Introduction to Knowledge-based Systems

Exercise Sheet 3: Nonmonotonic Reasoning

Exercise 1: **(1 point)**

A propositional theory, T, consists of the following formulas:

$$a \wedge (b \vee c), \qquad \neg a \vee ((b \vee c) \rightarrow d), \qquad (a \wedge d) \rightarrow e$$

where a, b, c, d, and e are atomic formulas.

1. Determine the set $T_{\text{asm}} = \{\neg P \mid P \text{ is an atomic formula such that } T \nvdash P\}$.
2. Determine $\text{CWA}(T)$.
3. Is $\text{CWA}(T)$ consistent? Justify your answer.

Exercise 2: **(1 point)**

Let T be a consistent propositional theory. Show that $\text{CWA}(T)$ is inconsistent iff there are atomic formulas $a_1, \ldots, a_n$ such that $T \models a_1 \vee \ldots \vee a_n$ and $T \not\models a_i$ for all $i = 1, \ldots, n$.

Hint: to show the only-if direction (left to right implication) you might first prove that if $\text{CWA}(T)$ is inconsistent, then T_{asm} is non-empty. Further on, you might find it useful to consider the propositional variables appearing in T_{asm} and to use the Contradiction Theorem.

Exercise 3: **(1 point)**

Let $T = (W, \Delta)$ be a default theory, where $W = \{a \wedge (c \rightarrow b)\}$, $\Delta = \{\frac{a:\neg b}{c}, \frac{a:\neg c}{b}\}$, and a, b, and c are propositional atoms. Determine the defaults applicable to $Cn(W)$ relative to E, as well as the classical reduct Δ_E for

a) $E = Cn(\{a \wedge (c \rightarrow b), b\})$,
b) $E = Cn(\{a \wedge (c \rightarrow b), b, c\})$,
c) $E = Cn(\{a \wedge (c \rightarrow b)\})$.

Exercise 4: **(1 point)**

Let $T = (W, \Delta)$ be a default theory, where $W = \emptyset$, $\Delta = \{\frac{:\neg b \vee \neg c}{a}, \frac{a:a \wedge \neg c}{b}, \frac{a:a, \neg b}{c}\}$, and a, b, and c are propositional atoms. Determine the defaults applicable to $Cn(W)$ relative to E, as well as the classical reduct Δ_E for

a) $E = Cn(\{a\})$,
b) $E = Cn(\{a, b\})$,
c) $E = Cn(\emptyset)$.

Exercise 5: **(1 point)**
Let $T = (W, \Delta)$ as given in Exercise 3. Determine $\Gamma_T(E)$ for

a) $E = Cn(\{a \wedge (c \rightarrow b), b\})$,
b) $E = Cn(\{a \wedge (c \rightarrow b), b, c\})$,
c) $E = Cn(\{a \wedge (c \rightarrow b)\})$.

Exercise 6: **(1 point)**
Let $T = (W, \Delta)$ as given in Exercise 4. Determine $\Gamma_T(E)$ for

a) $E = Cn(\{a\})$,
b) $E = Cn(\{a, b\})$,
c) $E = Cn(\emptyset)$.

Exercise 7: **(1 point)**
Determine the extensions of the default theory $T = (W, \Delta)$

a) given in Exercise 3,
b) given in Exercise 4.

Exercise 8: **(2 points)**
You have the following information:

1. When there is seafood for dinner, Charles usually prefers white wine.
2. If there is seafood paella for dinner, then Charles does not prefer white wine.
3. There is seafood for dinner.

a) Formalise this situation in terms of a default theory. Use $Seafood$ for "there is seafood for dinner", $White$ for "Charles prefers white wine" and $SeafoodPaella$ for "there is seafood paella for dinner". What are the extensions of the formalised theory?

b) Add to the theory the following new information:

4. There is seafood paella for dinner.

What are the extensions now?

Exercise 9: **(2 points)**
You have the following information:

1. When there is seafood for dinner, Charles usually prefers white wine.
2. When there is paella for dinner, Charles usually does not prefer white wine.
3. There is seafood and paella for dinner.

a) Formalise the situation in terms of a default theory, using the same propositional symbols as above plus $Paella$ for "there is paella for dinner". Does this theory have extensions?

b) Modify your default theory replacing 1 by 1':
1'. When there is seafood for dinner, Charles usually prefers white wine, unless there is paella.

What can you say about the extensions of the theory now?

Exercise 10: **(2 points)**

You want to create a (simple) rule-based controller for a steam engine. The engine is equipped with three signal gauges which provide the current temperature of the engine, the pressure inside the engine, the status of the valve, which can be open or closed. You can also send signals to the engine: you can close or open the valve, and you can shut down the engine.

Your knowledge about running the engine is the following. A temperature higher than 105 is dangerously high, and the pressure higher than 7 is dangerously high. We must open the valve in case it is closed and the pressure or temperature are dangerously high. If the temperature or the pressure are dangerously high and the valve is open, then the engine is overheating. If the valve is closed and the pressure is lower than 2, then there is a leak. The engine must be shut down if either there is a leak or it is overheating.

1. Implement a rule-based system for ensuring safety of the engine, based on the knowledge above (pseudo-code rules with detailed preconditions and an explanation of the action is sufficient). Use integer variables for the temperature and pressure, use function calls to send signals. Use a global element `status` which describes the current state of the system, i.e., the values of all the variables. We give here an example of a possible rule (it is not part of the solution!):

   `example-rule`:
   when
   p: status($Temp > 1000$, $Press > 20$, $ValveOpen = true$)
   then
   change p to $p.imminentExplosion = true$, $p.evacuate()$

2. Consider three situations:

   (a) The pressure in the engine is 1.6, and the valve is closed.

   (b) The temperature of the engine is 110, and the valve is closed.

   (c) The pressure in the engine is 6, the temperature is 115 and the valve is open.

   For all three situations, initialise the `status`, and then apply the rules from 1. to obtain the final result. Hint: You can use a table.