Kernel for Domination
When the Stars Are Out

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Introduction

Afterwards: kernel
<table>
<thead>
<tr>
<th>Dominating Set</th>
<th>$K_{1, 1, 1}, K_{1, 2}$</th>
<th>$K_{1, 3}$</th>
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<tr>
<td>Trivial</td>
<td>NP-complete $9^k n^{O(1)}$ $O(k^4)$-kernel</td>
<td>NP-complete $36^k n^{O(1)}$ No poly kernel</td>
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Kernel

Afterwards: algorithm
Nice claw–free graphs

- should be claw–free
- should have $\alpha(G) > 3$
- should exclude certain structures
Nice claw-free graphs

What do you mean...nice?
Nice claw-free graphs

XX-graphs
Decomposition into strips

Proper interval graph

Proper circular-arc

Spot

...
Small or big
Small strips

- Induced part of strip graph has vertex cover of size 2k
  - Use Buss’ kernel
  - Number of strips is $O(k^4)$

Big strips

- Induce 2k vertices in strip graph
  - Number of strips is $4k^2$
Make big strips less big
Influence of boundary

Difference at most 1

No jump

Jump: (and reduce k)
What about tubes?

Case: unit interval graph
What about tubes?

Remaining cases
$O(k^4)$-vertex kernel

Next: algorithm
Want to guess which clique (vertex of strip graph) has vertex of optimal DS

**Big strips**
- Induce 2k vertices in strip graph

**Small strips**
- Induced part of strip graph has vertex cover of size 2k
$g^k \; n^{O(1)}$

Next: take-away
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Questions?

Thank you!