Reasoning with Inconsistent and Uncertain Ontologies

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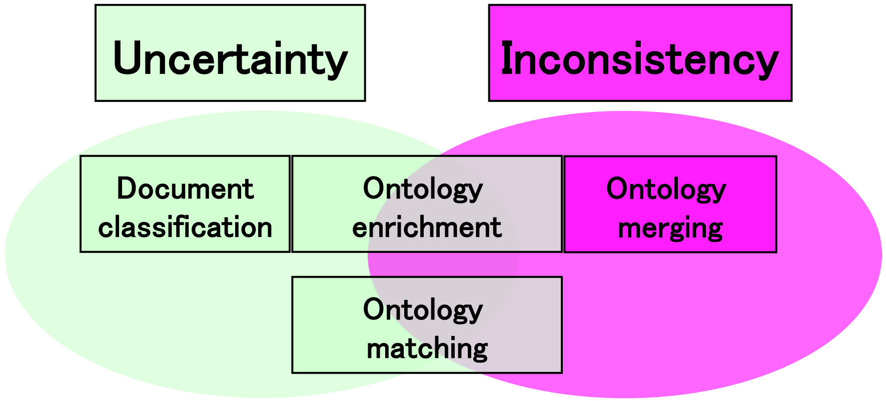
Motivation

❑ Ontologies are not always perfect



❑ Challenging problem

– Deal with both uncertainty and inconsistency

❖ Ontology learning, ontology matching

❖ Application domain: medicine and biology

❑ Solutions: probabilistic or possibilistic description logics

Outline

- ❑ Probabilistic logic vs possibilistic logic
- ❑ Probabilistic description logics
- ❑ Possibilistic description logics and its extension
- ❑ Revising ontologies in description logics
- ❑ Mapping repair in description logics

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Probabilistic Logic

- There are several versions of probabilistic logic
- We consider Nilsson's probabilistic logic (AI'86)
- Consider a set of sentences $\mathscr{L}$
  - Each sentence S is attached with two sets of *possible worlds*
    - Worlds in which S is true and worlds in which S is false
- Probability of a sentence S
  - Ω is the set of all possible worlds
  - $P(S)=P(\{\omega \in \Omega \mid S \text{ is true in } \omega\})$
- Formula: (S,a)
  - $P(S)=a$
  - Usually, we only know probabilities of some sentences

Probabilistic Logic (Cont.)

- Example

Consider {p, p→q, q}, there are four worlds

	ω_1	ω_2	ω_3	ω_4
p	true	true	false	false
p→q	true	false	true	true
q	true	false	true	false

Probabilistic Logic (Cont.)

- Probabilistic entailment
  - Given: probabilities of sentences in a set of sentences $\mathcal{B}$
  - Determine: probability of a sentence S
- Special case: $\mathcal{B}=\{p, p\rightarrow q\}$ and $S=q$

$$P(p\rightarrow q)+P(p)-1 \leq P(q) \leq P(p\rightarrow q)$$

- The probabilistic entailment problem can be solved by linear programming methods

Probabilistic Logic (Cont.)

- Properties
  - $P(\Omega)=1$, Ω is the set of all possible worlds
  - $P(\perp)=0$
  - If $Mod(S \wedge T)=\varnothing$, then $P(S \vee T)=P(S)+P(T)$
  - $P(S \vee T)=P(S)+P(T) - P(S \wedge T)$
  - $P(\neg S)=1-P(S)$
- Conditional probability
  - $P(T \mid S)=P(S \wedge T) \setminus P(S)$

Interval Probabilistic Logic

- Events
  - A set of basic events $\Phi=\{p_1,\cdots,p_l\}$
  - p_i | $\perp$, $\top$ | $\neg\phi$ | $\phi\wedge\psi$
- Conditional constraint: $(\psi|\phi)$ [l,u], l, u$\in$ [0,1]
  - E.g. (fly|bird)[0.95,1]
- Probabilistic formula
  - $(\psi|\phi)$ [l,u] | $\neg\phi$ | $\phi\wedge\psi$
  - E.g. $\neg$(bird|ostrich)[1,1]
- Logical constraint: $\psi \Leftarrow \phi$
- Probabilistic knowledge base KB=(L,P)
  - L is a finite set of logical constraints
  - P is a finite set of conditional constraints
  - $\varepsilon_1 \neq \varepsilon_2$ for any two distinct $(\varepsilon_1)[l_1,u_1]$, $(\varepsilon_2)[l_2,u_2] \in P$

Interval Probabilistic Logic

- Example
  - KB=({bird ⇐ eagle, feathers ⇐ bird},{(fly|bird)[0.95,1]})
  - bird ⇐ eagle: all eagles are birds
  - feathers ⇐ bird: all birds have feathers
  - (fly|bird)[0.95,1]: birds fly with a probability of at least 0.95

Interval Probabilistic Logic

- World: a truth assignment to the basic events
  - Associates with every basic event a binary truth value
  - Can be extended to all events by induction as usual
  - I_Φ denotes the set of all worlds for Φ
- Model: I is a model of ϕ iff $I(\phi)$=true
  - Denoted as $I \vDash \phi$
  - I is a model of a set of events L ($I \vDash L$) iff I is a model of all $\phi \in L$
- Satisfiability: ϕ is satisfiable iff a model of ϕ exists
- Logical consequence: $\phi \vDash \psi$ iff $I(\phi)$=true implies $I(\psi)$=true

Interval Probabilistic Logic

- Probabilistic interpretation Pr: a probability function on I_Φ
  - $Pr(\phi)$: sum of all $Pr(I)$ such that $I \models \phi$
- Conditioning:
  - $Pr(\psi \mid \phi)$: $Pr(\psi \wedge \phi)/Pr(\phi)$ with $Pr(\phi)>0$
  - $Pr_\phi(I)$: $Pr(I)/Pr(\phi)$ with $I \models \phi$ and 0 for other I
- Truth of logical constraints and probabilistic formulas
  - $Pr \models \psi \Leftarrow \phi$ iff $Pr(\phi)=Pr(\psi \wedge \phi)$ iff $Pr \models (\psi \mid \phi)[1,1]$
  - $Pr \models (\psi \mid \phi)[l,u]$ iff $Pr(\phi)=0$ or $Pr(\psi \mid \phi) \in [l,u]$
  - $Pr \models \neg F$ iff not $Pr \models F$
  - $Pr \models F \wedge G$ iff $Pr \models F$ and $Pr \models G$
- Satisfiability and logical consequences can be defined as usual

Interval Probabilistic Logic

- Tightest logical consequence: KB $\Vdash_{tight}$ $(\psi \mid \phi)[l,u]$ iff
  - Every model of $L \cup P$ is a model of $(\psi \mid \phi)[l,u]$ and
  - l is the infimum of $Pr(\psi \mid \phi)$ subject to all models Pr of $L \cup P$ with $Pr(\phi)>0$
  - u is the supremum of $Pr(\psi \mid \phi)$ subject to all models Pr of $L \cup P$ with $Pr(\phi)>0$
- Note: when $L \cup P \Vdash \bot \Leftarrow \phi$ then [l,u] is [1,0]
- Property 1: a logical constraint $\psi \Leftarrow \phi$ has the same meaning as the conditional constraints $(\psi \mid \phi)[1,1]$
- Property 2: model-theoretical logical entailment in probabilistic logic generalizes model-theoretical entailment in ordinary propositional logic

Interval Probabilistic Logic

- KB=({bird $\Leftarrow$ eagle, feathers $\Leftarrow$ bird},{(fly|bird)[0.95,1]})
  - bird $\Leftarrow$ eagle: all eagles are birds
  - feathers $\Leftarrow$ bird: all birds have feathers
  - (fly|bird)[0.95,1]: birds fly with a probability of at least 0.95
- Logical consequences of KB
  - KB $\VDash$ (feathers | bird)[1,1]
  - KB $\VDash$ (fly|bird)[0.95,1]
  - KB $\VDash$ (feathers | eagle)[1,1]
  - KB $\VDash$ (fly | eagle)[0,1]

Probabilistic properties of being able to fly is not inherited from birds to eagles

Interval Probabilistic Logic

- KB=({bird ⇐ ostrich},{(legs|bird)[1,1], (fly|bird)[1,1], (fly|ostrich)[0,0.05]})
  – $(\psi \mid \phi)[1,1]$ is interpreted as $\psi \Leftarrow \phi$
- Logical consequences of KB
  – KB ⊫ (legs|bird)[1,1]
  – KB ⊫ (fly|bird)[1,1]
  – KB ⊫ (legs|ostrich)[1,0]
  – KB ⊫ (fly| ostrich)[1,0]

There is a local inconsistency

Interval Probabilistic Logic

- Solution: interpret $(\psi \mid \phi)[1,1]$ as a default rule
  - Define probability rankings
- Probability ranking κ maps Pr to $\{0,1,\ldots\}\cup\{\infty\}$
  - $\kappa(Pr)=0$ for at least one Pr
  - If F is satisfiable, $\kappa(F)=\min\{\kappa(Pr) \mid Pr \vDash F\}$
  - Otherwise, $\kappa(F)=\infty$
- Pr verifies $(\psi \mid \phi)[l,u]$ iff $Pr(\phi)>0$ and $Pr \vDash (\psi \mid \phi)[l,u]$
  - Pr falsifies $(\psi \mid \phi)[l,u]$ iff $Pr(\phi)>0$ and $Pr \nvDash (\psi \mid \phi)[l,u]$
- P tolerates C under L: $L \cup P$ has a model that verifies C
  - P is under L in conflict C with iff no model of $L \cup P$ verifies C

Interval Probabilistic Logic

- κ is admissible with $KB=(L,P)$
  - $\kappa(\neg(\psi \mid \phi)[1,1])=\infty$ for all $\psi \Leftarrow \phi \in L$ and
  - $\kappa(\phi>0)<\infty$ and $\kappa(\phi>0 \wedge (\psi \mid \phi)[l,u]) < \kappa(\phi>0 \wedge \neg(\psi \mid \phi)[l,u])$ for all $(\psi \mid \phi)[l,u] \in P$

Minimal $\kappa(Pr)$ of all Pr verifying $(\psi \mid \phi)[l,u]$

- z-partition of KB: unique ordered partition $(P_0,\ldots,P_k)$ of P su

F in a higher rank is more specific and preferred

  - Each P_i, $0\le i\le k$, is the set of all $C\in \bigcup_{j=i}^{k} P_j$ tolerated under L by $\bigcup_{j=i}^{k} P_j$
- Pr is lex-preferable to Pr' iff some i exists such that
  - $|\{C \in P_i \mid Pr \vDash C\}| > |\{C \in P_i \mid Pr' \vDash C\}|$ and
  - $|\{C \in P_j \mid Pr \vDash C\}| = |\{C \in P_j \mid Pr' \vDash C\}|$ for all $i<j\le k$
- Lex-minimal model Pr of F: ... del of F is lex-preferable to Pr

Pr satisfies more constraints in rank i than and Pr' and satisfies the same constraints in ranks higher than i as Pr'

Interval Probabilistic Logic

- Lex-entailment: $(\psi \mid \phi)[l,u]$ is a lex-consequence of KB, denoted KB $\Vdash^{lex} (\psi \mid \phi)[l,u]$ iff
  - Each lex-minimal model of $\mathcal{L} \cup \{\phi>0\}$ satisfies $(\psi \mid \phi)[l,u]$

- Tight lex-entailment: $(\psi \mid \phi)[l,u]$ is a tight lex-consequence of KB, denoted as KB $\Vdash^{lex,tight} (\psi \mid \phi)[l,u]$ iff
  - $l=\inf Pr(\psi \mid \phi)$ (resp., $u=\sup Pr(\psi \mid \phi)$) subject to all lex-minimal models Pr of $\mathcal{L} \cup \{\phi>0\}$

Interval Probabilistic Logic

- KB=({bird $\Leftarrow$ ostrich},{(legs|bird)[1,1], (fly|bird)[1,1], (fly|ostrich)[0,0.05]})
  - $(\psi \mid \phi)[1,1]$ is interpreted as $\psi \leftarrow \phi$ (default rule)
- Logical consequences of KB
  - KB $\Vdash$ (legs|ostrich)[1,0]
  - KB $\Vdash$ (fly| ostrich)[1,0]
- Lex-consequences of KB
  - KB $\Vdash$ (legs|ostrich)[1,1]
  - KB $\Vdash$ (fly| ostrich)[0,0.05]

Possibility Theory

- Possibility distribution $\pi: \Omega \to L$
  - Ω represents universe of discourse
  - $(L,<)$ is a bounded total ordered scale
  - $\pi(\omega) \geq \pi(\omega')$ means ω is a priori more plausible than ω'
- Possibility measure and necessity measure

$\Pi(A)=\sup\{\pi(\omega): \omega \in A\}$ $\quad$ $N(A)=1-\Pi(\neg A)$

- Property
  - $\Pi(A \cup B)=\max(\Pi(A), \Pi(B))$
  - $N(A \cap B)=\min(N(A), N(B))$

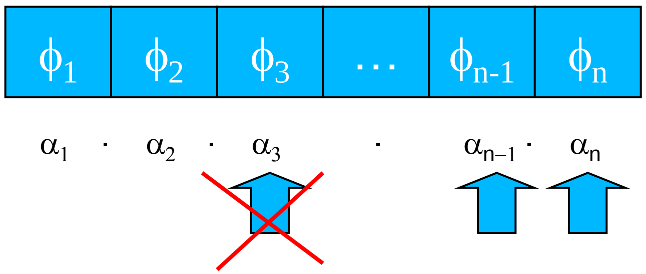
Possibilistic Logic

- **Syntax**
  - Possibilistic formula: (ϕ, a) denotes *certainty degree* of $\phi \geq a$
    - Example: (eat_fish(Tweety), 0.46) We are somewhat certain that Tweety eats fish
  - Possibilistic knowledge base: a set of possibilistic formulae

    $B=\{(\phi_i, a_i) : i=1,\cdots,n\}$
- **Example**
  - p: there were human beings in Mars before
  - q: scientists have detected some strange signals from outer space
  - r: there are aliens in other planets
  - s: the ancestors of human are gorillas
  - $B=\{(p, 0.4), (q \rightarrow r, 1), (s, 0.8), (\neg s \rightarrow \neg r, 0.9)\}$

Possibilistic Logic (Cont.)

- Inconsistency degree of B
  - B*: classical base of B
  - $B_{\geq a}=\{\phi \in B^* \mid (\phi,b) \in B, B \geq a\}$
  - $Inc(B)=\max\{a: B_{\geq a}$ is inconsistent$\}$



$Inc(B)=a_3$

- $B=\{(q,1), (q \rightarrow r, 1), (s \rightarrow \neg r, 0.9), (s, 0.8), (p, 0.4)\}$

What is Inc(B)?

Possibilistic Logic

❑ Possibilistic inference

- ϕ is a plausible consequence of B, denoted $B \vdash_P \phi$ iff $B_{>inc(B)} \vdash \phi$
- (ϕ, a) is a consequence of B, denoted $B \vdash (\phi, a)$ iff $a > Inc(B)$, and $B_{\geq a} \vdash \phi$
- ϕ is a possibilistic consequence of B to degree a, denoted $B \vdash_\pi (\phi, a)$ iff
  - ❖ $B_{\geq a}$ is consistent
  - ❖ $B_{\geq a} \vdash \phi$
  - ❖ For all $b > a$, $B_{\geq b} \nvdash \phi$

Only formulas whose weights are greater than or equal to the inconsistency degree are used

Possibilistic Logic (Cont.)

- Example

B={(q,1), (q→r, 1), (¬s→¬r, 0.9) , (s, 0.9) ,(p, 0.4)}

- $B \vdash_{\pi} (q, 1)$
- $B \vdash_{\pi} (q \rightarrow r, 1)$
- $B \nvdash_{\pi} (p, 0.4)$

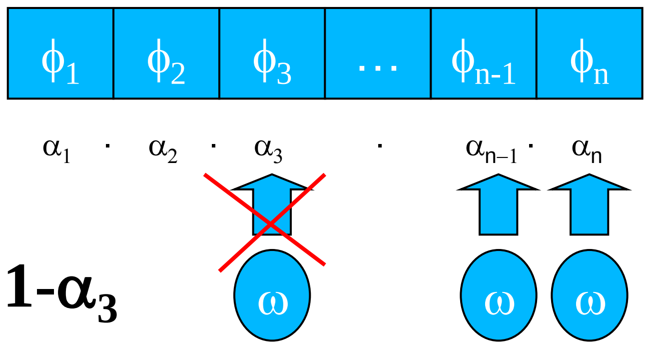
Drowning effect

Possibilistic Logic

- Semantics
  - Possibility distribution π: $\Omega \to [0,1]$
    - $\pi(\omega)=1$: ω is totally possible
    - $\pi(\omega)=0$: ω is impossible
    - $\pi(\omega)>0$: ω is possible
- Normal possibility distribution π: there exists one world ω such that $\pi(\omega)=1$
- Satisfaction: π satisfies (ϕ,a), denoted $\pi \models (\phi,a)$, iff $N(\phi) \geq a$
  - $\pi \models$ B iff $\pi \models (\phi,a)$, for all $(\phi,a) \in$ B
  - B$\models(\phi,a)$ iff for every $\pi \models$ B, we have $\pi \models (\phi,a)$

Possibilistic Logic (Cont.)

- Possibilistic knowledge base $B=\{(\phi_1,a_1),\cdots,(\phi_n,a_n)\} \Rightarrow$ *a* unique possibility distribution



$\pi(\omega)=1-\alpha_3$

- A possibility distribution $\Rightarrow$ a possibilistic knowledge base
  - $\Pi(\phi)=\max\{\pi(\omega): \omega \vDash \phi\}$
  - $N(\phi)=1-\Pi(\neg\phi)$

Possibilistic Logic

- Soundness and completeness
  - $B \vdash (\phi, a)$ iff $B \vDash (\phi, a)$
  - $B \vdash_P \phi$ iff $N_B(\phi) > Inc(B)$
  - $B \vdash_\pi (\phi, a)$ iff $N_B(\phi) \geq a$ and $a > Inc(B)$

Generalizations of Possibilistic Logic

- Linear-order inference
  - B is stratified as $(S_1,\ldots, S_k)$
    - Formulas in S_i have the same weights
    - The weight of formulas in S_i is greater than that of formulas in S_j with i<j
    - $K_{LO,B}=\cup S'_i$ with $S'_i=S_i$ if $S'_1\cup\ldots\cup S'_{n-1} \cup S_i$ is consistent

    and $S'_i=\varnothing$ otherwse
    - $B\vdash_{LO} \phi$ iff $K_{LO,B} \vdash\phi$
- Example: $B=\{(q,1), (q\rightarrow r, 1), (s\rightarrow\neg r, 0.9), (s, 0.9), (t,0.9), (p, 0.4)\}$
  - $B\vdash_{LO} p$
  - $B\nvdash_{LO} t$

Generalizations of Possibilistic Logic

- Lexicographic inference
  - B is stratified as $(S_1,\dots, S_k)$
  - For $(S'_1,\dots, S'_k)$ and $(S''_1,\dots, S''_k)$ which are subsets of $(S_1,\dots, S_k)$
  - $(S'_1,\dots, S'_k)$ is preferred to $(S''_1,\dots, S''_k)$ iff some i exists such that
    - ❖ $|S'_i| > |S''_i|$ and
    - ❖ $|S'_j| = |S''_j|$ for all $i<j\leq k$
  - $B\vdash_{lex} \phi$ iff $S'_1\cup\dots \cup S'_k \vdash\phi$ for all lexi-preferred subest $(S'_1,\dots, S'_k)$ of $(S_1,\dots, S_k)$
- Example: $B=\{(q,1), (q\rightarrow r, 1), (s\rightarrow\neg r, 0.9) , (s, 0.9) ,(t,0.9), (p, 0.4)\}$
  - $B\vdash_{Lex} p$
  - $B\vdash_{Lex} t$

What are lexi-preferred subsets of $(S_1,\dots, S_k)$?

Comparison

- **Properties**
  - Probabilistic logic
    - If $Mod(S \wedge T)=\varnothing$, then $P(S \vee T)=P(S)+P(T)$
    - $P(S \vee T)=P(S)+P(T) - P(S \wedge T)$
    - $P(\neg S)=1-P(S)$
  - Possbilistic logic
    - $\Pi(A \cup B)=\max(\Pi(A), \Pi(B))$
    - $N(A \cap B)=\min(N(A), N(B))$
- **Types of uncertainty**
  - Probabilistic logic: quantitative
  - Possibilistic logic: qualitative

Comparison

- Inconsistency
  - Probabilistic logic
    - Use probabilistic semantics
  - Possbilistic logic
    - Use standard first-order semantics
- Example
  - KB=({bird ⇐ ostrich},{(legs|bird)[1,1], (fly|bird)[1,1], (fly|ostrich)[0,0.05]}) is inconsistent under probabilistic semantics
    - (fly|ostrich)[0,0.05] can be inferred under probabilistic default semantics
  - KB′={(ostrich→bird, 1),(bird→legs,1), (bird→fly, 1), (ostrichfly→fly, 0.05) is not inconsistent under possibilistic semantics
    - (ostrich→fly,1) can be inferred under possibilistic semantics

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Description Logics

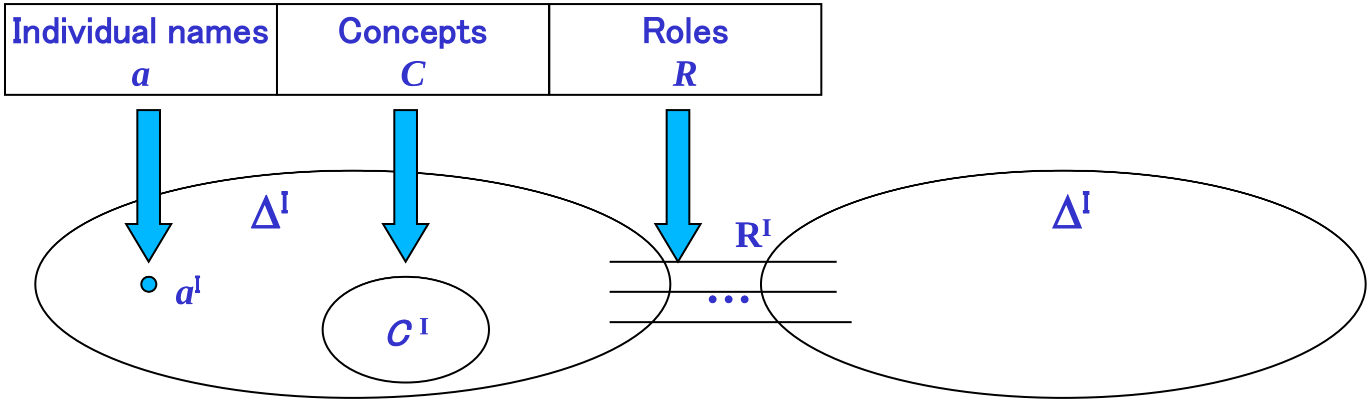
- Description logics
  - Are (mostly) decidable fragments of first-order predicate logic
  - Provide logical underpinning of W3C standard OWL
- Building blocks
  - Concepts (unary predicates/formulae with one free variable)
    - E.g., Person, Lawer $\sqcup$ Doctor
  - Roles (binary predicates/formulae with two free variables)
    - E.g., hasChild
  - Individuals (constants)
    - E.g., John, Mary

Description Logics (Syntax)

- Description languages
  - Defining complex concepts: sets of individuals
  - Defining complex roles: binary relations on individuals
- Complex concepts are built by
  - Atomic concepts: Tissue, Heart
  - Constructors: $\text{Tissue} \sqcap \exists \text{part-of.Heart}$
- Complex roles are built by
  - Atomic roles: part-of, has-location
  - Constructors: HasFather^{-}

Description Logics (Semantics)

- Interpretation: $I=(\Delta^I, .^I)$
  - Domain: Δ^I
  - Assignment function $.^I$



Description Logics (Cont.)

- Interpretation: $I=(\Delta^I, \cdot^I)$

Construct	Syntax	Example	Semantics
Atomic concept	**A**	**Heart**	$A^I \subseteq \Delta^I$
Atomic role	**R**	**part-of**	$R^I \subseteq \Delta^I \times \Delta^I$
Negation	$\neg$ **C**	$\neg$ **Heart**	$\Delta^I \setminus C^I$
Conjunction	**C** $\sqcap$ **D**	**Lawyer**$\sqcap$**Doctor**	$C^I \setminus D^I$
Value restriction	$\forall$ **R.C**	$\forall$ **part-of.Wood**	$\{a \mid \forall b.\ (a,b) \in R^I, (a,b) \in C^I\}$
…	…	…	…

Description Logics (Ontology)

- TBox T: defining terminology of application domain
  - Inclusion assertion on concept : $C \sqsubseteq D$

    Pericardium ⊑ Tissue ⊓ ∃ part-of.Heart

  - Inclusion assertion on roles: $R \sqsubseteq S$

    Part-of ⊑ has-location

- ABox A: stating facts about a specific "world"
  - membership assertion: C(a) or R(a,b)

    HappyMan(Bob), HasChild(Bob, Mary)

Description Logics(Semantics)

- Given an interpretation $\mathcal{I}$
- Semantics of TBox axioms
  - $\mathcal{I} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
  - $\mathcal{I} \models R \sqsubseteq S$ if $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
- Semantics of ABox assertions
  - $\mathcal{I} \models C(a)$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
  - $\mathcal{I} \models R(a,b)$ if $(a^{\mathcal{I}},b^{\mathcal{I}}) \in R^{\mathcal{I}}$

Description Logics(Semantics)

- Model of an ontology $O = \langle T, A \rangle$
  - I is a model of O if it satisfies all axioms in T and all assertions in A
- Concept satisfiability
  - Concept C is satisfiable in O if C^{I} is nonempty for some model I of O
- Ontology Entailment
  - $O \models \phi$ iff $I \models \phi$ for all models I of O

Description Logics (Semantics)

- Incoherent ontology: ontology with at least one unsatisfiable concept
  - Example: {PhDStudent $\sqsubseteq$ Student,

    PhDStudent $\sqsubseteq$ Employee,

    Student $\sqsubseteq \neg$Employee
- Inconsistent ontology: ontology without a model
  - Example: {PhDStudent $\sqsubseteq$ Student,

    PhDStudent $\sqsubseteq$ Employee,

    Student $\sqsubseteq \neg$Employee,

    PhDStudent(John)}

Incoherent ontology can be consistent!

Description Logics

- Example: DICE ontology
  - Brain $\sqsubseteq$ CentralNervousSystem $\sqcap$ $\exists$systempart.NervousSystem $\sqcap$ BodyPart $\sqcap$ $\exists$ region.HeadAndNeck $\sqcap$ $\forall$region.HeadAndNeck
  - CentralNervousSystem $\sqsubseteq$ NervousSystem
  - BodyPart $\sqsubseteq \neg$NervousSystem or

    DisjointWith(BodyPart,NervousSystem)

Description Logics

- Example from Foaf
  - Person(timbl)
  - Homepage(timbl, http://w3.org/)
  - Homepage(w3c, http://w3.org/)
  - Organization(w3c)
  - InverseFunctionalProperty(Homepage)
  - DisjointWith(Organization, Person)
- Example from OpenCyc
  - ArtifactualFeatureType(PopulatedPlace)
  - ExistingStuffType(PopulatedPlace)
  - DisjointWith(ExistingObjectType,ExistingStuffType)
  - ArtifactualFeatureType $\sqsubseteq$ ExistingObjectType

Description Logics

- Deficiency of DLs
  - Cannot express uncertain information

    I am quite sure that a heart patient has a private health insurance

    I am a little certain that Tom is a heart patient
  - Cannot deal with inconsistency
- Syntax and semantics of DLs need to be extended

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Probabilistic Description Logics
–Syntax

- Classification of individuals: a set of classical individuals $\mathbf{I}_C$ and a finite set of probabilistic individuals $\mathbf{I}_P$
- Basic classification concept (basic c-concept): DL concepts that are free of probabilistic individuals from $\mathbf{I}_P$
- c-concept:
  - Every basic concept is a c-concept
  - If C and D are C-concept, $\neg C$ and $C \sqcap D$ are c-concepts
- Conditional constraint: $(D\,|\,C)[l,u]$
  - C and D are c-concepts
  - Meaning: probability of D given C lies between l and u

Probabilistic Description Logics

❑ Example

A terminological probabilistic knowledge base
ax_1: (HighBloodPressure | Heartpatient) [1,1]
ax_2: (¬HasHighBloodPressure | PacemakerPatient) [1,1]
ax_3: (MalePacemakerPatient | PacemakerPatient) [0.4,1]
ax_4: (Male | PacemakerPatient) [0.4,1]
ax_5: PacemakerPatient $\sqsubseteq$ Heartpatient

Probabilistic Description Logics
–Syntax

- PTBox $PT=(T,P)$
  - T: classical DL knowledge base
  - P: is a finite set of conditional constraints
- PABox P_o
  - o is a probabilistic individual in $\mathbf{I}_P$
  - P_o is a finite set of conditional constraints
    - $(D\,|\,C)[l,u] \in P_o$: if $C(o)$ holds then $D(o)$ holds with a probability between l and u
    - $(D\,|\,\top)[l,u] \in P_o$: $D(o)$ holds with a probability between l and u
    - $(\exists R.\{o'\}\,|\,C)[l,u] \in P_o$: if $C(o)$ holds then $R(o,o')$ holds with a probability between l and u

Probabilistic Description Logics

❑ Example

A terminological probabilistic knowledge base
ax_1: (HighBloodPressure | Heartpatient) [1,1] ax_2: (¬HasHighBloodPressure | PacemakerPatient) [1,1] ax_3: (MalePacemakerPatient | PacemakerPatient) [0.4,1] ax_4: (Male | PacemakerPatient) [0.4,1] ax_5: PacemakerPatient $\sqsubseteq$ Heartpatient
A PABox for the probabilistic individual Tom
ax_6: (PacemakerPatient | $\top$) [0.8,1]

Probabilistic Description Logics
–Syntax

- Probabilistic knowledge base KB=($T, P, (P_o)_{o \in \mathbf{I}_P}$)
  - P : probabilistic knowledge about randomly chosen individuals
    - Conditional constraints in P are default statements
  - P_o: probabilistic knowledge about the concrete individual o
    - Conditional constraints in every P_o with $o \in \mathbf{I}_P$ are strict statements

Probabilistic Description Logics
–Semantics

- World $\mathcal{I}$: a set of basic c–concepts $C \in \mathcal{C}$ such that $\{C(i) \mid C \in I\} \cup \{\neg C(i) \mid C \in \mathcal{C} \setminus \mathcal{I}\}$ is satisfiable
  - $\mathcal{C}$ is the set of all basic c–concepts
  - i is a new individual
  - $\mathcal{J}_{\mathcal{C}}$ denotes the set of all worlds relative to $\mathcal{C}$
- Model of a TBox: $\mathcal{I}$ is a model of $\mathcal{T}$ ($\mathcal{I} \models \mathcal{T}$) iff $\mathrm{T} \cup \{C(i) \mid C \in \mathcal{I}\} \cup \{\neg C(i) \mid C \in \mathcal{C} \setminus \mathcal{I}\}$ is satisfiable
- Model of a c–concept: $\mathcal{I} \models C$
  - $\mathcal{I} \models C$ iff $C \in \mathcal{I}$, C is a basic c–concept
  - $\mathcal{I} \models \neg C$ iff $\mathcal{I} \models C$ does not hold
  - $\mathcal{I} \models C \sqcap D$ iff $\mathcal{I} \models C$ and $\mathcal{I} \models D$

Probabilistic Description Logics
–Semantics

- Probabilistic interpretation Pr : a probability function on $\mathcal{I}_{\mathcal{C}}$
  - $\mathcal{I}_{\mathcal{C}}$ denotes the set of all worlds relative to $\mathcal{C}$
  - $Pr(C)$: sum of all $Pr(I)$ such that $I \models C$
- $Pr \models T$: $I \models T$ for every $I \in \mathcal{I}_{\mathcal{C}}$ such that $Pr(I)>0$
- $Pr \models (D|C)[l,u]$: $Pr(C)=0$ or $Pr(D|C) \in [l,u]$:
  - $Pr(D|C) = Pr(D \sqcap C) / Pr(C)$
- $Pr \models \mathcal{F}$: $I \models F$ for every $F \in \mathcal{F}$
  - $\mathcal{F}$ is a set of conditional constraints

Probabilistic Description Logics –Semantics

- Pr verifies $(D|C)[l,u]$ iff $Pr(C)=1$ and $Pr \models (D|C)[l,u]$
  - Pr falsifies $(D|C)[l,u]$ iff $Pr(C)=1$ and $Pr \not\models (D|C)[l,u]$
- $\mathcal{F}$ tolerates $(D|C)[l,u]$ under T: $T \cup \mathcal{F}$ has a model that verifies $(D|C)[l,u]$
- Consistency of a PTBox $PT=(T,P)$
  - T is satisfiable
  - There exists an ordered partition $(P_0,\ldots,P_k)$ of P such that
    - Each P_i, $0 \le i \le k$, is the set of all $F \in P_i \cup \ldots \cup P_k$ tolerated under T by $P_i \cup \ldots \cup P_k$
    - Idea of the partition: when in conflict, remove conditional constraints in the lower strata
    - The partition follows the rule of maximum specificity
    - Called z-partition of PT

Probabilistic Description Logics

- Example

	A terminological probabilistic knowledge base
P_0	ax_1: (HighBloodPressure | Heartpatient) [1,1]
P_1	ax_2: (¬HasHighBloodPressure | PacemakerPatient) [1,1] ax_3: (MalePacemakerPatient | PacemakerPatient) [0.4,1] ax_4: (Male | PacemakerPatient) [0.4,1] ax_5: PacemakerPatient $\sqsubseteq$ Heartpatient
	A PABox for the probabilistic individual Tom
	ax_6: (PacemakerPatient | $\top$) [0.8,1]

Probabilistic Description Logics
-Semantics

- Pr is lex-preferable to Pr' iff some i exists such that
  - $|\{F \in P_i \mid Pr \models F\}| > |\{F \in P_i \mid Pr' \models F\}|$ and
  - $|\{F \in P_j \mid Pr \models F\}| = |\{F \in P_j \mid Pr' \models F\}|$ for all $j<i\leq k$
- Lex-minimal model Pr of F: no model of F is lex-preferable to Pr
- Lex-entailment: $(D|C)[l,u]$ is a lex-consequence of $\mathcal{F}$ under PT, denoted $\mathcal{F} \Vdash^{lex} (D|C)[l,u]$ iff
  - Each lex-minimal model of $T \cup \mathcal{F} \cup \{(C \mid \top)[1,1]\}$ satisfies $(D|C)[l,u]$
- Tight lex-entailment: $(D|C)[l,u]$ is a tight lex-consequence of $\mathcal{F}$ under PT, denoted as $\mathcal{F} \Vdash^{lex,tight} (D|C)[l,u]$ iff
  - $l=\inf Pr(\psi \mid \phi)$ (resp., $u=\sup Pr(\psi \mid \phi)$) subject to all lex-minimal models Pr of $L \cup \{\phi>0\}$

Probabilistic Description Logics

- Example: the following are tight lex-consequence of *PT*

A terminological probabilistic knowledge base	
P_0	ax_1: (HighBloodPressure | Heartpatient) [1,1]
P_1	ax_2: (¬HasHighBloodPressure | PacemakerPatient) [1,1]
	ax_3: (MalePacemakerPatient | PacemakerPatient) [0.4,1]
	ax_4: (Male | PacemakerPatient) [0.4,1]
	ax_5: PacemakerPatient $\sqsubseteq$ Heartpatient

  – (HighBloodPressure | Male $\sqcap$ Heartpatient) [1,1]

  – (MalePacemakerPatient | PacemakerPatient) [0.4,1]

- But (HasHighBloodPressure | PacemakerPatient) [1,1] is not inferred

Log-linear Description Logics

- Knowledge base $K=(K^D, K^U)$
  - K^D: a classical DL knowledge base
  - K^U : a set of weighted axioms (like possibilistic DLs)
- Semantics
  - Based on probability distributions over consistent knowledge bases

$$K^D \subseteq K' \subseteq K^D \bigcup \{c:(c, w_c \in K^U\}$$

$$\Pr_K(K') = \begin{cases} \frac{1}{Z}\exp(\sum_{\{c \in K' \setminus K^D\}} w_c) & \text{if } K' \text{ consistent} \\ 0 & \text{otherwise} \end{cases}$$

  - Marginal probability of an axiom: sum of the probabilities of theconsistent knowledge bases containing it

Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
- **Possibilistic description logics and its extension**
- Revising ontologies in description logics
- Mapping repair in description logics

Possibilistic Description Logics

A possibilistic DL knowledge base

Tool(amilcare): 1.0
Tool(cavido): 0.46
Application(cavido): 0.46
Tool $\sqsubseteq$ Application : 0.3

Tool(cavido): 0.46

Tool $\sqsubseteq$ Application : 0.3

Note : Inference is non-trivial when inconsistency exists
Certainty degree is attached to inferred axiom

Possibilistic Description Logics
–Syntax

- Possibilistic axiom: (ϕ, a), ϕ is a DL axiom
- Possibilistic DL knowledge base $B=\{(\phi_i, a_i): i=1,\cdots,n\}$
- Classical DL Base $B^*=\{\phi: (\phi, a)\in B\}$
- α–cut of B: $B_{\geq\alpha}=\{\phi\in B^*: (\phi, b)\in B$, and $b\geq a\}$
- Inconsistency degree: $Inc(B)=\max\{a: B_{\geq a}$ is inconsistent$\}$
- Example:

  B={(Tool(amilcare), 1), (Application(cavido), 0.46),

  (Tool(cavido), 0.46), (disjoint(Tool, Application), 0.3)}

Possibilistic Description Logics
–Semantics

- **Possibility distribution** π: **I** $\rightarrow$ [0,1], **I is the set of all interpretations**
  - $\pi(I)$ represents the degree of compatibility of I with available information
  - $\pi(I_1)>\pi(I_2)$: I_1 is preferred to I_2
- **Possibility measure** Π

  $\Pi(\phi)=\max\{\pi(I): I\in \mathbf{I}, I \vDash \phi\}$

- **Necessity Measure** N:

  $N(\phi)=1-\max\{\pi(I): I\nvDash\phi\}$

Possibilistic Description Logics

- Example

A possibilistic DL knowledge base
ax_1: (Heartpatient $\sqsubseteq$ HighBloodPressure, 1.0)
ax_2: (PacemakerPatient $\sqsubseteq$ $\neg$HighBloodPressure, 1.0)
ax_3: (HeartPatient $\sqsubseteq$ $\exists$HasHealthInsurance.PrivateHealth,0.9)
ax_4: (PacemakerPatient(Tom), 0.8)

Possibilistic Description Logics

- Inference services: instance checking

A possibilistic DL knowledge base
ax_1: (Heartpatient $\sqsubseteq$ HighBloodPressure, 1)
ax_2: (PacemakerPatient $\sqsubseteq$ ¬HighBloodPressure, 1)
ax_3: (HeartPatient $\sqsubseteq$ ∃HasHealthInsurance.PrivateHealth,0.9)
ax_4: (PacemakerPatient(Tom), 0.8)
ax_5: (HeartPatient(Tom),0.5)
ax_6: (HeartPatient $\sqsubseteq$ MalePacemakerPatient, 0.4)

¬ HighBloodPressure(Tom)

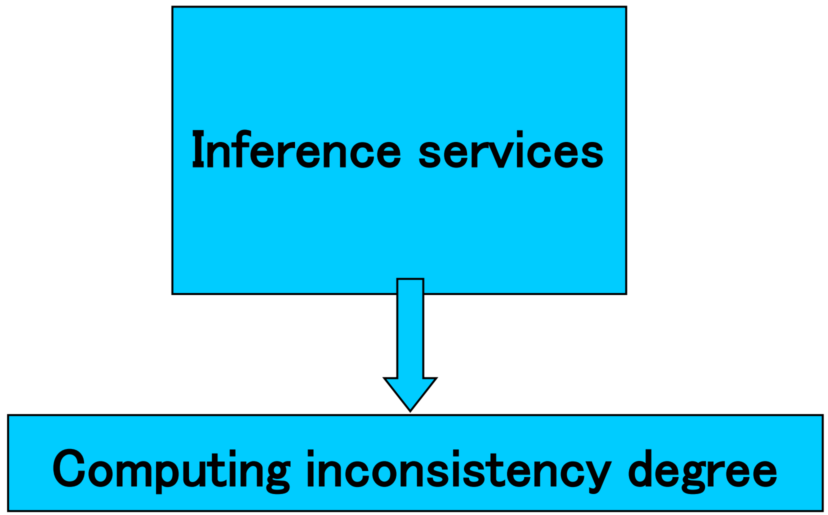
Possibilistic Description Logics

- ❑ Inference services: instance checking with weight

A possibilistic DL knowledge base
ax_1: (Heartpatient $\sqsubseteq$ HighBloodPressure, 1)
ax_2: (PacemakerPatient $\sqsubseteq$ $\neg$HighBloodPressure, 1)
ax_3: (HeartPatient $\sqsubseteq$ $\exists$HasHealthInsurance.PrivateHealth,0.9)
ax_4: (PacemakerPatient(Tom), 0.8)
ax_5: (HeartPatient(Tom),0.5)
ax_6: (HeartPatient $\sqsubseteq$ MalePacemakerPatient, 0.4)

$\neg$ HighBloodPressure(Tom): 0.8

Reduction



Algorithms

- A black-box algorithm (Qi et.al. ECSQARU2007, IJIS 2011)
  - Idea: search the weights by a binary search
    - Call a standard DL reasoner to check inconsistency
  - A system called PossDL has been implemented
- A tableaux algorithm (Qi and Pan ASWC 2008)
  - Idea: extending classical tableaux algorithm for DL ALC
    - A weight is attached to a concept name or a role name
  - No implementation is done

Generalizations of Possibilistic Description Logics

- Linear order inference (Qi et.al. ECSQARU2007, IJIS 2011)
  - Algorithm idea: compute the inconsistency degree and remove axioms whose weights are equal to it
    - Call a standard DL reasoner to check inconsistency
  - PossDL provides functionalities to compute consequences of linear order inference
- Lexicographic inference (Du and Qi RR 2008)
  - Algorithm idea: compile the DL axioms to propositional programs
  - No implementation has been done