Introduction to Knowledge-based Systems
Exercise Sheet 1: Search and CSP

Exercise 1 (2 pts):
Let $b > 1$ be the maximal branching degree in the search tree and let d be its depth. Estimate the number of nodes, $n_{bfs}(d)$, generated during a bfs[1] with depth d. Show that $n_{bfs}(d) = O(b^d)$ and estimate the constant c_{bfs}.

Exercise 2 (2 pts):
Let $b > 1$ be the maximal branching degree in the search tree and let d be its depth. Estimate the number of nodes, $n_{dfid}(d)$, generated during a dfid with depth d. Show that $n_{dfid}(d)$ is $O(b^d)$ and estimate the constant c_{dfid}. What can you say about the overhead induced by dfid?

Exercise 3 (1 pt):
Consider dfid again. Analyze the behavior of

$$\frac{1}{(1-\frac{1}{b})^2}$$

if the branching factor b increases (you may draw a curve!). For which kind of problems is the overhead induced by dfid low? Compare it to the behaviour of

$$\frac{1}{(1-\frac{1}{b})}.$$

Exercise 4 (1 pt):
Give an example that A^* on graphs (with admissible heuristics) is not optimal. The pseudo code of A^* can be found at `http://en.wikipedia.org/wiki/A*_search_algorithm`

Exercise 5 (2 pts):
Show the following statement: Consistent heuristics are admissible.

Exercise 6 (2 pts):
Prove that the set of nodes expanded by A^* is a subset of the set of nodes expanded by ucs (operators have the same cost).

Exercise 7 (2 pts):
Let $f(n) = c_g g(n) + c_h h(n)$ be an evaluation function, where c_g, c_h be constants.

a. Define $c_g, c_h, h(\cdot), g(\cdot)$ such that A^* with this evaluation function is bfs.

b. Define $c_g, c_h, h(\cdot), g(\cdot)$ such that A^* with this evaluation function is dfs.

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[1]bfs: breadth-first search; dfs: depth-first search; dfid: depth-first iterative deepening; ucs: uniform cost search

Exercise 8 (3 pts):
An evaluation function $\tilde{f}$ is called *monotone* if for all nodes n and n', n' successor of n,

$$\tilde{f}(n) \quad \leq \quad \tilde{f}(n')$$

holds. Show that the function

$$\tilde{f}(n) = \begin{cases} f(n) & \text{if } n \text{ is the start node,} \\ \max\{f(n), \tilde{f}(m)\} & \text{if } n \text{ is the successor of } m \end{cases}$$

is (i) monotone, (ii) the corresponding heuristic $\tilde{h}$ is admissible and (iii) $\tilde{h}$ dominates the admissible heuristic h.

Exercise 9:
In four houses, each with a different colour, live four persons of different nationalities, each of whom grows a different kind of plants and prefers a different food. Given the following facts, the goal is to find out which house is yellow, where the Spanish person lives, who eats cheese and who grows roses.

1. The Italian grows cactuses.
2. The orchids grow in front of house three.
3. The person who grows the orchids likes rice.
4. The Norwegian does not live in house four.
5. The third house is pink.
6. The German lives directly next to the person who eats steaks.
7. The person eating pancakes lives directly to the right of the gray house.
8. The dahlias grow directly to the right of the pink house.
9. The person eating steaks lives directly next to the blue house.

Formulate this problem as a CSP **(1 pt)** and give a solution **(1 pt)**.