

Generating Shortest Synchronizing Sequences using Answer Set Programming

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Abstract. For a finite state automaton, a synchronizing sequence is an input sequence that takes all the states to the same state. Checking the existence of a synchronizing sequence and finding a synchronizing sequence, if one exists, can be performed in polynomial time. However, the problem of finding a shortest synchronizing sequence is known to be NP-hard. In this work, the usefulness of Answer Set Programming to solve this optimization problem is investigated, in comparison with brute-force algorithms and SAT-based approaches.

Keywords: finite automata, shortest synchronizing sequence, ASP

1 Introduction

For a state based system that reacts to events from its environment by changing its state, a synchronizing sequence is a specific sequence of events that brings the system to a particular state no matter where the system initially is. Synchronizing sequences have found applications in many practical settings. In model based testing, it can be used to bring the unknown initial state of an implementation to a specific state to start testing [1, 2]. As Natarajan [3] and Eppstein [4] explain, it can be used to orient a part to a certain position on a conveyor belt. Another interesting application is from biocomputing, where one can use a DNA molecule encoding a synchronizing sequence to bring huge number of identical automata (in the order of 10^{12} automata/ μ l) to a certain restart state [5].

Such a state based system can be formalized as a *finite state automaton* (FA). We restrict ourselves to *deterministic* FA, which is defined as a tuple $A = (Q, \Sigma, \delta)$, where Q is a finite set of states, Σ is a finite input alphabet, and $\delta : Q \times \Sigma \mapsto Q$ is a transition function, defining how each state of A is changed by the application of inputs. The transition function δ is extended to words in Σ^* naturally as $\delta(q, \varepsilon) = q$, $\delta(q, wx) = \delta(\delta(q, w), x)$, where $q \in Q$, $w \in \Sigma^*$, $x \in \Sigma$, and ε is the empty word. A FA $A = (Q, \Sigma, \delta)$ is called *completely specified* when δ is a total function. We will only consider completely specified FA in this work. Figure 1 is an example of a FA.

We can now define a synchronizing sequence formally. Given an FA $A = (Q, \Sigma, \delta)$, an input sequence $w \in \Sigma^*$ is called a *synchronizing sequence* for A if $\forall q, q' \in Q$, $\delta(q, w) = \delta(q', w)$. As an example, *baab* is a synchronizing sequence for A_1 given in Figure 1.

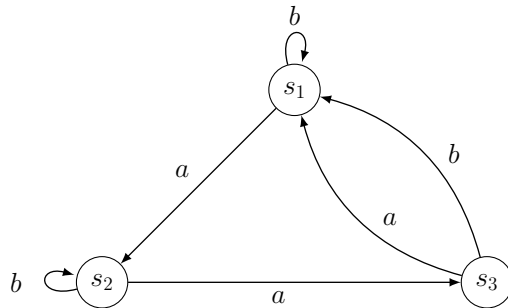


Fig. 1: An example FA A_1

Synchronizing sequences attracted much attention from a theoretical point of view as well. In the literature, a synchronizing sequence is also referred to as a synchronizing word, reset sequence, or a reset word. Not every FA has a synchronizing sequence, and one can check the existence of a synchronizing sequence for a given FA in polynomial time. On the other hand, the problem of finding a shortest synchronizing sequence is known to be NP-hard [4]. For this reason, several heuristic approaches have been suggested to compute short synchronizing sequences [4, 6–8]. These algorithms guarantee a synchronizing sequence of length $O(n^3)$ where n is the number of states in the FA. The best known upper bound is $n(7n^2 + 6n - 16)/48$ [9]. However, it has been conjectured by Černý almost half a century ago that this upper bound is $(n-1)^2$ [10, 11] after providing a class of FA with n states whose shortest synchronizing sequence is of length $(n-1)^2$. The conjecture is shown to hold for certain classes of automata [4, 5, 12–15]. However, the conjecture is still open in general, and it is one of the oldest open problems of finite state automata theory.

Despite the fact that it is NP-hard, considering the computation of shortest synchronizing sequences is still useful. Such attempts are valuable both for understanding the characteristics of shortest synchronizing sequence problem (see e.g. [16]) and for forming a base line for the performance evaluation of heuristics for computing short synchronizing sequences.

In this work, we formulate the problem of computing a shortest synchronizing sequence in Answer Set Programming (ASP) [17, 18]—a knowledge representation and reasoning paradigm with an expressive formalism and efficient solvers. The idea of ASP is to formalize a given problem as a “program” and to solve the problem by computing models (called “answer sets” [19]) of the program using “ASP solvers”, such as CLASP [20].

After we represent the shortest synchronizing sequence problem in ASP, we experimentally evaluate the performance and effectiveness of ASP, in comparison with two other approaches, one based on SAT [16] and the other on a brute-force algorithm [21]. For our experiments with the SAT-based approach, we extend the SAT formulation of the existence of a synchronizing sequence of a given length [16], to FA with more than two input symbols.

The rest of the paper is organized as follows. In Section 2 we present four different ASP formulations for the problem. An existing SAT formulation [16] is extended to FAs with more than two inputs in Section 3. The experimental results are given in Section 4 to compare the approaches. Finally, in Section 5 we give concluding remarks and some future research directions.

2 ASP Formulations of the Shortest Synchronizing Sequence Problem

Let us first consider the decision version of the shortest synchronizing sequence problem: For an FA $A = (Q, \Sigma, \delta)$ and a positive integer constant c , decide whether A has a synchronizing word w of length c .

Without loss of generality, we represent states and input symbols of an FA $A = (Q, \Sigma, \delta)$, by the range of numbers $1..n$ and $1..k$ ($n = |Q|$, $k = |\Sigma|$), respectively. Then an FA $A = (Q, \Sigma, \delta)$ can be described in ASP by three forms of atoms given below:

- $state(s)$ ($1 \leq s \leq n$) describing the states in Q ,
- $symbol(j)$ ($1 \leq j \leq k$) describing the input symbols in Σ , and
- $transition(s, j, s')$ ($1 \leq s, s' \leq n$, $1 \leq j \leq k$) describing the transitions $\delta(s, j) = s'$.

We represent possible lengths i of sequences by atoms of the form $step(i)$ ($1 \leq i \leq c$).

A synchronizing sequence of length c is characterized by atoms of the form $synchro(i, x)$ ($1 \leq i \leq c$, $1 \leq x \leq k$) describing that the i 'th symbol of the word is x .

Using these atoms, we can represent the decision version of the shortest synchronizing sequence problem with a “generate-and-test” methodology used in various ASP formulations. In the following, we present two different ASP formulations based on this approach.

In these ASP formulations, we use an auxiliary concept of a *path in A characterized by a sequence w_1, w_2, \dots, w_x of symbols in Σ* , which is defined as a sequence q_1, q_2, \dots, q_{x+1} of states in Q such that $\delta(q_i, w_i) = q_{i+1}$ for every i ($1 \leq i \leq x$). The existence of such a path of length i in A from a state s to a state q (i.e., the reachability of a state q from a state s by a path of length i in A) characterized by the first i symbols of a word w is represented by atoms of the form $path(s, i + 1, q)$ defined as follows:

$$\begin{aligned} path(s, 1, s) &\leftarrow state(s) \\ path(s, i + 1, q) &\leftarrow path(s, i, r), synchro(i, x), transition(r, x, q), \\ &state(s), state(r), state(q), symbol(x), step(i) \end{aligned} \quad (1)$$

2.1 Connecting All States to a Sink State

In the first ASP formulation, which we call ASP_1 , first we “generate” a sequence w of c symbols by the following choice rule:

$$1\{synchro(i, j) : symbol(j)\}1 \leftarrow step(i) \quad (2)$$

where $step(i)$ is defined by a set of facts:

$$step(i) \leftarrow (1 \leq i \leq c) \quad (3)$$

Next, we ensure that it is a synchronizing sequence by “testing” that it does not violate the condition:

C_1 There exists a sink state $f \in Q$ such that every path in A characterized by w ends at f .

by adding the following constraints

$$\leftarrow sink(f), not\ path(s, c + 1, f), state(s), state(f) \quad (4)$$

where $sink(f)$ describes a sink state:

$$1\{sink(f) : state(f)\}1 \leftarrow . \quad (5)$$

The union of the program ASP_1 that consists of the rules (2), (3), (4), (5), (1), with a set of facts describing an FA A has an answer set iff there exists a synchronizing sequence of length c for A .

2.2 Merging States Pairwise

In the second ASP formulation, which we call ASP_2 , first we “generate” a sequence w of c symbols by the choice rule (2).

Next, we ensure that it is a synchronizing sequence by “testing” that it does not violate the following condition, instead of constraint C_1 :

D_1 For every pair q_i, q_{i+1} of states in $Q = \{q_1, q_2, \dots, q_n\}$, $\delta(q_i, w) = \delta(q_{i+1}, w)$.

by adding the following cardinality constraints

$$\leftarrow 1\{merged(r) : state(r), r < n\}n - 2 \quad (6)$$

where $merged(r)$ describes that there exists a state s reachable from the states r and $r+1$ by paths characterized by the first i symbols of w for some i ($1 \leq i \leq c$):

$$\begin{aligned} merged(r) \leftarrow path(r, i, s), path(r + 1, i, s), \\ state(s), state(r), state(r + 1), step(i) \end{aligned} \quad (7)$$

The union of the program ASP_2 that consists of the rules (2), (3), (6), (7), (1), with a set of facts describing an FA A has an answer set iff there exists a synchronizing sequence of length c for A .

2.3 Optimization

The ASP formulations given in Section 2.1 and Section 2.2, with a set of facts describing an FA A , have answer sets if the given FA A has a synchronizing sequence of length c . In order to find the length of the shortest synchronizing sequence, one can perform a binary search on possible values of c .

In this section, we present another ASP formulation where we let the ASP solver first decide the length l of a shortest synchronizing sequence, where $l \leq c$:

$$1\{shortest(l) : 1 \leq l \leq c\}1 \leftarrow \quad (8)$$

and declare possible lengths of sequences:

$$step(j) \leftarrow shortest(i) \quad (1 \leq j \leq i \leq c). \quad (9)$$

Next, we ensure that l is indeed the optimal value, by the following optimization statement

$$\#minimize[shortest(l) = l] \leftarrow \quad (10)$$

We denote by ASP_1^{opt} (resp. ASP_2^{opt}) the ASP formulation obtained from ASP_1 (resp. ASP_2) by adding the rules (8) and (10), and replacing the rules (3) by the rules (9). If ASP_1^{opt} (resp. ASP_2^{opt}) with a set of facts describing an FA A has an answer set X then X characterizes a shortest synchronizing sequence for A .

3 SAT Formulation of the Shortest Synchronizing Sequence Problem

In [16], a SAT formulation of the problem of checking if an FA A has a synchronizing sequence of a certain length is presented. However, this formulation is given only for FA with two input symbols. We extend this SAT formulation to FA with any number of input symbols as follows.

We first define a boolean operator ∇ that will simplify the description of our SAT formulation. For a given set of boolean variables $\{r_1, r_2, \dots, r_k\}$, we define $\nabla\{r_1, r_2, \dots, r_k\}$ as follows:

$$\nabla\{r_1, r_2, \dots, r_k\} \equiv ((r_1 \Rightarrow (\neg r_2 \wedge \neg r_3 \wedge \dots \wedge \neg r_k)) \wedge (r_2 \Rightarrow (\neg r_1 \wedge \neg r_3 \wedge \dots \wedge \neg r_k)) \wedge \dots \wedge (r_k \Rightarrow (\neg r_1 \wedge \neg r_2 \wedge \dots \wedge \neg r_{k-1}))) \wedge (r_1 \vee r_2 \vee \dots \vee r_k)$$

Intuitively, $\nabla\{r_1, r_2, \dots, r_k\}$ is true with respect to an interpretation I iff exactly one of the variables r_i is true and all the others are false with respect to I .

Checking the existence of a synchronizing sequence of length c is converted into a SAT problem by considering the following boolean formulae. Below we use the notation $[c]$ to denote the set $\{1, 2, \dots, c\}$.

- F_1 : An input sequence of length c has to be created. At each step of this input sequence, there should be exactly one input symbol being used. For this purpose, we generate Boolean variables $X_{l,x}$ which should be set to true (by an interpretation) only if at step l the input symbol x is used. The following formulae make sure that only one input symbol is picked for each step l .

$$\sigma_1 = \bigwedge_{l \in [c]} (\nabla \{X_{l,x} \mid x \in X\})$$

- F_2 : Similar to what we accomplish in ASP formulations by atoms of the form $path(s, i, q)$, we need to trace the state reached when the input sequence guessed by formula σ_1 is applied. For this purpose, boolean variables $S_{i,j,k}$ (which we call *state tracing variables*) are created which are set to true (by an interpretation) only if we are at state q_k at step j when we start from state q_i . We first make sure that for each starting state and at any step, there will always be exactly one current state.

$$\sigma_2 = \bigwedge_{i \in [n], l \in [c]} (\nabla \{S_{i,l,j} \mid j \in n\})$$

- F_3 : The initial configuration of the FA A must be indicated. For this purpose state tracing variables should be initialized for their first step.

$$\sigma_3 = \bigwedge_{i \in [n]} (S_{i,1,i})$$

- F_4 : Again, similar to the constraints in ASP formulations, over atoms of the form $path(s, i, q)$, we have the corresponding SAT formulae to make sure that state tracing variables are assigned according to the transitions of the FA A . For each state q_j and input x of A , if we have $\delta(q_j, x) = q_k$ in A , then we generate the following formulae:

$$\sigma_4 = \bigwedge_{i,j \in [n], l \in [c], x \in X} ((S_{i,l,j} \wedge X_{l,x}) \Rightarrow S_{i,l+1,k})$$

- F_5 : A synchronizing sequence w merges all the states at a sink state after the application of w . We use boolean variable Y_i to pick a sink state. Since only one of the states has to be a sink state, we introduce the following formulae:

$$\sigma_5 = (\nabla \{Y_i \mid i \in [n]\})$$

- F_6 : Finally, we need to make sure that all the states reach the sink state picked by F_5 at the end of the last step after the application of the synchronizing sequence guessed by formulae F_1 .

$$\sigma_6 = \bigwedge_{i,j \in [n]} (Y_i \Rightarrow S_{j,c+1,i})$$

The conjunction of all formulae introduced above is a Boolean formula that is satisfiable iff there exists a synchronizing sequence of FA A of length c .

4 Experimental Study

In this section, we present the experimental study carried out to compare the performance of the ASP formulations, the SAT formulation, and the brute-force algorithm for generating a shortest synchronizing sequence.

We first present our experiments using finite automata that are generated randomly. Given the number of states and the number of input symbols, an FA is generated by assigning the next state of each transition randomly. If the FA generated in this way does not have a synchronizing sequence, then it is discarded. Otherwise, it is included in the set of FAs used in our experiments. We generated 100 random FAs this way for each number of states we used in the experiments (except for the biggest set of tests with 50 states and 4-6 input symbols, where we use only 50 FAs to speed up the experiments).

The implementation of the brute-force algorithm in the tool COMPAS [21] is used. The brute-force algorithm could be used for FAs with up to 27 states. Beyond this number of states, COMPAS could not complete the computation due to memory restrictions.

We implemented tools that create ASP and SAT formulations from a given FA and an integer constant c as explained in Section 2, Section 3, and also the SAT formulation given in [16] for FAs with two inputs only.

In the results given below, the formulations ASP_1 , ASP_2 , ASP_1^{opt} , and ASP_2^{opt} refer to the ASP formulations given in Section 2. SAT_1 and SAT_2 refer to the SAT formulations given in [16] and Section 3, respectively. BF refers to the brute-force algorithm.

Note that the ASP formulations ASP_1^{opt} and ASP_2^{opt} report the length of a shortest synchronizing sequence, provided that the constant c given is not smaller than the length of a shortest synchronizing sequence. When c is not big enough, another experiment is performed by doubling the value of c . We report only the results from successful ASP_1^{opt} and ASP_2^{opt} experiments, where a sufficiently large c is given. An experimental study is presented in [16] where the length of the shortest synchronizing sequence is reported to be around $2\sqrt{n}$ on the average for an n state automaton with two input symbols. We therefore initially take the value of c as $2\sqrt{n}$.

On the other hand, the ASP formulations ASP_1 and ASP_2 , and also the SAT formulations SAT_1 and SAT_2 , only report if a synchronizing sequence of length c exists or not. Therefore, one has to try several c values to find the length of the shortest synchronizing sequence using these formulations. In our experiments with these formulations, we find the length of a shortest synchronizing sequence by applying a binary search on the value of c , by using a script that invokes the ASP solver for each attempt on a possible value of c separately. We similarly take the initial value of c to be $2\sqrt{n}$ as explained above. The time reported is the total time taken by all the attempts until the length of a shortest synchronizing sequence is found. The memory reported is the average memory usage in these attempts.

The experiments are carried out using MiniSat 2.2.0 [22] and Clingo 3.0.3 [23] running on Ubuntu Linux where the hardware is a 2.4Ghz Intel Core-i3 machine.

Table 1: Experiments on FAs with 2 input symbols (time - secs)

n	ASP_1	ASP_2	ASP_1^{opt}	ASP_2^{opt}	SAT_1	BF
5	0	0	0	0	7	0
10	2	2	2	11	10	0
15	1	2	2	5	10	0
20	4	6	5	8	12	4
25	6	12	7	14	13	73
26	7	11	8	15	14	145
27	9	12	8	14	15	289

Table 2: Experiments on FAs with 2 input symbols (memory - kBytes)

n	ASP_1	ASP_2	ASP_1^{opt}	ASP_2^{opt}	SAT_1
5	7750	7731	7622	7620	7677
10	8169	7278	8154	8160	7983
15	6284	6566	6465	6300	7847
20	6909	6911	6943	6947	7810
25	6769	6775	6937	6779	8066
26	7151	6798	7136	6822	8213
27	7123	7106	6744	7119	8113

In Table 1 and Table 2, the time and the memory performance of the formulations ASP_1 , ASP_2 , SAT_1 , and the brute-force algorithm are given. We could not get a report on the memory usage of COMPAS for the brute-force algorithm, hence no data is given for the brute-force algorithm in Table 2. In this set of experiments, the number of states n ranges between 5 and 27, and the number of input symbols is fixed to 2.

In Table 3 and Table 4, the time and the memory performance of the formulations ASP_1^{opt} , ASP_2^{opt} , and SAT_2 are given on FAs with the number of states $n \in \{30, 40, 50\}$ and the number of inputs $k \in \{2, 4, 6\}$.

Table 1 shows that the brute-force approach uses much more time than the other approaches, especially as the size of the FA gets bigger. Therefore, after a certain threshold size, the brute-force approach is not an alternative.

By investigating the results given in Table 1 and Table 3, one can see that the ASP formulation approach of ASP_1 and ASP_1^{opt} perform better than ASP_2 and ASP_2^{opt} , in general. This may be due to that the number of ground instances of (4) and (5) is smaller than that of (6) and (7). However, after intelligent grounding of Clingo, the program sizes of ASP_1^{opt} and ASP_2^{opt} become comparable. On the other hand, we have observed that ASP_2^{opt} leads to more backtracking and restarts compared to ASP_1^{opt} . For example, for an instance of 50 states and 2 input symbols, ASP_1^{opt} leads to 82878 choices and no restarts, whereas ASP_2^{opt} leads to 137276 choices and 4 restarts. This may be due to that, in ASP_1^{opt} with respect to (4) and (5), once a sink node is selected, for every state,

Table 3: Experiments on FAs with different number of inputs (time - secs)

n	k	ASP_1	ASP_2	ASP_1^{opt}	ASP_2^{opt}	SAT_2
30	2	4	17	4	18	49
30	4	66	80	45	57	101
30	6	208	405	160	231	490
40	2	33	45	71	122	222
40	4	348	380	244	311	472
40	6	1158	1400	707	980	2133
50	2	93	120	117	146	430
50	4	902	1101	835	833	2975
50	6	3205	4010	2705	3032	7492

Table 4: Experiments on FAs with different number of inputs (memory - kBytes)

n	k	ASP_1	ASP_2	ASP_1^{opt}	ASP_2^{opt}	SAT_2
30	2	6063	7143	4973	6140	42764
30	4	7309	7438	5278	7457	49681
30	6	7735	7621	7496	10457	52250
40	2	6709	6029	5616	7448	67362
40	4	7050	7073	7880	8550	78697
40	6	7764	8024	7983	10234	84671
50	2	7222	8336	8965	16072	86453
50	4	8438	9056	9931	12843	85157
50	6	8773	9228	10729	14092	93118

existence of a path of a fixed length is checked; on the other hand, in ASP_2^{opt} with respect to (6) and (7), for every state, existence of two paths of the same length is checked, which may intuitively lead to more backtracking and restarts. On the other hand, the memory performances of all ASP approaches are similar, as displayed by Table 2 and Table 4.

As for the comparison of the ASP and SAT approaches, one can see that the ASP approaches are both faster and uses less memory than the SAT approach, in general. However, the ASP approach seems to have a faster increase in the running time compared to the SAT approach. This trend needs to be confirmed by further experiments.

We also experimented with finite state automata from MCNC'91 benchmarks [24]. We used only those finite state machine examples in this benchmark set that correspond to completely specified finite state automata. The results of these experiments are given in Table 5 for the time comparison. We obtained similar results to what we have observed in our experiments on random finite state automata. The time performance of the ASP approaches are better than the SAT approach in these experiments as well. We note that the benchmark example "dk16", which is also the automaton having the largest number of states, has the longest running time among the automata in the benchmark set.

Table 5: Experiments on FAs from MCNC benchmarks (time - msecs)

Name	n	k	ASP_1	ASP_2	ASP_1^{opt}	ASP_2^{opt}	SAT_2
bbtas	6	4	15	18	10	14	52
beecount	7	8	18	18	10	9	89
dk14	7	8	18	20	15	17	62
dk15	4	8	13	14	7	7	20
dk17	8	4	27	26	8	8	69
dk27	7	2	22	21	6	6	45
dk512	15	2	33	27	11	12	278
dk16	27	4	191	231	132	127	21253
lion9	9	4	91	138	36	137	449
MC	4	8	18	18	7	7	53

However, the running time does not depend only on the number of states. The number of input symbols and the length of the shortest synchronizing sequence would also have an effect. For the comparison of the memory used for these examples, all ASP approaches used around 6 MBytes of memory, whereas the SAT approach used minimum 6 MBytes and maximum 38 MBytes memory for these experiments.

5 Conclusion and Future Work

In this paper, the problem of finding a shortest synchronizing sequence for a FA is formulated in ASP. Four different ASP formulations are given. Also an extension of the SAT formulation of the same problem given in [16] is suggested.

The performance of these formulations are compared by an experimental evaluation. The ASP and SAT formulations are shown to scale better than the brute-force approach. The experiments indicate that the ASP formulations perform better than the SAT approach. However this needs to be further investigated with an extended set of experiments.

Based on the encouraging results obtained from this work, using ASP to compute some other special sequences used in finite state machine based testing can be considered as a future research direction. For example checking the existence of, and computing a Preset Distinguishing Sequence is a PSPACE-hard problem [1]. Although checking the existence and computing an Adaptive Distinguishing Sequence [1] can be performed in polynomial time, generating a minimal Adaptive Distinguishing Sequence is an NP-hard problem. These hard problems can be addressed by using ASP.

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