Revisiting Postulates for Inconsistency Measures

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Abstract

We discuss postulates for inconsistency measures as proposed in the literature.We examine them both individually and as a collection. Although we criticize two of the original postulates, we mostly focus on the meaning of the postulates as a whole. Also and accordingly, we discuss a number of new postulates as substitutes and/or as alternative families.

Introduction

In (Hunter and Konieczny 2008; Hunter and Konieczny 2010), Hunter and Konieczny have introduced postulates for inconsistency measures over knowledge bases. Let us first make it clear that the phrase "inconsistency measure" refers to the informal meaning of a measure, not to the usual formal definition whose countable additivity requirement would leave no choice for an inconsistency measure, making all minimal inconsistent knowledge bases in each cardinality to count as equally inconsistent (unless making some *consistent* formulas to count as more *inconsistent* than others!). However, we stick with the usual range $R^+ \cup \{\infty\}$ (so, the range is totally ordered and 0 is the least element). The intuition is: The higher the amount of inconsistency in the knowledge base, the greater the number returned by the inconsistency measure.

Let us emphasize that we deal with postulates for inconsistency measures that account for a raw amount of inconsistency: E.g., it will clearly appear below that an inconsistency measure I satisfying the (Monotony) postulate due to Hunter-Konieczny precludes I to be a ratio (except for quite special cases, see (Hunter and Konieczny 2010)).

HK Postulates

Hunter and Konieczny refer to a propositional language[1] $\mathcal{L}$ for classical logic $\vdash$. Belief bases are finite sequences over $\mathcal{L}$. $\mathcal{K}_\mathcal{L}$ is comprised of all belief bases over $\mathcal{L}$, in set-theoretic form (i.e., a member of $\mathcal{K}_\mathcal{L}$ is an ordinary set[2]).

According to Hunter and Konieczny, a function I over belief bases is an inconsistency measure if it satisfies the following properties, $\forall K, K' \in \mathcal{K}_\mathcal{L}, \forall \alpha, \beta \in \mathcal{L}$

- $I(K) = 0$ iff $K \not\vdash \bot$ (Consistency Null)
- $I(K \cup K') \geq I(K)$ (Monotony)
- If α is free[3] for K then $I(K \cup \{\alpha\}) = I(K)$ (Free Formula Independence)
- If $\alpha \vdash \beta$ and $\alpha \not\vdash \bot$ then $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$ (Dominance)

We start by arguing against (Free Formula Independence) and (Dominance) in the next section. We browse in the subsequent section several consequences of HK postulates, stressing the need for more general principles in each case. We then introduce various postulates supplementing the original ones, ending with a new axiomatization. We also devote a full section to a major principle, replacement of equivalent subsets. The section preceding the conclusion can be viewed as a kind of rejoinder backing (Monotony) and (Free Formula Independence) via the main new postulate.

Objections to HK Postulates

Objection to (Dominance)

In contrapositive form, (Dominance) says:

$$\text{For } \alpha \vdash \beta, \text{ if } I(K \cup \{\alpha\}) < I(K \cup \{\beta\}) \text{ then } \alpha \vdash \bot \quad (1)$$

but it makes sense that the lefthand side holds while $\alpha \not\vdash \bot$. An example is as follows. Let $K = \{a \wedge b \wedge c \wedge \cdots \wedge z\}$. Take $\beta = \neg a \vee (\neg b \wedge \neg c \wedge \cdots \wedge \neg z)$ while $\alpha = \neg a$. We may hold $I(K \cup \{\alpha\}) < I(K \cup \{\beta\})$ on the following grounds:

- The inconsistency in $I(K \cup \{\alpha\})$ is $\neg a$ vs a.
- The inconsistency in $I(K \cup \{\beta\})$ is either as above (i.e., $\neg a$ vs a) or it is $\neg b \wedge \neg c \wedge \cdots \wedge \neg z$ vs $b \wedge c \wedge \cdots \wedge z$ that may be viewed as more inconsistent than the case $\neg a$ vs a, hence, $\{a \wedge b \wedge c \wedge \cdots \wedge z\} \cup \{\neg a \vee (\neg b \wedge \neg c \wedge \cdots \wedge \neg z)\}$ can be taken as more inconsistent overall than $\{a \wedge b \wedge c \wedge \cdots \wedge z\} \cup \{\neg a\}$ thereby violating (1) because $\alpha \not\vdash \bot$ here.

Objection to (Free Formula Independence)

Unfolding the definition, (Free Formula Independence) is:

$$\text{If } K' \cup \{\alpha\} \vdash \bot \text{ for no consistent subset } K' \text{ of } K \text{ then } I(K \cup \{\alpha\}) = I(K) \quad (2)$$

[1] For simplicity, we use a language based on the complete set of connectives $\{\neg, \wedge, \vee\}$.

[2] In the conclusion, we mention the case of multisets.

[3] A formula φ is free for X iff $Y \cup \{\alpha\} \vdash \bot$ for no consistent subset Y of X.

(Hunter and Konieczny 2010) has an example of a consistent free formula whose rightmost conjunct contradicts a *consistent* part of a formula of K and so does its leftmost conjunct. A different case (where no minimal inconsistent subset is a singleton set) is $K = \{a \wedge c, b \wedge \neg c\}$ and $\alpha = \neg a \vee \neg b$. Atoms a and b are compatible but $a \wedge b$ is contradicted by α, and $K \cup \{\alpha\}$ may be regarded as more inconsistent than K: (2) is failed.

Consequences of HK Postulates

Proposition 1 *(Monotony) entails*

- *if* $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\alpha, \beta\})$ *then* $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\beta\})$

Proof Assume $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\alpha, \beta\})$. However, (Monotony) ensures $I(K \cup \{\alpha, \beta\}) \geq I(K \cup \{\beta\}$. Hence the result. ■

That is, if I conforms with adjunction (roughly speaking, it means identifying $\{\alpha, \beta\}$ with $\{\alpha \wedge \beta\}$) then I respects the idea that adding a conjunct cannot make the amount of inconsistency to decrease.

Notation. $\alpha \equiv \beta$ denotes that both $\alpha \vdash \beta$ and $\beta \vdash \alpha$ hold. Also, $\alpha \equiv \beta \vdash \gamma$ is an abbreviation for $\alpha \equiv \beta$ and $\beta \vdash \gamma$ (so, $\alpha \equiv \beta \not\vdash \gamma$ means that $\alpha \equiv \beta$ and $\beta \not\vdash \gamma$).

Proposition 2 *(Free Formula Independence) entails*

- *if* $\alpha \equiv \top$ *then* $I(K \cup \{\alpha\}) = I(K)$ (Tautology Independence)

Proof A tautology is trivially a free formula for any K. ■

Unless $\beta \not\vdash \bot$, there is however no guarantee that the following holds:

- if $\alpha \equiv \top$ then $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta\})$ ($\top$-conjunct Independence)

Proposition 3 *(Dominance) entails*

- $I(K \cup \{\alpha_1, \ldots, \alpha_n\}) = I(K \cup \{\beta_1, \ldots, \beta_n\})$ *whenever* $\alpha_i \equiv \beta_i \not\vdash \bot$ *for* $i = 1..n$ (Swap)

Proof For $i = 1..n$, $\alpha_i \equiv \beta_i$ so that (Dominance) can be applied in both directions. As a consequence, for $i = 1..n$, it clearly holds that $I(K \cup \{\beta_1, \ldots, \beta_{i-1}, \alpha_i, \ldots, \alpha_n\}) = I(K \cup \{\beta_1, \ldots, \beta_i, \alpha_{i+1}, \ldots, \alpha_n\})$. ■

Proposition 3 fails to guarantee that I be independent of any consistent subset of the knowledge base being replaced by an equivalent (consistent) set of formulas:

- if $K' \not\vdash \bot$ and $K' \equiv K''$ then $I(K \cup K') = I(K \cup K'')$ (Exchange)

Proposition 3 guarantees that any consistent formula of the knowledge base can be replaced by an equivalent formula without altering the result of the inconsistency measure. Clearly, postulates for inconsistency measures are expected *not* to entail $I(K \cup \{\alpha\}) = I(K \cup \{\beta\})$ for $\alpha \equiv \beta \vdash \bot$. However, some subcases are desirable: $I(K \cup \{\alpha \vee \alpha\}) = I(K \cup \{\alpha\})$, $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta \wedge \alpha\})$, and so on, in full generality (i.e., even for $\alpha \vdash \bot$) but Proposition 3 fails to ensure any of these.

Proposition 4 *(Dominance) entails*

- *if* $\alpha \wedge \beta \not\vdash \bot$ *then* $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\beta\})$

Proof Apply (Dominance) to the valid inference $\alpha \wedge \beta \vdash \beta$ and the result ensues. ■

Proposition 4 means that I respects the idea that adding a conjunct cannot make amount of inconsistency to decrease, in the case of a consistent conjunction (however, one really wonders why this not guaranteed to hold in more cases?).

Proposition 5 *Due to (Dominance) and (Monotony)*

- *For* $\alpha \in K$, *if* $\alpha \not\vdash \bot$ *and* $\alpha \vdash \beta$ *then* $I(K \cup \{\beta\}) = I(K)$

Proof $I(K \cup \{\alpha\}) = I(K)$ as $\alpha \in K$. By (Dominance), $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$. Therefore, $I(K) \geq I(K \cup \{\beta\})$. The converse holds due to (Monotony). ■

Proposition 5 guarantees that a consequence of a consistent formula of the knowledge base can be added without altering the result of the inconsistency measure. What about a consequence of a consistent subset of the knowledge base? Indeed, Proposition 5 is a special case of

(A_n) For $\{\alpha_1, \ldots, \alpha_n\} \subseteq K$, if $\{\alpha_1, \ldots, \alpha_n\} \not\vdash \bot$ and $\{\alpha_1, \ldots, \alpha_n\} \vdash \beta$ then $I(K \cup \{\beta\}) = I(K)$

That is, Proposition 5 guarantees (A_n) only for $n = 1$ but what is the rationale for stopping there?

Example 1 *Let* $K = \{\neg b, a \wedge b, b \wedge c\}$. *Proposition 5 ensures that* $I(K \cup \{a, c\}) = I(K \cup \{a\}) = I(K \cup \{c\}) = I(K)$. *Although* $a \wedge c$ *behaves as* a *and* c *with respect to all contradictions in* K *(i.e.,* $a \wedge b$ *vs* $\neg b$ *and* $b \wedge c$ *vs* $\neg b$*), HK postulates fail to ensure* $I(K \cup \{a \wedge c\}) = I(K)$.

Replacement of Equivalent Subsets

The value of (Exchange)

Firstly, (Exchange) is not a consequence of (Dominance) and (Monotony). An example is $K_1 = \{a \wedge c \wedge e, b \wedge d \wedge \neg e\}$ and $K_2 = \{a \wedge e, c \wedge e, b \wedge d \wedge \neg e\}$. Due to (Exchange), $I(K_1) = I(K_2)$ but HK postulates do not impose equality. Next are a few results showing properties of (Exchange).

Proposition 6 *(Exchange) is equivalent to each of these:*

- *The family* $(A_n)_{n \geq 1}$
- *If* $K' \equiv K''$, $K' \not\vdash \bot$ *then* $I(K \cup K') = I((K \setminus K') \cup K'')$
- *If* $K' \equiv K''$ *and* $K' \not\vdash \bot$ *and* $K \cap K' = \emptyset$ *then* $I(K \cup K') = I(K \cup K'')$
- *If* $\{K_1, \ldots, K_n\}$ *is a partition of* $K \setminus K_0$ *where* K_0 *is defined as* $K_0 = \{\alpha \in K \mid \alpha \vdash \bot\}$ *such that* $K_i \not\vdash \bot$ *and* $K'_i \equiv K_i$ *for* $i = 1..n$ *then* $I(K) = I(K_0 \cup K'_1 \cup \cdots \cup K'_n)$

Proof Assume (A_n) for all $n \geq 1$ and $K' \equiv K'' \not\vdash \bot$. (i) Let $K' = \{\alpha_1, \ldots, \alpha_m\}$. Define $\langle K'_j \rangle_{j \geq 0}$ where $K'_0 = K \cup K''$ and $K'_{j+1} = K'_j \cup \{\alpha_{j+1}\}$. It is clear that $K'' \not\vdash \bot$ and $K'' \vdash \alpha_{j+1}$ and $K'' \subseteq K'_j$. Hence, (A_n) can be applied to K'_j and this gives $I(K'_j) = I(K'_j \cup \{\alpha_{j+1}\}) = I(K'_{j+1})$. Overall, $I(K'_0) = I(K'_m)$. I.e., $I(K \cup K'') = I(K \cup K' \cup K'')$. (ii) Let $K'' = \{\beta_1, \ldots, \beta_p\}$. Consider the sequence $\langle K''_j \rangle_{j \geq 0}$ where $K''_0 = K \cup K'$ and $K''_{j+1} = K''_j \cup \{\beta_{j+1}\}$. Clearly, $K' \not\vdash \bot$ and $K' \vdash \beta_{j+1}$

and $K' \subseteq K''_j$. Hence, (A_n) can be applied to K''_j and this gives $I(K''_j) = I(K''_j \cup \{\beta_{j+1}\}) = I(K''_{j+1})$. Overall, $I(K''_0) = I(K''_p)$. I.e., $I(K \cup K') = I(K \cup K' \cup K'')$. Combining the equalities, $I(K \cup K') = I(K \cup K'')$. That is, the family $(A_n)_{n\geq 1}$ entails (Exchange).

We now show that the family $(A_n)_{n\geq 1}$ is entailed by the second item in the statement of Proposition 6, denoted (Exchange′), which is:

If $K' \not\vdash \bot$ and $K' \equiv K''\}$
then $I(K \cup K') = I((K \setminus K') \cup K'')$

Let $\{\alpha_1, \ldots, \alpha_n\} \subseteq K$ such that $\{\alpha_1, \ldots, \alpha_n\} \not\vdash \bot$ and $\{\alpha_1, \ldots, \alpha_n\} \vdash \beta$. So, $\{\alpha_1, \ldots, \alpha_n\} \equiv \{\alpha_1, \ldots, \alpha_n, \beta\}$. For $K' = \{\alpha_1, \ldots, \alpha_n\}$, $K'' = \{\alpha_1, \ldots, \alpha_n, \beta\}$ (Exchange) gives $I(K) = I((K \setminus \{\alpha_1, \ldots, \alpha_n\}) \cup \{\alpha_1, \ldots, \alpha_n, \beta\} = I(K \cup \{\beta\})$.

By transitivity, we have thus shown that (Exchange) is entailed by (Exchange′). Since the converse is obvious, the equivalence between (Exchange), (Exchange′) and the family $(A_n)_{n\geq 1}$ holds.

It is clear that the third item in the statement of Proposition 6 is equivalent with (Exchange).

Consider now (Exchange″), the last item in the statement of Proposition 6:

If $\{K_1, \ldots, K_n\}$ is a partition of $K \setminus K_0$ where
$K_0 = \{\alpha \in K \mid \alpha \vdash \bot\}$ such that
$K_i \not\vdash \bot$ and $K'_i \equiv K_i$ for $i = 1..n$ then
$I(K) = I(K_0 \cup K'_1 \cup \cdots \cup K'_n)$.

(i) Assume (Exchange′). We now prove (Exchange″). Let $\{K_1, \ldots, K_n\}$ be a partition of $K \setminus K_0$ satisfying the conditions of (Exchange″). Trivially, $I(K) = I(K_0 \cup K \setminus K_0) = I(K_0 \cup K_1 \cup \cdots \cup K_n)$. Then, $K_i \setminus K_n = K_i$ for $i = 1..n-1$. Applying (Exchange′) yields $I(K_0 \cup K_1 \cup \cdots \cup K_n) = I(K_0 \cup K_1 \cup \cdots \cup K'_n)$ hence $I(K) = I(K_0 \cup K_1 \cup \cdots \cup K'_n)$. Applying (Exchange′) iteratively upon K_{n-1}, K_{n-2}, …, K_1 gives $I(K) = I(K_0 \cup K'_1 \cup \cdots \cup K'_n)$.

(ii) Assume (Exchange″). We now prove (Exchange′). Let $K' \not\vdash \bot$ and $K'' \equiv K'$. Clearly, $(K \cup K')_0 = K_0$ and $(K \cup K') \setminus (K \cup K')_0 = (K \setminus K_0) \cup K'$. As each formula in $K \setminus K_0$ is consistent, $K \setminus K_0$ can be partitioned into $\{K_1, \ldots, K_n\}$ such that $K_i \not\vdash \bot$ for $i = 1..n$ (take $n = 0$ in the case that $K = K_0$). Then, $\{K_1 \setminus K', \ldots, K_n \setminus K', K'\}$ is a partition of $(K \setminus K_0) \cup K'$ satisfying the conditions in (Exchange″). Now, $I(K \cup K') = I(K_0 \cup (K_1 \setminus K') \cup \cdots \cup (K_n \setminus K') \cup K')$. Applying (Exchange″) with each K_i substituting itself and K'' substituting K', we obtain $I(K \cup K') = I(K_0 \cup (K_1 \setminus K') \cup \cdots \cup (K_n \setminus K') \cup K'')$. That is, $I(K \cup K') = I((K \setminus K') \cup K'')$. ■

Proposition 7 *(Exchange) entails (Swap).*

Proof Taking advantage of transitivity of equality, it will be sufficient to prove $I(K \cup \{\beta_1, \ldots, \beta_{i-1}, \alpha_i, \ldots, \alpha_n\}) = I(K \cup \{\beta_1, \ldots, \beta_i, \alpha_{i+1}, \ldots, \alpha_n\})$ for $i = 1..n$. Due to $\alpha_i \equiv \beta_i$ and $\beta_i \not\vdash \bot$, it holds that $\{\alpha_i\} \not\vdash \bot$ and $\{\alpha_i\} \equiv \{\alpha_i, \beta_i\}$. As a consequence, (Exchange) can be applied to $K \cup \{\beta_1, \ldots, \beta_{i-1}, \alpha_{i+1}, \ldots, \alpha_n\}$ for $K' = \{\alpha_i\}$ and $K'' = \{\alpha_i, \beta_i\}$. Accordingly, $I(K \cup \{\beta_1, \ldots, \beta_{i-1}, \alpha_i, \ldots, \alpha_n\})$ is then equal to $I(((K \cup \{\beta_1, \ldots, \beta_{i-1}, \alpha_{i+1}, \ldots, \alpha_n\}) \setminus \{\alpha_i\}) \cup \{\alpha_i, \beta_i\})$ and the latter is $I(K \cup \{\beta_1, \ldots, \beta_i, \alpha_{i+1}, \ldots, \alpha_n\})$. ■

That (Exchange) entails (Swap) is natural. Surprisingly, (Exchange) also entails (Tautology Independence).

Proposition 8 *(Exchange) gives (Tautology Independence).*

Proof The non-trivial case is $\alpha \notin K$. Apply (Exchange′) for $K' = \{\alpha\}$ and $K'' = \emptyset$, so, $I(K \cup \{\alpha\}) = I((K \setminus \{\alpha\}) \cup \emptyset)$ ensues. I.e., $I(K \cup \{\alpha\}) = I(K)$. ■

The value of an adjunction postulate

In keeping with the meaning of the conjunction connective in classical logic, consider a dedicated postulate in the form

- $I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha \wedge \beta\})$ (Adjunction Invariancy)

Proposition 9 *(Adjunction Invariancy) entails*

- $I(K \cup \{\alpha, \beta\}) = I((K \setminus \{\alpha, \beta\}) \cup \{\alpha \wedge \beta\})$ (Disjoint Adjunction Invariancy)
- $I(K) = I(\{\bigwedge K\})$ (Full Adjunction Invariancy)
  where $\bigwedge K$ denotes $\alpha_1 \wedge \ldots \wedge \alpha_n$ for any enumeration $\alpha_1, \ldots, \alpha_n$ of K.

Proof Let $K = \{\alpha_1, \ldots, \alpha_n\}$. Apply iteratively (Adjunction Invariancy) as $I(\{\alpha_1 \wedge \ldots \wedge \alpha_{i-1}, \alpha_i, \ldots, \alpha_n\}) = I(\{\alpha_1 \wedge \ldots \wedge \alpha_i, \alpha_{i+1}, \ldots, \alpha_n\})$ for $i = 2..n$. ■

Proposition 10 *Assuming $I(\{\alpha \wedge (\beta \wedge \gamma)\}) = I(\{(\alpha \wedge \beta) \wedge \gamma\})$ and $I(\{\alpha \wedge \beta\}) = I(\{\beta \wedge \alpha\})$, (Disjoint Adjunction Invariancy) and (Full Adjunction Invariancy) are equivalent.*

Proof Assume (Full Adjunction Invariancy). $K \cup \{\alpha, \beta\} = (K \setminus \{\alpha, \beta\}) \cup \{\alpha, \beta\}$ yields $I(K \cup \{\alpha, \beta\}) = I((K \setminus \{\alpha, \beta\}) \cup \{\alpha, \beta\})$. By (Full Adjunction Invariancy), $I((K \setminus \{\alpha, \beta\}) \cup \{\alpha, \beta\}) = I(\{\bigwedge((K \setminus \{\alpha, \beta\}) \cup \{\alpha, \beta\})\})$ and the latter can be written $I(\{\gamma_1 \wedge \ldots \wedge \gamma_n \wedge \alpha \wedge \beta\})$ for some enumeration $\gamma_1, \ldots, \gamma_n$ of $K \setminus \{\alpha, \beta\}$. I.e., $I(K \cup \{\alpha, \beta\}) = I(\{\gamma_1 \wedge \ldots \wedge \gamma_n \wedge \alpha \wedge \beta\})$. By (Full Adjunction Invariancy), $I((K \setminus \{\alpha, \beta\}) \cup \{\alpha \wedge \beta\}) = I(\{\bigwedge((K \setminus \{\alpha, \beta\}) \cup \{\alpha \wedge \beta\})\})$ that can be written $I(\{\gamma_1 \wedge \ldots \wedge \gamma_n \wedge \alpha \wedge \beta\})$ for the same enumeration $\gamma_1, \ldots, \gamma_n$ of $K \setminus \{\alpha, \beta\}$. So, $I(K \cup \{\alpha, \beta\}) = I((K \setminus \{\alpha, \beta\}) \cup \{\alpha \wedge \beta\})$. As to the converse, it is trivial to use (Disjoint Adjunction Invariancy) iteratively to get (Full Adjunction Invariancy). ■

A counter-example to the purported equivalence of (Adjunction Invariancy) and (Full Adjunction Invariancy) is as follows. Let $K = \{a, b, \neg b \wedge \neg a\}$. Obviously, $I(K \cup \{a, b\}) = I(K)$ since $\{a, b\} \subseteq K$. (Full Adjunction Invariancy) gives $I(K) = I(\{\bigwedge_{\gamma \in K} \gamma\})$ i.e. $I(K \cup \{a, b\}) = I(\{\bigwedge_{\gamma \in K} \gamma\}) = I(\{a \wedge b \wedge \neg b \wedge \neg a\})$. A different case of applying (Full Adjunction Invariancy) gives $I(K \cup \{a \wedge b\}) = I(\{\bigwedge_{\gamma \in K \cup \{a \wedge b\}} \gamma\}) = I(\{a \wedge b \wedge \neg b \wedge \neg a \wedge a \wedge b\})$. However, HK postulates do not provide grounds to infer $I(\{a \wedge b \wedge \neg b \wedge \neg a\}) = I(\{a \wedge b \wedge \neg b \wedge \neg a \wedge a \wedge b\})$ hence (Adjunction Invariancy) may fail here.

(Adjunction Invariancy) provides a natural equivalence between (Monotony) and a principle which expresses that adding a conjunct cannot make the amount of inconsistency to decrease:

Proposition 11 *Assuming (Consistency Null), (Adjunction Invariancy) yields that (Monotony) is equivalent with*

- $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\alpha\})$ (Conjunction Dominance)

Proof Assume (Monotony), a simple instance of which is $I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha, \beta\})$. (Adjunction Invariancy) gives $I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha \wedge \beta\})$. As a consequence, $I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha \wedge \beta\})$. This inequality shows that (Conjunction Dominance) holds.
Assume (Conjunction Dominance). First, consider $K \neq \emptyset$. Let $\alpha \in K$. Thus, $I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha \wedge \beta\}$ by (Conjunction Dominance). (Adjunction Invariancy) gives $I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha \wedge \beta\})$. Hence, $I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha, \beta\})$. I.e., $I(K) \leq I(K \cup \{\beta\})$ because $\alpha \in K$. For $K' \in \mathcal{K}_{\mathcal{L}}$, it is enough to iterate this finitely many times (one for every β in $K' \setminus K$) to obtain $I(K) \leq I(K \cup K')$. Now, consider $K = \emptyset$. By (Consistency Null), $I(K) = 0$ hence $I(K) \leq I(K \cup K')$. ■

(Free Formula Independence) yields (Tautology Independence) by Proposition 2 although a more general principle (e.g., ($\top$-conjunct Independence) or the like) ensuring that I be independent of tautologies is to be expected. The next result shows that (Adjunction Invariancy) is the way to get both postulates at once.

Proposition 12 *Assuming (Consistency Null), (Adjunction Invariancy) yields that ($\top$-conjunct Independence) and (Tautology Independence) are equivalent.*

Proof For $\alpha \equiv \top$, (Adjunction Invariancy) and (Tautology Independence) give $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\alpha, \beta\}) = I(K \cup \{\beta\})$. As to the converse, let $\beta \in K$. Therefore, $I(K) = I(K \cup \{\beta\}) = I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha\})$. At to the case $K = \emptyset$, it is settled by means of (Consistency Null). ■

(Adjunction Invariancy) provides for free various principles related to (idempotence, commutativity, and associativity of) conjunction as follows.

Proposition 13 *(Adjunction Invariancy) entails*

- $I(K \cup \{\alpha \wedge \alpha\}) = I(K \cup \{\alpha\})$
- $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta \wedge \alpha\})$
- $I(K \cup \{\alpha \wedge (\beta \wedge \gamma)\}) = I(K \cup \{(\alpha \wedge \beta) \wedge \gamma\})$

Proof (i) $I(K \cup \{\alpha \wedge \alpha\}) = I(K \cup \{\alpha, \alpha\}) = I(K \cup \{\alpha\})$. (ii) $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\alpha, \beta\}) = I(K \cup \{\beta, \alpha\}) = I(K \cup \{\beta \wedge \alpha\})$. (iii) $I(K \cup \{\alpha \wedge (\beta \wedge \gamma)\}) = I(K \cup \{\alpha, \beta \wedge \gamma\}) = I(K \cup \{\alpha, \beta, \gamma\}) = I(K \cup \{\alpha \wedge \beta, \gamma\}) = I(K \cup \{(\alpha \wedge \beta) \wedge \gamma\})$. ■

(Adjunction Invariancy) and (Exchange) are two principles devoted to ensuring that replacing a subset of the knowledge base with an equivalent subset does not change the value given by the inconsistency measure. The contexts that these two principles require for the replacement to be safe differ:

1. For $K' \nvdash \perp$, (Exchange) is more general than (Adjunction Invariancy) since (Exchange) guarantees $I(K \cup K') = I(K \cup K'')$ for every $K'' \equiv K'$ but (Adjunction Invariancy) ensures it only for $K'' = \{\bigwedge K'_i \mid \mathfrak{K} = \{K'_1, .., K'_n\}\}$ where $\mathfrak{K}$ ranges over the partitions of K'.
2. For $\alpha \vdash \perp$, (Adjunction Invariancy) is more general than (Exchange) because (Adjunction Invariancy) guarantees $I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha \wedge \beta\})$ but (Exchange) does not guarantee it.

Revisiting HK Postulates

Sticking with (Consistency Null) and (Monotony)

First, (Consistency Null) or a like postulate is indispensable because there seems to be no way to have a sensible inconsistency measure that would not be able to always discriminate between consistency and inconsistency.

(Monotony) is to be kept since contradictions in classical logic (and basically all logics) are monotone (Besnard 2010) wrt information: That is, extra information cannot make a contradiction to vanish.

We will not retain (Monotony) as an explicit postulate, because it ensues from our schematic postulate (see later).

Intended postulates

(Tautology Independence) and ($\top$-conjunct Independence) are due postulates. More generally, it would make no sense, when determining how inconsistent a theory is, to take into account any inessential difference in which a formula can be written (e.g., $\alpha \vee \beta$ instead of $\beta \vee \alpha$). Define α' to be a *prenormal form* of α if α' is obtained from α by applying (posibly repeatedly) one or more of the following principles: commutativity, associativity and distribution for $\wedge$ and $\vee$, De Morgan laws, double negation equivalence. Henceforth the next[4] postulate:

- If β is a prenormal form of α, $I(K \cup \{\alpha\}) = I(K \cup \{\beta\})$ (Rewriting)

As (Monotony) essentially means that extra information cannot make amount of inconsistency to decrease, the same idea must apply to conjunction because $\alpha \wedge \beta$ cannot involve less information than α. Thus, another due postulate is:

- $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\alpha\})$ (Conjunction Dominance)

Indeed, it does not matter whether α or β or both be inconsistent: It definitely cannot be rational to hold that there is a case (even a single one) where extending K with a conjunction would result in *less* inconsistency than extending K with one of the conjuncts.

Taking care of disjunction

It is very difficult to assess how inconsistent a disjunction is, but bounds can be set. Indeed, a disjunction expresses two alternative possibilities; so, accrual across these would make little sense. That is, amount of inconsistency in $\alpha \vee \beta$ cannot exceed amount of inconsistency in either α or β, depending on which one involves a higher amount of inconsistency. Hence the following postulate.

[4]Insharp contrast to (Irrelevance of Syntax) that allows for destructive transformation from α to β when both are inconsistent, (Rewriting) takes care of inhibiting purely deductive transformations (the most important one is presumably from $\alpha \wedge \perp$ to $\perp$).

- $I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$
(Disjunct Maximality)

Two alternative formulations for (Disjunct Maximality) are as follows.

Proposition 14 *Assume* $I(K \cup \{\alpha \vee \beta\}) = I(K \cup \{\beta \vee \alpha\})$. *(Disjunct Maximality) is equivalent with each of*

- *if* $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$ *then* $I(K \cup \{\alpha\}) \geq I(K \cup \{\alpha \vee \beta\})$
- *either* $I(K \cup \{\alpha \vee \beta\}) \leq I(K \cup \{\alpha\})$ *or* $I(K \cup \{\alpha \vee \beta\}) \leq I(K \cup \{\beta\})$

Proof Let us prove that (Disjunct Maximality) entails the first item. Assume $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$. I.e., $I(K \cup \{\alpha\}) = \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$. Using (Disjunct Maximality), $I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$, i.e. $I(K \cup \{\alpha\})) \geq I(K \cup \{\alpha \vee \beta\})$. As to the converse direction, assume that if $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$ then $I(K \cup \{\alpha\}) \geq I(K \cup \{\alpha \vee \beta\})$. Consider the case $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) = I(K \cup \{\alpha\})$. Hence, $I(K \cup \{\alpha\}) \geq I(K \cup \{\beta\})$. According to the assumption, it follows that $I(K \cup \{\alpha\}) \geq I(K \cup \{\alpha \vee \beta\})$. That is, $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) \geq I(K \cup \{\alpha \vee \beta\})$. Similarly, the case $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) = I(K \cup \{\beta\})$ gives $I(K \cup \{\beta\}) \geq I(K \cup \{\beta \vee \alpha\})$. Then, $I(K \cup \{\beta\}) \geq I(K \cup \{\alpha \vee \beta\})$ in view of the hypothesis in the statement of Proposition 14. That is, $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) \geq I(K \cup \{\alpha \vee \beta\})$. Combining both cases, (Disjunct Maximality) holds.
The equivalence of (Disjunct Maximality) with the last item is due to the fact that the codomain of I is totally ordered. ■

Although it is quite unclear how to weigh inconsistencies out of a disjunction, they must weigh no more than out of both disjuncts (whether tied together by a conjunction or not), which is the reason for holding

- $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\alpha \vee \beta\})$
($\wedge$-over-$\vee$ Dominance)

and its conjunction-free counterpart

- $I(K \cup \{\alpha, \beta\}) \geq I(K \cup \{\alpha \vee \beta\})$

Proposition 15 *Assume* $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta \wedge \alpha\})$. *(Conjunction Dominance) and (Disjunct Maximality) entail ($\wedge$-over-$\vee$ Dominance).*

Proof Given $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta \wedge \alpha\})$, (Conjunction Dominance) gives $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\alpha\})$ and $I(K \cup \{\alpha \wedge \beta\}) \geq I(K \cup \{\beta\})$. Therefore, $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) \leq I(K \cup \{\alpha \wedge \beta\})$. In view of (Disjunct Maximality), $I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$, and it accordingly follows that $I(K \cup \{\alpha \vee \beta\}) \leq I(K \cup \{\alpha \wedge \beta\})$ holds. ■

Proposition 16 *(Monotony) and (Disjunct Maximality) entail*

- $I(K \cup \{\alpha, \beta\}) \geq I(K \cup \{\alpha \vee \beta\})$

Proof Due to (Monotony), $I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha, \beta\})$ and $I(K \cup \{\beta\}) \leq I(K \cup \{\alpha, \beta\})$. As a consequence, $\max(I(K \cup \{\alpha\}), I(K \cup \{\beta\})) \leq I(K \cup \{\alpha, \beta\})$. Then, $I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$ due to (Disjunct Maximality). $I(K \cup \{\alpha, \beta\}) \geq I(K \cup \{\alpha \vee \beta\})$ easily ensues. ■

A schematic postulate

This is to be presented in two steps.

1. (Monotony) expresses that adding information cannot result in a decrease of the amount of inconsistency in the knowledge base. Considering a notion of primitive conflicts that underlies amount of inconsistency, (Monotony) is a special case of a postulate stating that amount of inconsistency is monotone with respect to the set of primitive conflicts $\mathcal{C}(K)$ of the knowledge base K: If $\mathcal{C}(K) \subseteq \mathcal{C}(K')$ then $I(K) \leq I(K')$.
Clearly, I is to admit different postulates depending on what features are required for primitive conflicts (see Table 1).

2. Keep in mind that an inconsistency measure refers to logical content of the knowledge base, *not* other aspects whether subject matter of contradiction, source of information,... This is because an inconsistency measure is only concerned with *quantity*, i.e. amount of inconsistency (of course, it is possible for example that a contradiction be more worrying than another -and so, making more pressing *to act* (Gabbay and Hunter 1993) about it- but this has nothing to do with amount of inconsistency). Now, what characterizes logical content is uniform substitutivity. Hence a postulate called (Substitutivity Dominance) stating that renaming cannot make the amount of inconsistency to decrease: If $\sigma K = K'$ for some substitution σ then $I(K) \leq I(K')$.

Combining these two ideas, we obtain the next postulate

- If $\mathcal{C}(\sigma K) \subseteq \mathcal{C}(K')$ for some substitution σ, $I(K) \leq I(K')$
(Subsumption Orientation)

Fact 1 *Every postulate of the form*

- $I(X) \leq I(Y)$ *for all* $X \in \mathcal{K}_{\mathcal{L}}$ *and* $Y \in \mathcal{K}_{\mathcal{L}}$ *such that condition* $C_{X,Y}$ *holds*

or of the form

- $I(X) = I(Y)$ *for all* $X \in \mathcal{K}_{\mathcal{L}}$ *and* $Y \in \mathcal{K}_{\mathcal{L}}$ *such that condition* $C_{X,Y}$ *holds*

is derived from (Subsumption Orientation) and from any property of $\mathcal{C}$ *ensuring that condition* C *holds.*

Individual properties of $\mathcal{C}$ ensuring condition C for a number of postulates, including all those previously mentioned in the paper, can be found in Table 1.

(Variant Equality) in Table 1 is named after the notion of a variant (Church 1956):

- If σ and σ' are substitutions s.t. $\sigma K = K'$ and $\sigma' K' = K$ then $I(K) = I(K')$
(Variant Equality)

New system of postulates (basic and strong versions)

All the above actually suggests a new system of postulates, which consists simply of (Consistency Null) and (Subsumption Orientation). The system is parameterized by the properties imposed upon $\mathcal{C}$ in the latter. In the range induced by $\mathcal{C}$, a basic system emerges, which amounts to the next list:

Specific property for $\mathcal{C}$	*Specific postulate entailed by (Subsumption Orientation)*
No property needed	(Variant Equality)
No property needed	(Substitutivity Dominance)
$\mathcal{C}(K \cup \{\alpha\}) = \mathcal{C}(K)$ for $\alpha \equiv \top$	(Tautology Independence)
$\mathcal{C}(K \cup \{\alpha \wedge \beta\}) = \mathcal{C}(K \cup \{\beta\})$ for $\alpha \equiv \top$	($\top$-conjunct Independence)
$\mathcal{C}(K \cup \{\alpha\}) = \mathcal{C}(K \cup \{\alpha'\})$ for α' prenormal form of α	(Rewriting)
$\mathcal{C}(K) \subseteq \mathcal{C}(K \cup \{\alpha\})$	(Instance Low)
$\mathcal{C}(K) \subseteq \mathcal{C}(K \cup \{\alpha\})$	(Monotony)
$\mathcal{C}(K \cup \{\alpha \vee \beta\}) \subseteq \mathcal{C}(K \cup \{\alpha \wedge \beta\})$	($\wedge$-over-$\vee$ Dominance)
$\mathcal{C}(K \cup \{\alpha\}) \subseteq \mathcal{C}(K \cup \{\alpha \wedge \beta\})$	(Conjunction Dominance)
$\mathcal{C}(K \cup \{\alpha, \beta\}) = \mathcal{C}(K \cup \{\alpha \wedge \beta\})$	(Adjunction Invariancy)
$\mathcal{C}(K \cup \{\alpha \vee \beta\}) \subseteq \mathcal{C}(K \cup \{\alpha\})$ or $\mathcal{C}(K \cup \{\beta\})$	(Disjunct Maximality)
$\mathcal{C}(K \cup \{\alpha \vee \beta\}) \supseteq \mathcal{C}(K \cup \{\alpha\})$ or $\mathcal{C}(K \cup \{\beta\})$	(Disjunct Minimality)
$\mathcal{C}(K \cup K') = \mathcal{C}(K \cup K'')$ for $K'' \equiv K' \nvdash \perp$	(Exchange)
$\mathcal{C}(K \cup \{\alpha_1, ..., \alpha_n\}) = \mathcal{C}(K \cup \{\beta_1, .., \beta_n\})$ if $\alpha_i \equiv \beta_i \nvdash \perp$	(Swap)
$\mathcal{C}(K \cup \{\beta\}) \subseteq \mathcal{C}(K \cup \{\alpha\})$ for $\alpha \vdash \beta$ and $\alpha \nvdash \perp$	(Dominance)
$\mathcal{C}(K \cup \{\alpha\}) = \mathcal{C}(K)$ for α free for K	(Free Formula Independence)

Table 1: Conditions for postulates derived from (Subsumption Orientation).

Basic System

$I(K) = 0$ iff $K \nvdash \perp$ (Consistency Null)

If α' is a prenormal form of α then $I(K \cup \{\alpha\}) = I(K \cup \{\alpha'\})$ (Rewriting)

If $\sigma K \subseteq K'$ for some substitution σ then $I(K) \leq I(K')$ (Instance Low)

$I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$ (Disjunct Maximality)

If $\alpha \equiv \top$ then $I(K) = I(K \cup \{\alpha\})$ (Tautology Independence)

If $\alpha \equiv \top$ then $I(K \cup \{\alpha \wedge \beta\}) = I(K \cup \{\beta\})$ ($\top$-conjunct Independence)

$I(K \cup \{\alpha\}) \leq I(K \cup \{\alpha \wedge \beta\})$ (Conjunction Dominance)

At the other end of the range is the strong system below (except for (Dominance) and (Free Formula Independence), it captures all postulates listed in Table 1).

Strong System

$I(K) = 0$ iff $K \nvdash \perp$ (Consistency Null)

If α' is a prenormal form of α then $I(K \cup \{\alpha\}) = I(K \cup \{\alpha'\})$ (Rewriting)

If $\sigma K \subseteq K'$ for some substitution σ then $I(K) \leq I(K')$ (Instance Low)

$I(K \cup \{\alpha \vee \beta\}) \leq \max(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$ (Disjunct Maximality)

$I(K \cup \{\alpha \vee \beta\}) \geq \min(I(K \cup \{\alpha\}), I(K \cup \{\beta\}))$ (Disjunct Minimality)

If $K'' \equiv K' \nvdash \perp$ then $I(K \cup K') = I(K \cup K'')$ (Exchange)

$I(K \cup \{\alpha, \beta\}) = I(K \cup \{\alpha \wedge \beta\})$ (Adjunction Invariancy)

HK Postulates as (Subsumption Orientation)

Time has come to make sense[5] of the HK choice of (Free Formula Independence) together with (Monotony), by means of Theorem 1 and Theorem 2.

[5] Still not defending the choice of (Free Formula Independence).

Theorem 1 *Let $\mathcal{C}$ be such that for every $K \in \mathcal{K}_\mathcal{L}$ and for every $X \subseteq \mathcal{L}$ which is minimal inconsistent, $X \in \mathcal{C}(K)$ iff $X \subseteq K$. If I satisfies both (Monotony) and (Free Formula Independence) then I satisfies (Subsumption Orientation) restricted to its non-substitution part, namely*

$$\textit{if } \mathcal{C}(K) \subseteq \mathcal{C}(K') \textit{ then } I(K) \leq I(K').$$

Proof Let $\mathcal{C}(K) \subseteq \mathcal{C}(K')$. Should K be a subset of K', (Monotony) yields $I(K) \leq I(K')$ as desired. So, let us turn to $K \nsubseteq K'$. Consider $\varphi \in K \setminus K'$. If φ were not free for K, there would exist a minimal inconsistent subset X of K such that $\varphi \in X$. Clearly, $X \nsubseteq K'$. The constraint imposed on $\mathcal{C}$ in the statement of the theorem would then yield both $X \in \mathcal{C}(K)$ and $X \notin \mathcal{C}(K')$, contradicting the assumption $\mathcal{C}(K) \subseteq \mathcal{C}(K')$. Hence, φ is free for K. In view of (Free Formula Independence), $I(K) = I(K \setminus \{\varphi\})$. The same reasoning applied to all the (finitely many) formulas in $K \setminus K'$ gives $I(K) = I(K \cap K')$. However, $K \cap K'$ is a subset of K' so that using (Monotony) yields $I(K \cap K') \leq I(K')$ hence $I(K) \leq I(K')$. ■

Define $\Xi = \{X \in \mathcal{K}_\mathcal{L} \mid \forall X' \subseteq X, X' \vdash \perp \Leftrightarrow X = X'\}$. Then, $\mathcal{C}$ is said to be *governed by minimal inconsistency* iff $\mathcal{C}$ satisfies the following property

$$\text{if } \mathcal{C}(K) \cap \Xi \subseteq \mathcal{C}(K') \cap \Xi \text{ then } \mathcal{C}(K) \subseteq \mathcal{C}(K').$$

It means that those Z in $\mathcal{C}(K)$ which are not minimal inconsistent cannot override set-inclusion induced by minimal inconsistent subsets —i.e., no such Z can, individually or collectively, turn $\mathcal{C}(K) \cap \Xi \subseteq \mathcal{C}(K') \cap \Xi$ into $\mathcal{C}(K) \nsubseteq \mathcal{C}(K')$.

Theorem 2 *Let $\mathcal{C}$ be governed by minimal inconsistency and be such that for all $K \in \mathcal{K}_\mathcal{L}$ and all $X \subseteq \mathcal{L}$ which is minimal inconsistent, $X \in \mathcal{C}(K)$ iff $X \subseteq K$. I satisfies (Monotony) and (Free Formula Independence) whenever I satisfies (Subsumption Orientation) restricted to its non-substitution part, namely*

$$\textit{if } \mathcal{C}(K) \subseteq \mathcal{C}(K') \textit{ then } I(K) \leq I(K').$$

Proof Trivially, if $X \subseteq K$ then $X \subseteq K \cup \{\alpha\}$. By the constraint imposed on $\mathcal{C}$ in the statement of the theorem, it follows that if $X \in \mathcal{C}(K)$ then $X \in \mathcal{C}(K \cup \{\alpha\})$. Since $\mathcal{C}$ is governed by minimal inconsistency, $\mathcal{C}(K) \subseteq \mathcal{C}(K \cup \{\alpha\})$ ensues and (Subsumption Orientation) yields (Monotony). Let α be a free formula for K. By definition, α is in no minimal inconsistent subset of $K \cup \{\alpha\}$. So, $X \subseteq K$ iff $X \subseteq K \cup \{\alpha\}$ for all minimal inconsistent X. By the constraint imposed on $\mathcal{C}$ in the statement of the theorem, $X \in \mathcal{C}(K)$ iff $X \in \mathcal{C}(K \cup \{\alpha\})$ ensues for all minimal inconsistent X. In symbols, $\mathcal{C}(K) \cap \Xi = \mathcal{C}(K \cup \{\alpha\}) \cap \Xi$. Since $\mathcal{C}$ is governed by minimal inconsistency, it follows that $\mathcal{C}(K) = \mathcal{C}(K \cup \{\alpha\})$. Thus, (Free Formula Independence) holds, due to (Subsumption Orientation). ■

These theorems mean that, *if substitutivity is left aside*, (Subsumption Orientation) is equivalent with (Free Formula Independence) and (Monotony) when primitive conflicts are essentially minimal inconsistent subsets. These postulates form a natural pair *if it is assumed that* minimal inconsistent subsets must be the basis for inconsistency measuring.

Conclusion

We have proposed a new system of postulates for inconsistency measures, i.e.

$I(K) = 0$ *iff K is consistent* (Consistency Null)

If $\mathcal{C}(\sigma K) \subseteq \mathcal{C}(K')$ *for a substitution* σ *then* $I(K) \leq I(K')$ (Subsumption Orientation)

parameterized by the requirements imposed on $\mathcal{C}$.

Even in its strong version, the new system omits both (Dominance) and (Free Formula Independence), which we have argued against. We have investigated various postulates, absent from the HK set, giving grounds to include them in the new system. We have shown that (Subsumption Orientation) accounts for the other postulates and provides a justification for (Free Formula Independence) together with (Monotony), through focussing on minimal inconsistent subsets.

We do not hold that the new system, in basic or strong version, captures all desirable cases, we more modestly claim for improving over the original HK set. In particular, we believe that the HK postulates suffer from over-commitment to minimal inconsistent subsets. Crucially, such a comment applies to *postulates* (they would exclude all approaches that are not based upon minimal inconsistent subsets) but it does not apply to *measures* themselves: There are excellent reasons to develop a specific measure (Knight 2002) (Mu, Liu and Jin 2012) (Jabbour and Raddaoui 2013) ...

As to future work, we must mention taking seriously belief bases as multisets –giving a counterpart to the idea that e.g. $\{a \wedge b \wedge \neg a \wedge \neg b \wedge a \wedge b \wedge \neg a \wedge \neg b\}$ might be viewed as more inconsistent than $\{a \wedge b \wedge \neg a \wedge \neg b\}$.

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