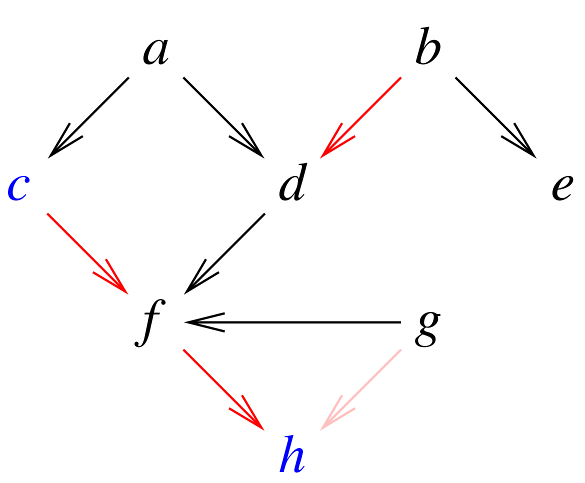
$$\cancel{(\bar{a} \vee \bar{e} \vee h)} \wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$$

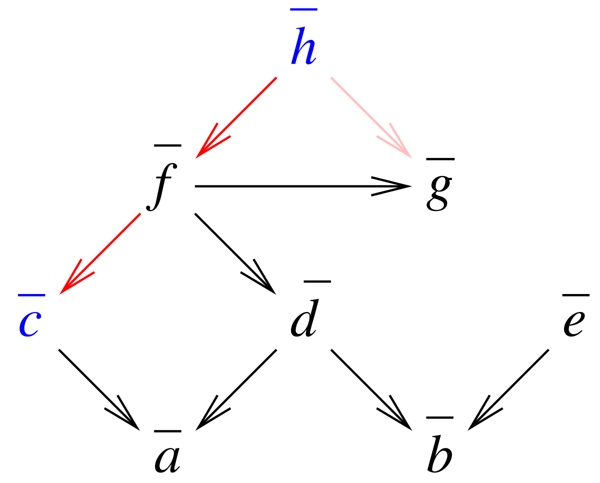
HTE

$a \to d \to f \to h$

# Hidden Tautology Elimination HTE

[HeuleJärvisaloBiere LPAR'2010]




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$

$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$

$$\cancel{(\bar{b} \vee \bar{c} \vee h)} \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$$

HTE

$c \rightarrow f \rightarrow h$

# Self-Subsuming Resolution SSR

[EénBiere-SAT'05]

$$\frac{C \vee l \qquad D \vee \bar{l}}{D} \quad C \subseteq D \qquad\qquad \frac{a \vee b \vee l \qquad a \vee b \vee c \vee \bar{l}}{a \vee b \vee c}$$

resolvent $D$ subsumes second antecedent $D \vee \bar{l}$

assume given CNF contains both antecedents $\ldots(a \vee b \vee l)(a \vee b \vee c \vee \bar{l})\cdots$

if $D$ is added to CNF then $D \vee \bar{l}$ can be removed $\Downarrow$

which in essence *removes* $\bar{l}$ from $D \vee \bar{l}$ $\ldots(a \vee b \vee l)(a \vee b \vee c \quad )\ldots$

used in SATeLite preprocessor

now common in many SAT solvers

[HeuleJärvisaloBiere LPAR'2010]

hidden literal addition (HLA) uses SSR in reverse order

$$\frac{C \vee l \qquad D \vee \bar{l}}{D} \quad C \subseteq D \qquad\qquad \frac{a \vee b \vee l \qquad a \vee b \vee c \vee \bar{l}}{a \vee b \vee c}$$

assume given CNF contains resolvent and first antecedent $\ldots(a \vee b \vee l)(a \vee b \vee c)\cdots$

we can replace $D$ by $D \vee \bar{l}$ $\ldots(a \vee b \vee l)(a \vee b \vee c \vee \bar{l})\ldots$

which in essence *adds* $\bar{l}$ to $D$, repeat HLA until fix-point

keep remaining non-tautological clauses *after removing added literals again*

HTE = assume $C \vee l$ is a binary clauses more general versions in the paper

**remove clauses with a literal implied by negation of another literal in the clause**

HTE confluent and BCP preserving modulo equivalent variable renaming

# Hidden Literal Elimination HLE

[HeuleJärvisaloBiere SAT'11]

better explained on binary implication graph

**remove literal from a clause which implies another literal in the clause**

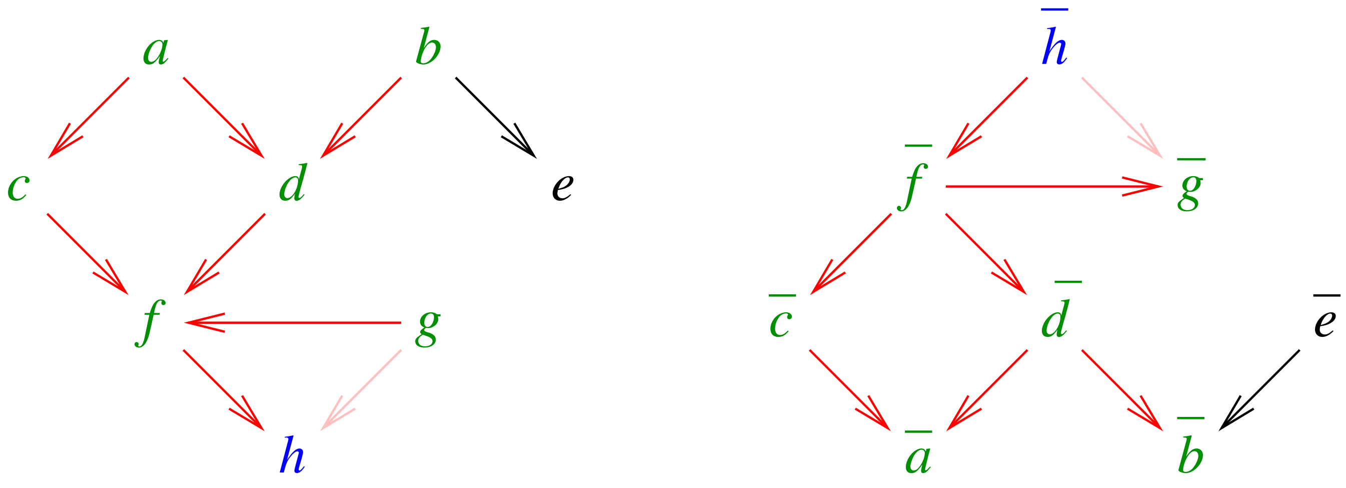
$$\ldots(\bar{a} \vee b)(\bar{b} \vee c)(a \vee c \vee d)\ldots \quad \Rightarrow \quad \ldots(\bar{a} \vee b)(\bar{b} \vee c)(c \vee d)\ldots$$

related work before all uses BCP:

- asymmetric branching — implemented in MiniSAT but switched off by default
- **distillation** — [JinSomenzi'05][HanSomenzi DAC'07]
- vivification — [PietteHamadiSais ECAI'08]
- caching technique in CryptoMiniSAT

HTE/HLE only uses the binary implication graph!

[HeuleJärvisaloBiere SAT'11]




$$(\bar{a} \lor c) \land (\bar{a} \lor d) \land (\bar{b} \lor d) \land (\bar{b} \lor e) \land$$

$$(\bar{c} \lor f) \land (\bar{d} \lor f) \land (\bar{g} \lor f) \land (\bar{f} \lor h) \land$$

$$(\cancel{a \lor b \lor c \lor d \lor} e \lor \cancel{f \lor g \lor} h)$$

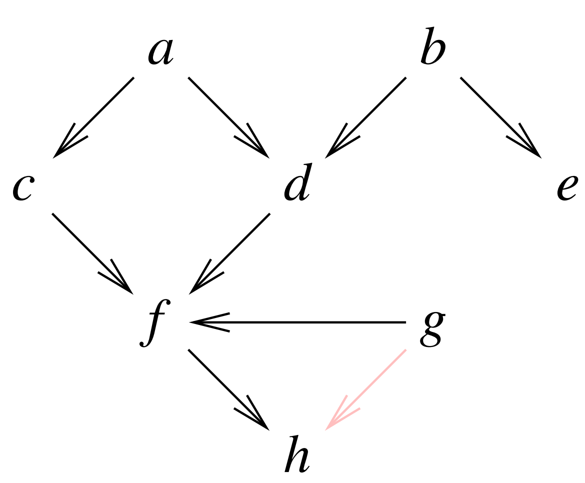
HLE

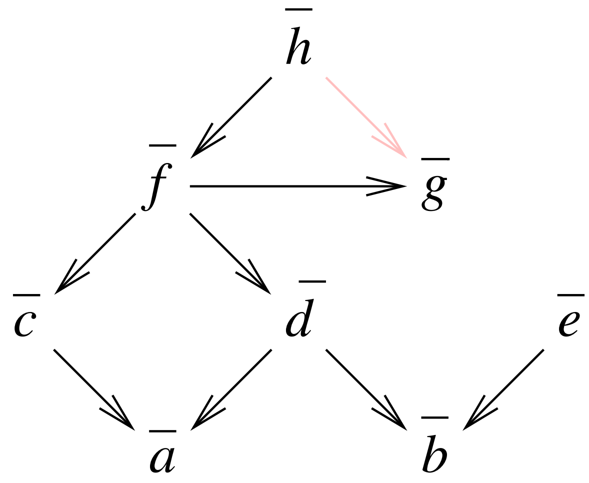
all but $e$ imply $h$

also $b$ implies $e$

# Hidden Literal Elimination HLE

[HeuleJärvisaloBiere SAT'11]



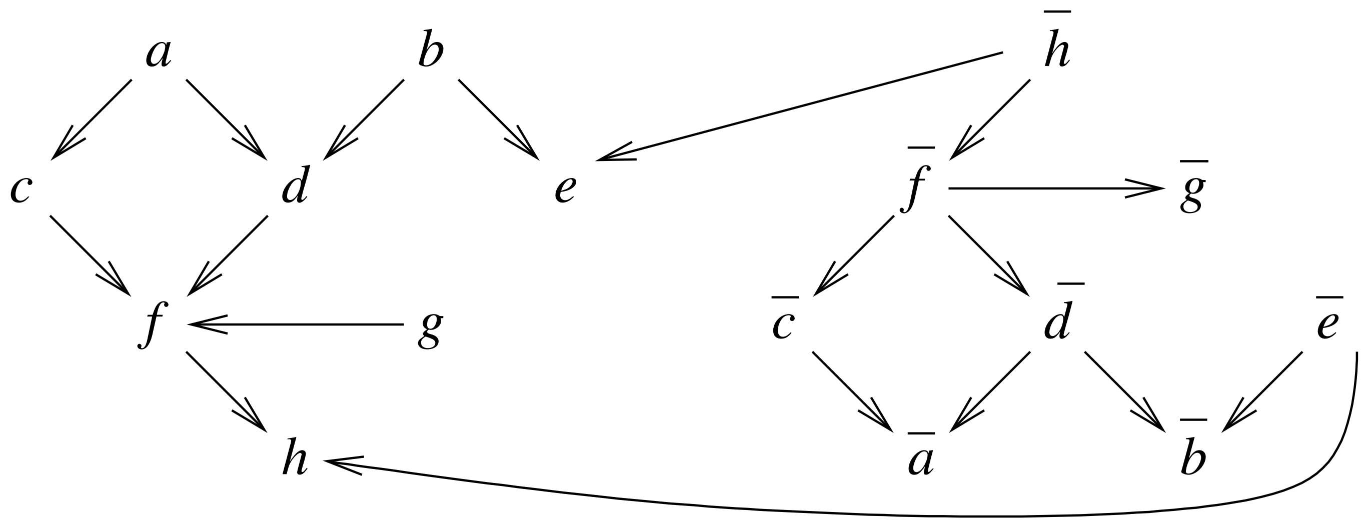


$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$

$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$

$$(\qquad\qquad e \vee \qquad h)$$

[HeuleJärvisaloBiere SAT'11]




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$
$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$
$$(e \vee h)$$

# Failed Literal Elimination FL



actually quite old technique . . . [Freeman PhdThesis'95] [LeBerre'01] . . .

$\boxed{\textit{assume} \text{ literal } l\text{, BCP, if conflict, add unit } \bar{l}}$

rather costly to run until completion conjecture: at least quadratic

one BCP is linear and also in practice can be quite expensive

need to do it for all variables and restart if new binary clause generated

useful in practice: lift common implied literals for assumption $l$ and assumption $\bar{l}$

**even on BIG (FL2) conjectured to be quadratic** [VanGelder'05]

$\ldots(\bar{a} \vee b)(\bar{b} \vee c)(\bar{c} \vee d)(\bar{d} \vee \bar{a})\ldots \quad \Rightarrow$ add unit clause $\bar{a}$

subsumed by running one HLA until completion

# Equivalent Literal Substitution ELS



[AspvallPlassTarjan'79] [Li'00] [Val'01] [Brafman'04] [VanGelder'05]

decompose BIG into strongly connect components (SCCs)

if there is an $l$ with $l$ and $\bar{l}$ in the same component $\Rightarrow$ *unsatisfiable*

otherwise replace all literals by a "representative"

**linear algorithm** can be applied routinely during garbage collection

but as with failed literal preprocessing may generate new binary clauses

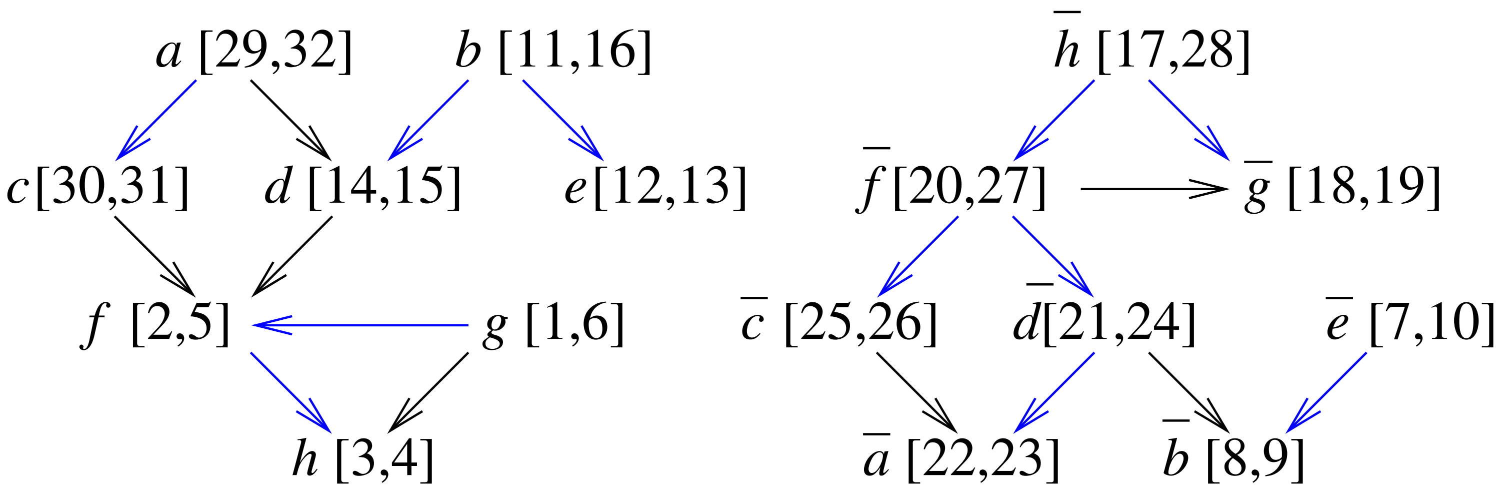
$$\ldots(\bar{a} \vee b)(\bar{b} \vee c)(\bar{c} \vee a)(a \vee b \vee c \vee d)\ldots \quad \Rightarrow \quad \ldots(a \vee d)\ldots$$

# Time Stamping



[HeuleJärvisaloBiere SAT'11]

DFS tree with discovered and finished times: $[\mathrm{dsc}(l), \mathrm{fin}(l)]$




tree edges

parenthesis theorem: $l$ ancestor in DFS tree of $k$ iff $[\mathrm{dsc}(k), \mathrm{fin}(k)] \subseteq [\mathrm{dsc}(l), \mathrm{fin}(l)]$

well known

ancestor relationship gives necessary conditions for (transitive) implication:

if $[\mathrm{dsc}(k), \mathrm{fin}(k)] \subseteq [\mathrm{dsc}(l), \mathrm{fin}(l)]$ then $l \to k$

if $[\mathrm{dsc}(\bar{l}), \mathrm{fin}(\bar{l})] \subseteq [\mathrm{dsc}(\bar{k}), \mathrm{fin}(\bar{k})]$ then $l \to k$

- time stamping in previous example does not cover $b \to h$
  $[11,16] = [\mathrm{dsc}(b), \mathrm{fin}(b)] \not\subseteq [\mathrm{dsc}(h), \mathrm{fin}(h)] = [3,4]$
  $[17,28] = [\mathrm{dsc}(\bar{h}), \mathrm{fin}(\bar{h})] \not\subseteq [\mathrm{dsc}(\bar{b}), \mathrm{fin}(\bar{b})] = [8,9]$

- in example still both HTE "unhidden", HLE works too (since $b \to e$)

- "coverage" heavily depends on DFS order

- as solution we propose multiple **randomized DFS** rounds/phases

- so we approximate a quadratic problem (reachability) randomly by a linear algorithm

- if BIG is a tree *one* time stamping covers everything

[HeuleJärvisaloBiere SAT'11]

*Unhiding* (formula $F$)

$stamp := 0$
**foreach** literal $l$ in BIG$(F)$ **do**
  $\mathrm{dsc}(l) := 0$; $\mathrm{fin}(l) := 0$
  $\mathrm{prt}(l) := l$; $\mathrm{root}(l) := l$
**foreach** $r \in \mathrm{RTS}(F)$ **do**
  $stamp :=$ *Stamp*$(r, stamp)$
**foreach** literal $l$ in BIG$(F)$ **do**
  **if** $\mathrm{dsc}(l) = 0$ **then**
    $stamp :=$ *Stamp*$(l, stamp)$
**return** *Simplify*$(F)$

*Stamp* (literal $l$, integer $stamp$)

$stamp := stamp + 1$
$\mathrm{dsc}(l) := stamp$
**foreach** $(\bar{l} \vee l') \in F_2$ **do**
  **if** $\mathrm{dsc}(l') = 0$ **then**
    $\mathrm{prt}(l') := l$
    $\mathrm{root}(l') := \mathrm{root}(l)$
    $stamp :=$ *Stamp*$(l', stamp)$
$stamp := stamp + 1$
$\mathrm{fin}(l) := stamp$
**return** $stamp$

*Simplify* (formula $F$)

**foreach** $C \in F$
  $F := F \setminus \{C\}$
  **if** *UHTE*$(C)$ **then continue**
  $F := F \cup \{$*UHLE*$(C)\}$
**return** $F$

# Unhiding HTE



[HeuleJärvisaloBiere SAT'11]

*UHTE* (clause $C$)

$l_{\text{pos}} :=$ first element in $S^{+}(C)$
$l_{\text{neg}} :=$ first element in $S^{-}(C)$
**while** *true*
  **if** $\text{dsc}(l_{\text{neg}}) > \text{dsc}(l_{\text{pos}})$ **then**
    **if** $l_{\text{pos}}$ is last element in $S^{+}(C)$ **then return** false
    $l_{\text{pos}} :=$ next element in $S^{+}(C)$
  **else if** $\text{fin}(l_{\text{neg}}) < \text{fin}(l_{\text{pos}})$ **or** $(|C| = 2$ **and** $(l_{\text{pos}} = \bar{l}_{\text{neg}}$ **or** $\text{prt}(l_{\text{pos}}) = l_{\text{neg}}))$ **then**
    **if** $l_{\text{neg}}$ is last element in $S^{-}(C)$ **then return** false
    $l_{\text{neg}} :=$ next element in $S^{-}(C)$
  **else return** true

$S^{+}(C)$ sequence of literals in $C$ ordered by $\text{dsc}()$
$S^{-}(C)$ sequence of negations of literals in $C$ ordered by $\text{dsc}()$

$$O(|C| \log |C|)$$

[HeuleJärvisaloBiere SAT'11]

*UHLE* (clause $C$)

```
finished := finish time of first element in S+_rev(C)
foreach l ∈ S+_rev(C) starting at second element
      if fin(l) > finished then C := C \ {l}
      else finished := fin(l)
finished := finish time of first element in S−(C)
foreach l̄ ∈ S−(C) starting at second element
      if fin(l̄) < finished then C := C \ {l}
      else finished := fin(l̄)
return C
```

$S^{+}_{\text{rev}}(C)$ reverse of $S^{+}(C)$

$$O(|C| \log |C|)$$

# Advanced Time Stamping

[HeuleJärvisaloBiere SAT'11]

*Stamp* (literal $l$, integer $stamp$)

BSC $\quad stamp := stamp + 1$

BSC $\quad \mathrm{dsc}(l) := stamp$; $\mathrm{obs}(l) := stamp$

ELS $\quad flag :=$ true // $l$ represents a SCC

ELS $\quad S$.push($l$) // push $l$ on SCC stack

BSC $\quad$ **for each** $(\bar{l} \vee l') \in F_2$

TRD $\quad\quad$ **if** $\mathrm{dsc}(l) < \mathrm{obs}(l')$ **then** $F := F \setminus \{(\bar{l} \vee l')\}$; **continue**

FLE $\quad\quad$ **if** $\mathrm{dsc}(\mathrm{root}(l)) \leq \mathrm{obs}(\bar{l}')$ **then**

FLE $\quad\quad\quad l_{\text{failed}} := l$

FLE $\quad\quad\quad$ **while** $\mathrm{dsc}(l_{\text{failed}}) > \mathrm{obs}(\bar{l}')$ **do** $l_{\text{failed}} := \mathrm{prt}(l_{\text{failed}})$

FLE $\quad\quad\quad F := F \cup \{(\bar{l}_{\text{failed}})\}$

FLE $\quad\quad\quad$ **if** $\mathrm{dsc}(\bar{l}') \neq 0$ **and** $\mathrm{fin}(\bar{l}') = 0$ **then continue**

BSC $\quad\quad$ **if** $\mathrm{dsc}(l') = 0$ **then**

BSC $\quad\quad\quad \mathrm{prt}(l') := l$

BSC $\quad\quad\quad \mathrm{root}(l') := \mathrm{root}(l)$

BSC $\quad\quad\quad stamp :=$ *Stamp*$(l', stamp)$

ELS $\quad\quad$ **if** $\mathrm{fin}(l') = 0$ **and** $\mathrm{dsc}(l') < \mathrm{dsc}(l)$ **then**

ELS $\quad\quad\quad \mathrm{dsc}(l) := \mathrm{dsc}(l')$; $flag :=$ false // $l$ is equivalent to $l'$

OBS $\quad\quad \mathrm{obs}(l') := stamp$ // set last observed time attribute

ELS $\quad$ **if** $flag =$ true **then** // if $l$ represents a SCC

BSC $\quad\quad stamp := stamp + 1$

ELS $\quad\quad$ **do**

ELS $\quad\quad\quad l' := S$.pop() // get equivalent literal

ELS $\quad\quad\quad \mathrm{dsc}(l') := \mathrm{dsc}(l)$ // assign equal discovered time

BSC $\quad\quad\quad \mathrm{fin}(l') := stamp$ // assign equal finished time

ELS $\quad\quad$ **while** $l' \neq l$

BSC $\quad$ **return** $stamp$

# Implementation

[HeuleJärvisaloBiere SAT'11]

- implemented as one inprocessing phase in our SAT solver Lingeling
  beside variable elimination, distillation, blocked clause elimination, probing, ...
- bursts of randomized DFS rounds and sweeping over the whole formula
- fast enough to be applicable to large learned clauses as well
  unhiding is particullary effective for learned clauses
- beside UHTE and UHLE we also have added hyper binary resolution UHBR
  not useful in practice

[HeuleJärvisaloBiere SAT'11]

| configuration | sol | sat | uns | unhd | simp | elim |
|---|---|---|---|---|---|---|
| adv.stamp (no uhbr) | 188 | 78 | 110 | 7.1% | 33.0% | 16.1% |
| adv.stamp (w/uhbr) | 184 | 75 | 109 | 7.6% | 32.8% | 15.8% |
| basic stamp (no uhbr) | 183 | 73 | 110 | 6.8% | 32.3% | 15.8% |
| basic stamp (w/uhbr) | 183 | 73 | 110 | 7.4% | 32.8% | 15.8% |
| no unhiding | 180 | 74 | 106 | 0.0% | 28.6% | 17.6% |

| configuration | hte | stamp | redundant | hle | redundant | units | stamp |
|---|---|---|---|---|---|---|---|
| adv.stamp (no uhbr) | 22 | 64% | 59% | 291 | 77.6% | 935 | 57% |
| adv.stamp (w/uhbr) | 26 | 67% | 70% | 278 | 77.9% | 941 | 58% |
| basic stamp (no uhbr) | 6 | 0% | 52% | 296 | 78.0% | 273 | 0% |
| basic stamp (w/uhbr) | 7 | 0% | 66% | 288 | 76.7% | 308 | 0% |
| no unhiding | 0 | 0% | 0% | 0 | 0.0% | 0 | 0% |

similar results for crafted and SAT'10 Race instances

- preprocessing is important for SAT solvers
- hard kernels do occur in practice
- inprocessing provides additional benefits
- new class of clause elimination procedures
- even quadratic algorithms are most of the time too expensive