Parameterized Complexity of Vertex Deletion into Perfect Graph Classes

Pim van 't Hof
University of Bergen

joint work with

Pinar Heggernes		*University of Bergen*
Bart M. P. Jansen		*Utrecht University*
Stefan Kratsch		*Utrecht University*
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WorKer 2011
Vienna, Austria, September 2–4, 2011

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Let $\mathcal{F}$ be a class of graphs.

$\mathcal{F}$-Vertex Deletion

Input : A graph G and an integer k.
Question : Is there a set $S \subseteq V(G)$ with $|S| \leq k$ such that $G - S$ is a member of $\mathcal{F}$?

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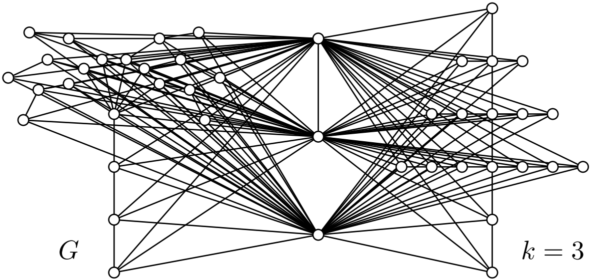

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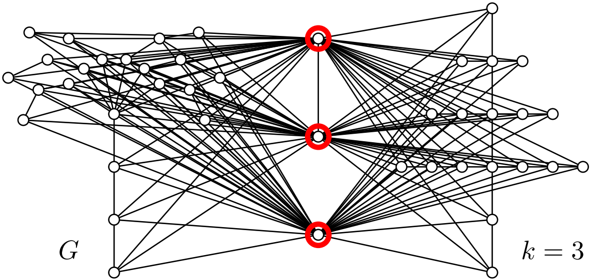

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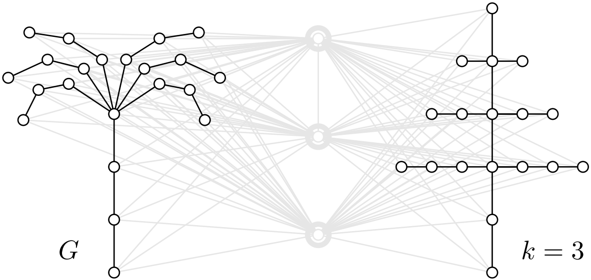

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$\mathcal{F}$	problem
edgeless	Vertex Cover
acyclic	Feedback Vertex Set
bipartite	Odd Cycle Transversal
planar	Planar Deletion
chordal	Chordal Deletion

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Theorem (Lewis & Yannakakis, 1980)

$\mathcal{F}$-DELETION *is NP-hard for every non-trivial, hereditary graph class $\mathcal{F}$.*

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When is $\mathcal{F}$-Deletion fixed-parameter tractable (FPT)?

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Theorem (Cai, 1996)

$\mathcal{F}$-DELETION is FPT for every graph class $\mathcal{F}$ that can be characterized by a finite set of forbidden induced subgraphs.

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$\mathcal{F}$-Deletion *is FPT for every graph class* $\mathcal{F}$ *that can be characterized by a finite set of forbidden induced subgraphs.*

Theorem (corollary of Robertson & Seymour, 1995, 2004)

$\mathcal{F}$-Deletion *is FPT for every minor-closed graph class* $\mathcal{F}$.

Is $\mathcal{F}$-DELETION FPT for every graph class $\mathcal{F}$ that is hereditary and can be recognized in polynomial time?

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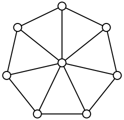
Theorem (Lokshtanov, 2008)

WHEEL-FREE DELETION *is W[2]-hard.*



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> *"...it would be interesting to see whether all of the "popular" graph classes, such as permutation graphs, AT-free graphs and perfect graphs, turn out to have fixed parameter tractable graph modification problems, or if some of these graph modification problems turn out to be hard for W[t] for some t."*

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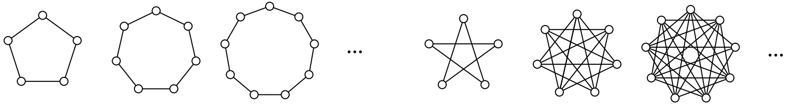
Strong Perfect Graph Theorem (Chudnovsky et al., 2006)

A graph is perfect if and only if it is (odd hole,odd antihole)-free.



Theorem

PERFECT DELETION *is W[2]-hard.*

Proof (sketch). "Hit" all odd holes and odd antiholes.

Theorem

PERFECT DELETION *is W[2]-hard.*

Proof (sketch). Reduction from HITTING SET (k).

HITTING SET (k)

Input : A set U, a family $\mathcal{H}$ of subsets of U, and an integer k.
Parameter : k.
Question : Is there a set $U' \subseteq U$ with $|U'| \leq k$ that contains a vertex from every set in $\mathcal{H}$?

Theorem (Downey & Fellows, 1999)

HITTING SET (k) *is W[2]-complete.*

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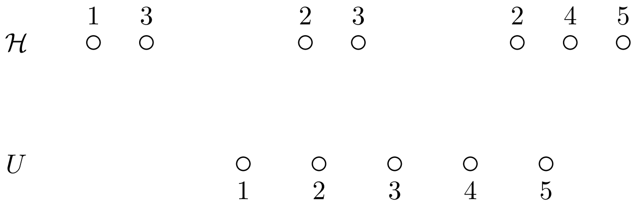
Given instance $(U, \mathcal{H}, k)$ of HITTING SET



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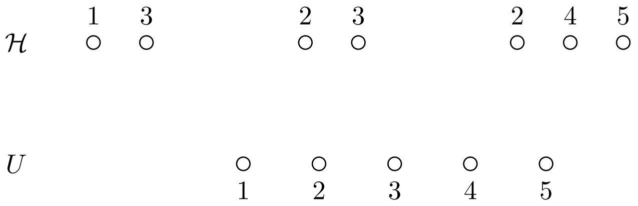

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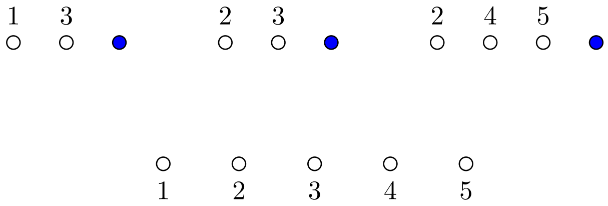

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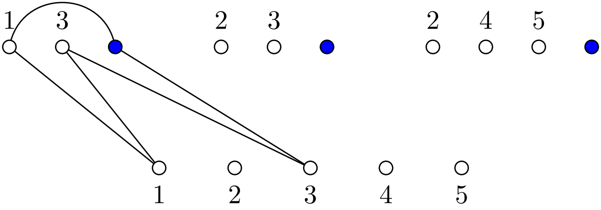

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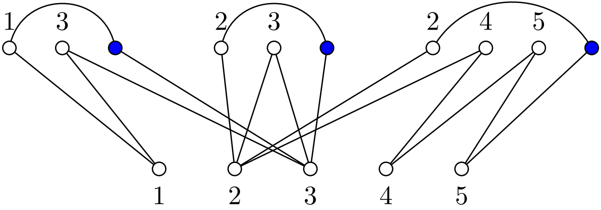

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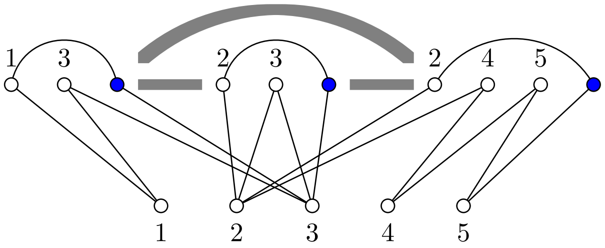

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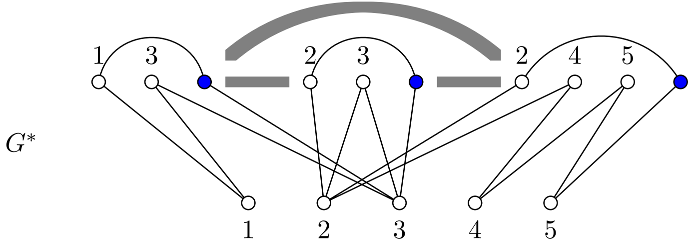

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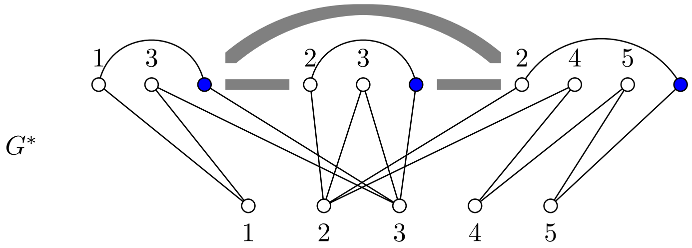

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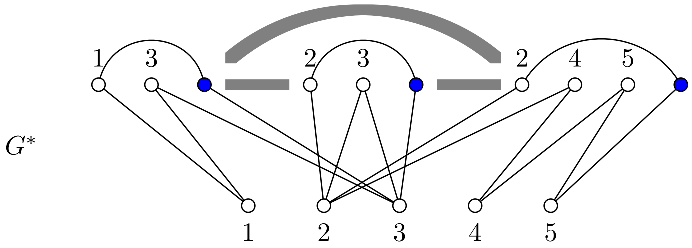

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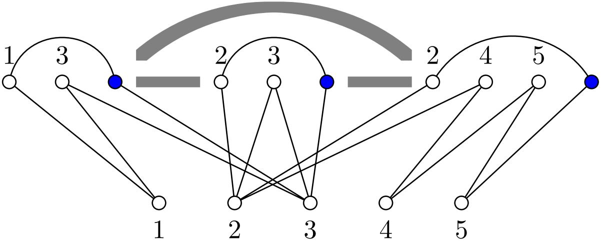

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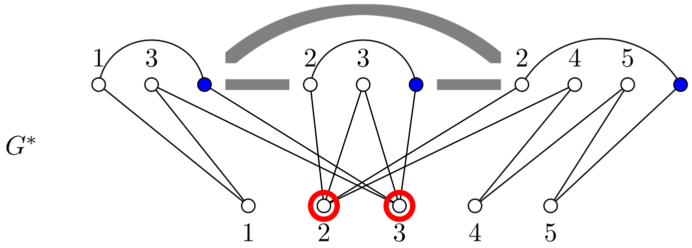

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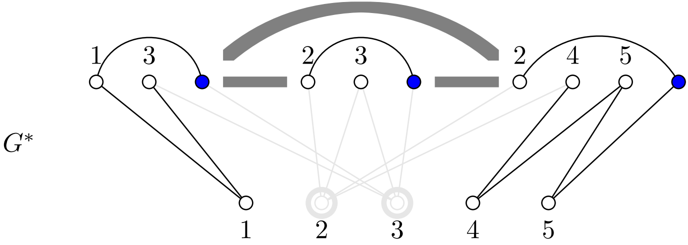

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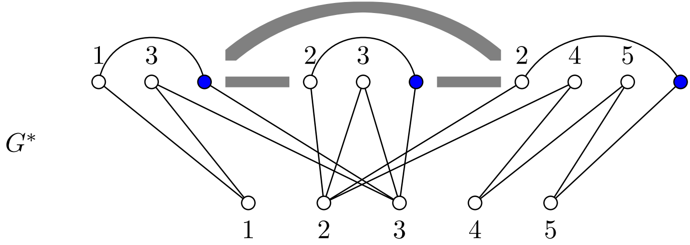

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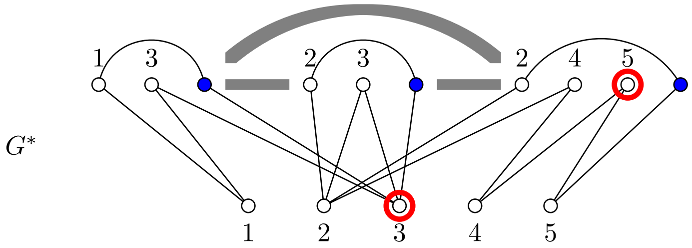

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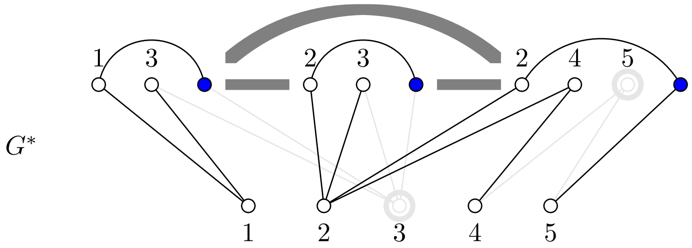

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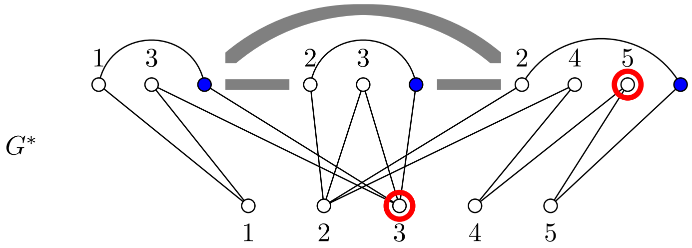

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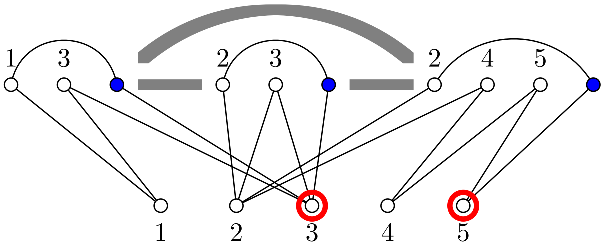

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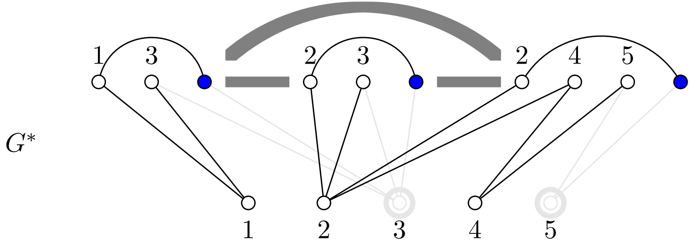

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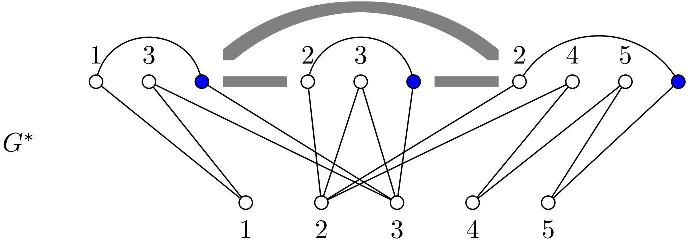

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chordal $\subset$ weakly chordal $\subset$ perfect

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weakly chordal	$\iff$	(hole,antihole)-free
chordal	$\iff$	(C_4,hole)-free

chordal	$\subset$	weakly chordal	$\subset$	perfect
FPT				W[2]-hard

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perfect	$\Longleftrightarrow$	(odd hole,odd antihole)-free
weakly chordal	$\Longleftrightarrow$	(hole,antihole)-free
chordal	$\Longleftrightarrow$	(C_4,hole)-free

chordal	$\subset$	weakly chordal	$\subset$	perfect
FPT		?		W[2]-hard

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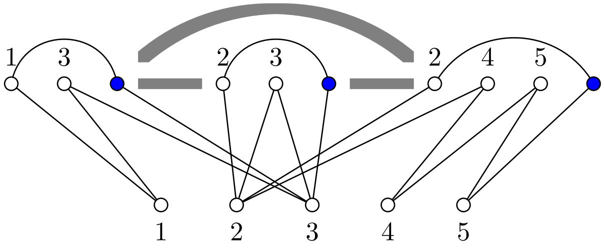
G^*



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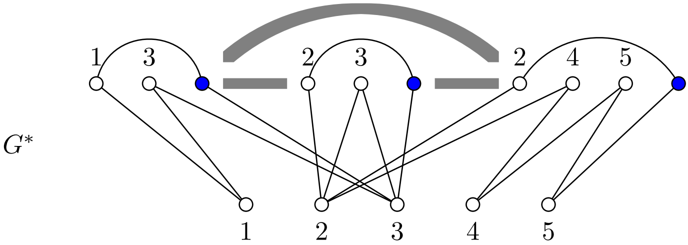

- Every hole or antihole in G^* is an odd hole.

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Corollary

WEAKLY CHORDAL DELETION *is W[2]-hard.*

$\mathcal{F}$	$\mathcal{F}$-Deletion is...
edgeless	FPT
acyclic	FPT
bipartite	FPT
chordal	FPT
planar	FPT
claw-free	FPT
cograph	FPT
split	FPT
outerplanar	FPT
bounded tw	FPT
wheel-free	W[2]-hard

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wheel-free	W[2]-hard
perfect	W[2]-hard
weakly chordal	W[2]-hard

Kernelization

Restricted $\mathcal{F}$-Deletion

Input : A graph G, a set $X \subseteq V(G)$ such that $G - X$ is in $\mathcal{F}$, and an integer k.

Parameter : $|X|$.

Question : Is there a set $S \subseteq X$ with $|S| \leq k$ such that $G - S$ is a member of $\mathcal{F}$?

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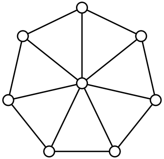
Example: $\mathcal{F}$ = class of forests, G is the graph below, $k = 2$.



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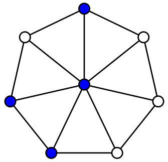

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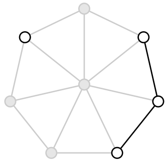

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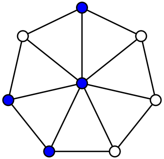

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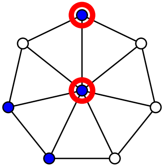
Parameter : $|X|$.

Question : Is there a set $S \subseteq X$ with $|S| \leq k$ such that $G - S$ is a member of $\mathcal{F}$?

Example: $\mathcal{F} =$ class of forests, G is the graph below, $k = 2$.



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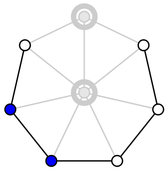

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Restricted $\mathcal{F}$-Deletion *is FPT for every graph class* $\mathcal{F}$ *that can be recognized in polynomial time.*

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What about polynomial kernels?

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weakly chordal	FPT	
perfect	FPT	

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Neither Restricted Perfect Deletion *nor* Restricted Weakly Chordal Deletion *admits a polynomial kernel, unless NP* $\subseteq$ *coNP/poly.*

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Hitting Set (k)

Input	:	A set U, a family $\mathcal{H}$ of subsets of U, and an integer k.
Parameter	:	k.
Question	:	Is there a set $U' \subseteq U$ with $\lvert U' \rvert \leq k$ that contains a vertex from every set in $\mathcal{H}$?

Theorem (Downey & Fellows, 1999)

Hitting Set (k) *is W[2]-complete.*

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Theorem (Dom, Lokshtanov & Saurabh, 2009)

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Restricted Chordal Deletion

Input	:	A graph G, a set $X \subseteq V(G)$ such that $G - X$ is chordal, and an integer k.
Parameter	:	$|X|$.
Question	:	Is there a set $S \subseteq X$ with $|S| \leq k$ such that $G - S$ is chordal?

Theorem

Restricted Chordal Deletion *admits a kernel with* $O(|X|^4)$ *vertices.*

Annotated Restricted Chordal Deletion

Input : A graph G, a set $X \subseteq V(G)$ such that $G - X$ is chordal, a set of critical pairs $C \subseteq \binom{X}{2}$, and an integer k.

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Question : Is there a set $S \subseteq X$ with $|S| \leq k$ such that $G - S$ is chordal, and S contains at least one vertex of each pair in C?

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Let (G, X, C, k) be an instance of ANNOTATED RESTRICTED CHORDAL DELETION. Let $F = G - X$; note that F is chordal.

Rule 1

If there is a vertex $x \in X$ such that $G[\{x\} \cup V(F)]$ is not chordal, then reduce to the instance $(G - \{x\}, X \setminus \{x\}, C', k)$, where C' is obtained from C by deleting all pairs which contain v.

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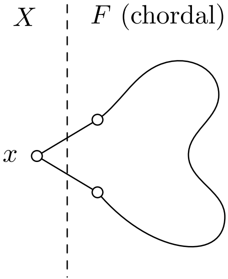

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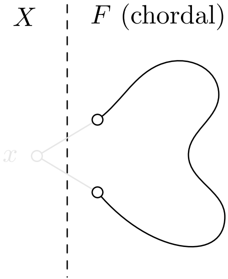

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Rule 2

If there are two vertices $x, y \in X$ with $\{x, y\} \notin C$ such that $G[\{x, y\} \cup V(F)]$ is not chordal, then reduce to the instance $(G, X, C \cup \{\{x, y\}\}, k)$.

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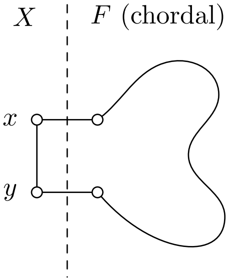

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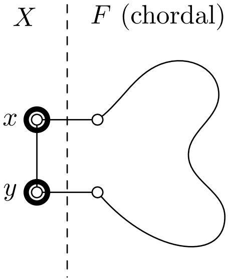

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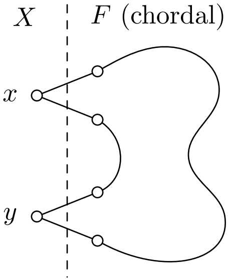

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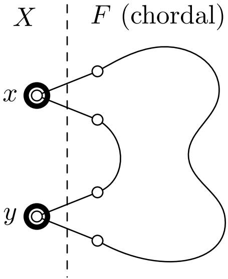

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If there is an edge $uv \in E(F)$ such that $N_G(u) \cap X = N_G(v) \cap X$, then reduce to the instance $(G/uv, X, C, k)$.

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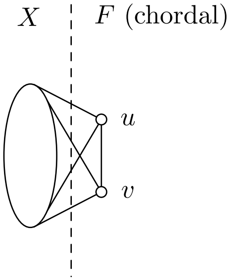

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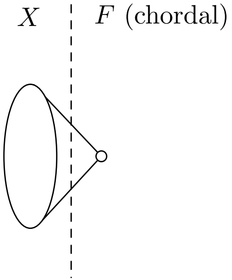

Lemma

If (G, X, C, k) is a reduced instance with respect to Rules 1–3, and P is an induced path in F, then P contains at most $2|X| + 1$ vertices.

Proof (sketch). Let $P = p_1 \cdots p_t$ be an induced path in F.

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Hence P has at most $2|X|$ edges, and $2|X| + 1$ vertices.

Let (G, X, C, k) be an instance of ANNOTATED RESTRICTED CHORDAL DELETION. Let $F = G - X$; note that F is chordal.

Rule 4

Repeat the following for each ordered triple (x, y, z) of distinct vertices in X: if there is an induced path P between x and z whose internal vertices are all in $F - N_G(y)$, then mark all the internal vertices of P. Let Y be the set of vertices that were not marked during this procedure. Reduce to the instance $(G - Y, X, C, k)$.

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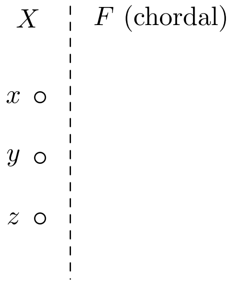

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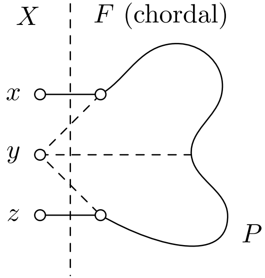

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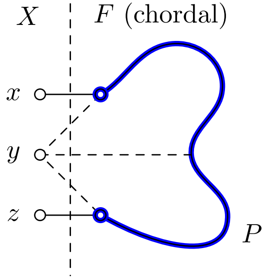

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Claim

Rule 4 is safe.

Let (G, X, C, k) be an instance of ANNOTATED RESTRICTED CHORDAL DELETION. Let $F = G - X$; note that F is chordal.

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Suppose (G, X, C, k) is a yes-instance, with solution S.

- Since $G - S$ is chordal, $G - Y - S$ is chordal.

Hence $(G - Y, X, C, k)$ is a yes-instance.

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Suppose $(G - Y, X, C, k)$ is a yes-instance, with solution S.

- $G - Y - S$ is chordal, and S intersects each pair in C.

Claim: S is a solution for (G, X, C, k).

Suppose, for contradiction, that S is *not* a solution for (G, X, C, k).

Theorem

ANNOTATED RESTRICTED CHORDAL DELETION *admits a kernel with* $O(|X|^4)$ *vertices.*

Proof (sketch). Let (G, X, C, k) be a reduced instance with respect to Rules 1–4. Let $F = G - X$.

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Since $V(G) = V(F) \cup X$, the result follows.

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- Let (G', X', C, k') be the obtained instance.

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G' has $O(|X|^4)$ vertices.

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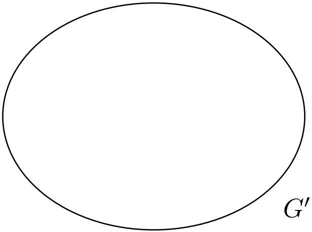

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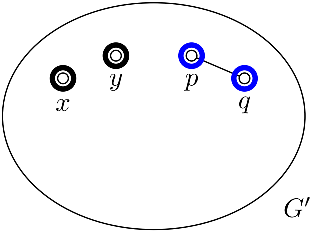

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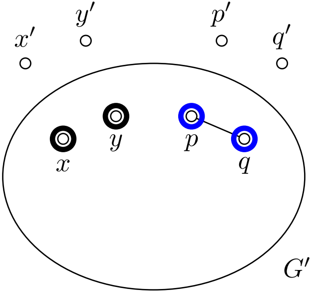

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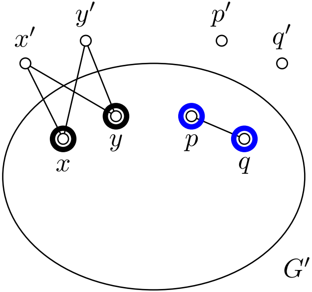

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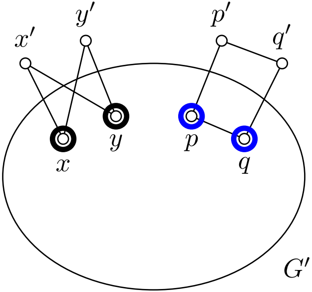

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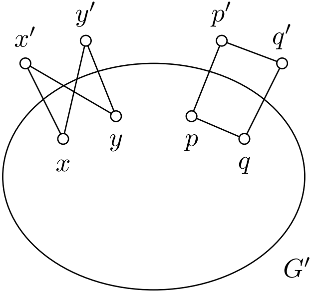

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Dank u wel!

Danke!

Takk!