Reasoning with Inconsistent and Uncertain Ontologies

Guilin Qi
Southeast University
China
gqi@seu.edu.cn

Reasoning Web 2012 September 05, 2012
Motivation

- Ontologies are not always perfect

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Inconsistency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Document classification</td>
<td>Ontology enrichment</td>
</tr>
<tr>
<td>Ontology matching</td>
<td></td>
</tr>
</tbody>
</table>

- Challenging problem
  - Deal with both uncertainty and inconsistency
    - Ontology learning, ontology matching
    - Application domain: medicine and biology

- Solutions: probabilistic or possibilistic description logics
Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
- Possibilistic description logics and its extension
- Revising ontologies in description logics
- Mapping repair in description logics
Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
- Possibilistic description logics and its extension
- Revising ontologies in description logics
- Mapping repair in description logics
Probabilistic Logic

- There are several versions of probabilistic logic
- We consider Nilsson’s probabilistic logic (AI’86)
- Consider a set of sentences $\mathcal{L}$
  - Each sentence $S$ is attached with two sets of possible worlds
    - Worlds in which $S$ is true and worlds in which $S$ is false

- Probability of a sentence $S$
  - $\Omega$ is the set of all possible worlds
  - $P(S) = P(\{\omega \in \Omega \mid S \text{ is true in } \omega\})$

- Formula: $(S,a)$
  - $P(S) = a$
  - Usually, we only know probabilities of some sentences
Example

Consider \{p, p\rightarrow q, q\}, there are four worlds

<table>
<thead>
<tr>
<th></th>
<th>(\omega_1)</th>
<th>(\omega_2)</th>
<th>(\omega_3)</th>
<th>(\omega_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>true</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>p\rightarrow q</td>
<td>true</td>
<td>false</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>q</td>
<td>true</td>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>
Probabilistic Logic (Cont.)

- **Probabilistic entailment**
  - Given: probabilities of sentences in a set of sentences $\mathcal{B}$
  - Determine: probability of a sentence $S$

- **Special case:** $\mathcal{B} = \{p, p \rightarrow q\}$ and $S = q$

  \[
  P(p \rightarrow q) + P(p) - 1 \leq P(q) \leq P(p \rightarrow q)
  \]

- The probabilistic entailment problem can be solved by linear programming methods
Probabilistic Logic (Cont.)

- **Properties**
  - $P(\Omega) = 1$, $\Omega$ is the set of all possible worlds
  - $P(\bot) = 0$
  - If $\text{Mod}(S \land T) = \emptyset$, then $P(S \lor T) = P(S) + P(T)$
  - $P(S \lor T) = P(S) + P(T) - P(S \land T)$
  - $P(\neg S) = 1 - P(S)$

- **Conditional probability**
  - $P(T | S) = \frac{P(S \land T)}{P(S)}$
Interval Probabilistic Logic

Events
- A set of basic events $\Phi = \{\rho_1, \ldots, \rho_i\}$
- $\rho_i | \bot, \top | \neg \phi | \phi \land \psi$

Conditional constraint: $(\psi | \phi) [l,u], l, u \in [0,1]$
- E.g. $(\text{fly} | \text{bird})[0.95,1]$

Probabilistic formula
- $(\psi | \phi) [l,u] | \neg \phi | \phi \land \psi$
- E.g. $\neg(\text{bird} | \text{ostrich})[1,1]$

Logical constraint: $\psi \iff \phi$

Probabilistic knowledge base $\text{KB} = (L, P)$
- $L$ is a finite set of logical constraints
- $P$ is a finite set of conditional constraints
- $\varepsilon_1 \neq \varepsilon_2$ for any two distinct $(\varepsilon_1)[l_1, u_1], (\varepsilon_2)[l_2, u_2] \in P$
Interval Probabilistic Logic

Example

- $KB=\{\text{bird} \iff \text{eagle}, \text{feathers} \iff \text{bird}\},\{(\text{fly} \mid \text{bird})[0.95,1]\})$
- bird $\iff$ eagle: all eagles are birds
- feathers $\iff$ bird: all birds have feathers
- $(\text{fly} \mid \text{bird})[0.95,1]$: birds fly with a probability of at least 0.95
Interval Probabilistic Logic

- **World:** a truth assignment to the basic events
  - Associates with every basic event a binary truth value
  - Can be extended to all events by induction as usual
  - $I_\Phi$ denotes the set of all worlds for $\Phi$

- **Model:** $I$ is a model of $\phi$ iff $I(\phi)=$true
  - Denoted as $I \models \phi$
  - $I$ is a model of a set of events $\mathcal{L}$ ($I \models \mathcal{L}$) iff $I$ is a model of all $\phi \in \mathcal{L}$

- **Satisfiability:** $\phi$ is satisfiable iff a model of $\phi$ exists

- **Logical consequence:** $\phi \models \psi$ iff $I(\phi)=$true implies $I(\psi)=$true
Interval Probabilistic Logic

- Probabilistic interpretation $Pr$: a probability function on $I_\Phi$
  - $Pr(\phi)$: sum of all $Pr(I)$ such that $I \models \phi$

- Conditioning:
  - $Pr(\psi \mid \phi)$: $Pr(\psi \land \phi)/Pr(\phi)$ with $Pr(\phi) > 0$
  - $Pr_\phi(I)$: $Pr(I)/Pr(\phi)$ with $I \models \phi$ and 0 for other $I$

- Truth of logical constraints and probabilistic formulas
  - $Pr \models \psi \iff \phi$ iff $Pr(\phi) = Pr(\psi \land \phi)$ iff $Pr \models (\psi \mid \phi)[1,1]$
  - $Pr \models (\psi \mid \phi)[1,u]$ iff $Pr(\phi) = 0$ or $Pr(\psi \mid \phi) \in [1,u]$
  - $Pr \models \neg F$ iff not $Pr \models F$
  - $Pr \models F \land G$ iff $Pr \models F$ and $Pr \models G$

- Satisfiability and logical consequences can be defined as usual
Interval Probabilistic Logic

- Tightest logical consequence: $\text{KB} \models_{\text{tight}} (\psi \mid \phi)[l,u]$ iff
  - Every model of $L \cup P$ is a model of $(\psi \mid \phi)[l,u]$ and
  - $l$ is the infimum of $Pr(\psi \mid \phi)$ subject to all models $Pr$ of $L \cup P$ with $Pr(\phi)>0$
  - $u$ is the supremum of $Pr(\psi \mid \phi)$ subject to all models $Pr$ of $L \cup P$ with $Pr(\phi)>0$

- Note: when $L \cup P \models \perp \iff \phi$ then $[l,u]$ is $[1,0]$

- Property 1: a logical constraint $\psi \iff \phi$ has the same meaning as the conditional constraints $(\psi \mid \phi)[1,1]$

- Property 2: model–theoretical logical entailment in probabilistic logic generalizes model–theoretical entailment in ordinary propositional logic
Interval Probabilistic Logic

- KB = ({bird ⇐ eagle, feathers ⇐ bird}, {(fly|bird)[0.95,1]})
  - bird ⇐ eagle: all eagles are birds
  - feathers ⇐ bird: all birds have feathers
  - (fly|bird)[0.95,1]: birds fly with a probability of at least 0.95

 Logical consequences of KB
  - KB ⊨ (feathers | bird)[1,1]
  - KB ⊨ (fly|bird)[0.95,1]
  - KB ⊨ (feathers | eagle)[1,1]
  - KB ⊨ (fly | eagle)[0,1]

Probabilistic properties of being able to fly is not inherited from birds to eagles
Interval Probabilistic Logic

- $\text{KB} = \{\text{bird} \leftrightarrow \text{ostrich}, \{(\text{legs} | \text{bird})[1,1], (\text{fly} | \text{bird})[1,1],
(\text{fly} | \text{ostrich})[0,0.05]\}\}$
  - $(\psi | \phi)[1,1]$ is interpreted as $\psi \leftrightarrow \phi$

- Logical consequences of KB
  - KB $\not\vdash (\text{legs} | \text{bird})[1,1]$
  - KB $\not\vdash (\text{fly} | \text{bird})[1,1]$
  - KB $\not\vdash (\text{legs} | \text{ostrich})[1,0]$
  - KB $\not\vdash (\text{fly} | \text{ostrich})[1,0]$

There is a local inconsistency
Interval Probabilistic Logic

- Solution: interpret \((\psi \mid \phi)[1,1]\) as a default rule
  - Define probability rankings

- Probability ranking \(\kappa\) maps \(Pr\) to \(\{0,1,\ldots\} \cup \{\infty\}\)
  - \(\kappa(Pr)=0\) for at least one \(Pr\)
  - If \(F\) is satisfiable, \(\kappa(F)=\min\{\kappa(Pr) \mid Pr \models F\}\)
  - Otherwise, \(\kappa(F)=\infty\)

- \(Pr\) verifies \((\psi \mid \phi)[l,u]\) iff \(Pr(\phi)>0\) and \(Pr \models (\psi \mid \phi)[l,u]\)
  - \(Pr\) falsifies \((\psi \mid \phi)[l,u]\) iff \(Pr(\phi)>0\) and \(Pr \nvdash (\psi \mid \phi)[l,u]\)

- \(P\) tolerates \(C\) under \(L\): \(L \cup P\) has a model that verifies \(C\)
  - \(P\) is under \(L\) in conflict \(C\) with iff no model of \(L \cup P\) verifies \(C\)
Interval Probabilistic Logic

- \( \kappa \) is admissible with \( KB = (L, P) \)
  - \( \kappa(\neg(\psi \mid \phi)[1,1]) = \infty \) for all \( \psi \leftarrow \phi \in L \) and
  - \( \kappa(\phi > 0) < \infty \) and \( \kappa(\phi > 0 \land (\psi \mid \phi)[l, u]) < \kappa(\phi > 0 \land \neg(\psi \mid \phi)[l, u]) \) for all
    \( (\psi \mid \phi)[l, u] \in P \)

- \( z \)-partition of \( KB \): unique ordered partition \((P_0, \ldots, P_k)\) of \( P \)
  - Each \( P_i, 0 \leq i \leq k \), is the set of all \( C \in \bigcup_{j=i}^{k} P_j \) tolerated under \( L \) by \( \bigcup_{j=i}^{k} P_j \)

- \( Pr \) is lex-preferable to \( Pr' \) iff some \( i \) exists such that
  - \( |\{C \in P_i \mid Pr \models C\}| > |\{C \in P_i \mid Pr' \models C\}| \) and
  - \( |\{C \in P_j \mid Pr \models C\}| = |\{C \in P_j \mid Pr' \models C\}| \) for all \( i < j \leq k \)

- Lex-minimal model \( Pr \) of \( F \): no model of \( F \) is lex-preferable to \( Pr \)
  - \( Pr \) satisfies more constraints in rank \( i \) than and \( Pr' \) and satisfies the same constraints in ranks higher than \( i \) as \( Pr' \)
Interval Probabilistic Logic

- **Lex–entailment:** $(\psi \mid \phi)[l,u]$ is a lex–consequence of KB, denoted $\text{KB} \models_{\text{lex}} (\psi \mid \phi)[l,u]$ iff
  - Each lex–minimal model of $\mathcal{L} \cup \{\phi > 0\}$ satisfies $(\psi \mid \phi)[l,u]

- **Tight lex–entailment:** $(\psi \mid \phi)[l,u]$ is a tight lex–consequence of KB, denoted as $\text{KB} \models_{\text{lex,tight}} (\psi \mid \phi)[l,u]$ iff
  - $l=\inf Pr(\psi \mid \phi)$ (resp., $u=\sup Pr(\psi \mid \phi)$) subject to all lex–minimal models $Pr$ of $\mathcal{L} \cup \{\phi > 0\}$
Interval Probabilistic Logic

- KB = (bird ⇐ ostrich), ((legs|bird)[1,1], (fly|bird)[1,1], (fly|ostrich)[0,0.05])
  - (ψ | φ)[1,1] is interpreted as ψ ← φ (default rule)

- Logical consequences of KB
  - KB ⊫ (legs|ostrich)[1,0]
  - KB ⊫ (fly| ostrich)[1,0]

- Lex–consequences of KB
  - KB ⊫ (legs|ostrich)[1,1]
  - KB ⊫ (fly| ostrich)[0,0.05]
Possibility Theory

- Possibility distribution $\pi: \Omega \rightarrow L$
  - $\Omega$ represents universe of discourse
  - $(L, <)$ is a bounded total ordered scale
  - $\pi(\omega) \geq \pi(\omega')$ means $\omega$ is a priori more plausible than $\omega'$

- Possibility measure and necessity measure
  \[
  \Pi(A) = \sup \{ \pi(\omega): \omega \in A \} \quad \text{N}(A) = 1 - \Pi(\neg A)
  \]

- Property
  - $\Pi(A \cup B) = \max(\Pi(A), \Pi(B))$
  - $\text{N}(A \cap B) = \min(\text{N}(A), \text{N}(B))$
Possibilistic Logic

- **Syntax**
  - Possibilistic formula: $(\phi, a)$ denotes *certainty degree of* $\phi \geq a$
    - Example: $(\text{eat_fish(Tweety)}, 0.46)$ We are somewhat certain that Tweety eats fish
  - Possibilistic knowledge base: a set of possibilistic formulae
    \[ B = \{(\phi_i, a_i) : i = 1, \ldots, n\} \]

- **Example**
  - p: there were human beings in Mars before
  - q: scientists have detected some strange signals from outer space
  - r: there are aliens in other planets
  - s: the ancestors of human are gorillas
  - B = {((p, 0.4), (q \rightarrow r, 1), (s, 0.8), (\neg s \rightarrow \neg r, 0.9))}
Possibilistic Logic (Cont.)

- Inconsistency degree of B
  - $B^*$: classical base of $B$
  - $B \geq a = \{ \phi \in B^* | (\phi, b) \in B, B \geq a \}$
  - $\text{Inc}(B) = \max\{a: B \geq a \text{ is inconsistent}\}$

\[\begin{array}{cccccc}
\phi_1 & \phi_2 & \phi_3 & \cdots & \phi_{n-1} & \phi_n \\
\alpha_1 \cdot & \alpha_2 \cdot & \alpha_3 \cdot & \cdots & \alpha_{n-1} \cdot & \alpha_n \\
\end{array}\]

$\text{Inc}(B) = a_3$

- $B = \{(q, 1), (q \rightarrow r, 1), (s \rightarrow \neg r, 0.9), (s, 0.8), (p, 0.4)\}$

What is $\text{Inc}(B)$?
Possibilistic Logic

- **Possibilistic inference**
  - $\phi$ is a plausible consequence of $B$, denoted $B \vdash_P \phi$ iff $B_{\text{inc}(B)} \vdash \phi$
  - $(\phi, a)$ is a consequence of $B$, denoted $B \vdash (\phi, a)$ iff $a > \text{Inc}(B)$, and $B_{\geq a} \vdash \phi$
  - $\phi$ is a possibilistic consequence of $B$ to degree $a$, denoted $B \vdash_{\pi} (\phi, a)$ iff
    - $B_{\geq a}$ is consistent
    - $B_{\geq a} \vdash \phi$
    - For all $b > a$, $B_{\geq b} \not\vdash \phi$

Only formulas whose weights are greater than or equal to the inconsistency degree are used
Possibilistic Logic (Cont.)

- Example

\[ \text{B} = \{(q, 1), (q \rightarrow r, 1), (\neg s \rightarrow \neg r, 0.9), (s, 0.9), (p, 0.4)\} \]

- \( \text{B} \models_{\pi} (q, 1) \)
- \( \text{B} \models_{\pi} (q \rightarrow r, 1) \)
- \( \text{B} \not\models_{\pi} (p, 0.4) \)

Drowning effect
Possibilistic Logic

- **Semantics**
  - Possibility distribution $\pi: \Omega \rightarrow [0,1]$
    - $\pi(\omega)=1$: $\omega$ is totally possible
    - $\pi(\omega)=0$: $\omega$ is impossible
    - $\pi(\omega)>0$: $\omega$ is possible

- Normal possibility distribution $\pi$: there exists one world $\omega$ such that $\pi(\omega)=1$

- **Satisfaction**: $\pi$ satisfies $(\phi,a)$, denoted $\pi \models (\phi,a)$, iff $N(\phi) \geq a$
  - $\pi \models B$ iff $\pi \models (\phi,a)$, for all $(\phi,a) \in B$
  - $B \models (\phi,a)$ iff for every $\pi \models B$, we have $\pi \models (\phi,a)$
Possibilistic Logic (Cont.)

- Possibilistic knowledge base \( B = \{(\phi_1, a_1), \cdots, (\phi_n, a_n)\} \Rightarrow \) a unique possibility distribution

\[ \pi(\omega) = 1 - \alpha_3 \]

- A possibility distribution \( \Rightarrow \) a possibilistic knowledge base
  - \( \Pi(\phi) = \max\{\pi(\omega) : \omega \models \phi\} \)
  - \( N(\phi) = 1 - \Pi(\neg \phi) \)
Possibilistic Logic

Soundness and completeness

- \( B \vdash (\phi, a) \) iff \( B \models (\phi, a) \)
- \( B \vdash_p \phi \) iff \( N_B(\phi) > \text{Inc}(B) \)
- \( B \vdash_\pi (\phi, a) \) iff \( N_B(\phi) \geq a \) and \( a > \text{Inc}(B) \)
Generalizations of Possibilistic Logic

- **Linear-order inference**
  - $B$ is stratified as $(S_1, ..., S_k)$
    - Formulas in $S_i$ have the same weights
    - The weight of formulas in $S_i$ is greater than that of formulas in $S_j$ with $i < j$
    - $K_{LO,B} = \bigcup S'_i$ with $S'_i = S_i$ if $S'_1 \cup ... \cup S'_{n-1} \cup S_i$ is consistent and $S'_i = \emptyset$ otherwise
    - $B \vdash_{LO} \phi$ iff $K_{LO,B} \vdash \phi$

- **Example:** $B = \{(q, 1), (q \rightarrow r, 1), (s \rightarrow \neg r, 0.9), (s, 0.9), (t, 0.9), (p, 0.4)\}$
  - $B \vdash_{LO} p$
  - $B \vdash_{LO} t$
Generalizations of Possibilistic Logic

- **Lexicographic inference**
  - B is stratified as \((S_1, \ldots, S_k)\)
  - For \((S'_1, \ldots, S'_k)\) and \((S''_1, \ldots, S''_k)\) which are subsets of \((S_1, \ldots, S_k)\)
  - \((S'_1, \ldots, S'_k)\) is preferred to \((S''_1, \ldots, S''_k)\) iff some \(i\) exists such that
    - | \(S'_i\) | > | \(S''_i\) | and
    - | \(S'_j\) | = | \(S''_j\) | for all \(i < j \leq k\)
  - \(B \vdash_{\text{lex}} \phi\) iff \(S'_1 \cup \ldots \cup S'_k \vdash \phi\) for all lexi-preferred subset \((S'_1, \ldots, S'_k)\) of \((S_1, \ldots, S_k)\)

- **Example:** \(B = \{(q, 1), (q \rightarrow r, 1), (s \rightarrow \neg r, 0.9), (s, 0.9), (t, 0.9), (p, 0.4)\}\)
  - \(B \vdash_{\text{lex}} p\)
  - \(B \vdash_{\text{lex}} t\)

What are lexi-preferred subsets of \((S_1, \ldots, S_k)\)?
Comparison

Properties

- **Probabilistic logic**
  - If \( \text{Mod}(S \land T) = \emptyset \), then \( P(S \lor T) = P(S) + P(T) \)
  - \( P(S \lor T) = P(S) + P(T) - P(S \land T) \)
  - \( P(\neg S) = 1 - P(S) \)

- **Possibilistic logic**
  - \( \Pi(A \cup B) = \max(\Pi(A), \Pi(B)) \)
  - \( N(A \cap B) = \min(N(A), N(B)) \)

Types of uncertainty

- **Probabilistic logic**: quantitative
- **Possibilistic logic**: qualitative
Comparison

- **Inconsistency**
  - Probabilistic logic
    - Use probabilistic semantics
  - Possibilistic logic
    - Use standard first-order semantics

- **Example**
  - $\text{KB} = \{\text{bird} \leftrightarrow \text{ostrich}\}, \{(\text{legs}|\text{bird})[1,1], (\text{fly}|\text{bird})[1,1], (\text{fly}|\text{ostrich})[0,0.05]\}$ is inconsistent under probabilistic semantics
    - $(\text{fly}|\text{ostrich})[0,0.05]$ can be inferred under probabilistic default semantics
  - $\text{KB}' = \{\text{ostrich} \rightarrow \text{bird}, 1\}, \{\text{bird} \rightarrow \text{legs}, 1\}, \{\text{bird} \rightarrow \text{fly}, 1\}, \{\text{ostrichfly} \rightarrow \text{fly}, 0.05\}$ is not inconsistent under possibilistic semantics
    - $(\text{ostrich} \rightarrow \text{fly}, 1)$ can be inferred under possibilistic semantics
Outline

- Probabilistic logic vs possibilistic logic

- Probabilistic description logics
  - Description logics
  - Probabilistic description logics

- Possibilistic description logics and its extension

- Revising ontologies in description logics

- Mapping repair in description logics
Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
  - Description logics
  - Probabilistic description logics
- Possibilistic description logics and its extension
- Revising ontologies in description logics
- Mapping repair in description logics
Description Logics

- Description logics
  - Are (mostly) decidable fragments of first-order predicate logic
  - Provide logical underpinning of W3C standard OWL

- Building blocks
  - Concepts (unary predicates/formulae with one free variable)
    - E.g., Person, Lawyer ⊔ Doctor
  - Roles (binary predicates/formulae with two free variables)
    - E.g., hasChild
  - Individuals (constants)
    - E.g., John, Mary
Description Logics (Syntax)

- Description languages
  - Defining complex concepts: sets of individuals
  - Defining complex roles: binary relations on individuals

- Complex concepts are built by
  - Atomic concepts: Tissue, Heart
  - Constructors: $Tissue \sqcap \exists part\cdot of. Heart$

- Complex roles are built by
  - Atomic roles: part-of, has-location
  - Constructors: HasFather
Description Logics (Semantics)

- **Interpretation:** \( I= (\Delta^I, .^I) \)
  - **Domain:** \( \Delta^I \)
  - **Assignment function:** \( .^I \)

![Diagram showing the interpretation of Description Logics](image)
Description Logics (Cont.)

- Interpretation: $I = (\Delta^I, \cdot^I)$

<table>
<thead>
<tr>
<th>Construct</th>
<th>Syntax</th>
<th>Example</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic concept</td>
<td>$A$</td>
<td>Heart</td>
<td>$A^I \subseteq \Delta^I$</td>
</tr>
<tr>
<td>Atomic role</td>
<td>$R$</td>
<td>part-of</td>
<td>$R^I \subseteq \Delta^I \times \Delta^I$</td>
</tr>
<tr>
<td>Negation</td>
<td>$\neg C$</td>
<td>$\neg$ Heart</td>
<td>$\Delta^I \setminus C^I$</td>
</tr>
<tr>
<td>Conjunction</td>
<td>$C \sqcap D$</td>
<td>Lawyer$\sqcap$Doctor</td>
<td>$C^I \setminus D^I$</td>
</tr>
<tr>
<td>Value restriction</td>
<td>$\forall R.C$</td>
<td>$\forall$ part-of.Wood</td>
<td>${a</td>
</tr>
</tbody>
</table>

...
Description Logics (Ontology)

- **TBox T**: defining terminology of application domain
  - Inclusion assertion on concept: $C \sqsubseteq D$
    
    \[
    \text{Pericardium} \sqsubseteq \text{Tissue} \sqcap \exists \text{part-of.Heart}
    \]
  
  - Inclusion assertion on roles: $R \sqsubseteq S$
    
    \[
    \text{Part-of} \sqsubseteq \text{has-location}
    \]

- **ABox A**: stating facts about a specific “world”
  - Membership assertion: $C(a)$ or $R(a,b)$
    
    \[
    \text{HappyMan}(Bob), \text{HasChild}(Bob, Mary)
    \]
Description Logics (Semantics)

- Given an interpretation I
- Semantics of TBox axioms
  - $I \models C \subseteq D$ if $C^I \subseteq D^I$
  - $I \models R \subseteq S$ if $R^I \subseteq S^I$
- Semantics of ABox assertions
  - $I \models C(a)$ if $a^I \in C^I$
  - $I \models R(a,b)$ if $(a^I,b^I) \in R^I$
Model of an ontology $O = \langle T, A \rangle$
- $I$ is a model of $O$ if it satisfies all axioms in $T$ and all assertions in $A$

Concept satisfiability
- Concept $C$ is satisfiable in $O$ if $C^I$ is nonempty for some model $I$ of $O$

Ontology Entailment
- $O \models \phi$ iff $I \models \phi$ for all models $I$ of $O$
Description Logics (Semantics)

- **Incoherent ontology:** ontology with at least one unsatisfiable concept
  - Example: \{PhDStudent ⊑ Student,
    
    PhDStudent ⊑ Employee,
    
    Student ⊑ ¬Employee\}

- **Inconsistent ontology:** ontology without a model
  - Example: \{PhDStudent ⊑ Student,
    
    PhDStudent ⊑ Employee,
    
    Student ⊑ ¬Employee,
    
    PhDStudent(John)\}

Incoherent ontology can be consistent!
Example: DICE ontology

- \( \text{Brain} \sqsubseteq \text{CentralNervousSystem} \sqcap \exists \text{systempart.NervousSystem} \sqcap \text{BodyPart} \sqcap \exists \text{region.HeadAndNeck} \sqcap \forall \text{region.HeadAndNeck} \)

- \( \text{CentralNervousSystem} \sqsubseteq \text{NervousSystem} \)

- \( \text{BodyPart} \sqsubseteq \neg \text{NervousSystem} \) or \( \text{DisjointWith} \) (BodyPart, NervousSystem)
Description Logics

- **Example from Foaf**
  - Person(timbl)
  - Homepage(timbl, http://w3.org/)
  - Homepage(w3c, http://w3.org/)
  - Organization(w3c)
  - InverseFunctionalProperty(Homepage)
  - DisjointWith(Organization, Person)

- **Example from OpenCyc**
  - ArtifactualFeatureType(PopulatedPlace)
  - ExistingStuffType(PopulatedPlace)
  - DisjointWith(ExistingObjectType, ExistingStuffType)
  - ArtifactualFeatureType ⊑ ExistingObjectType
Description Logics

- **Deficiency of DLs**
  - Cannot express uncertain information
    - *I am quite sure* that a heart patient has a private health insurance
    - *I am a little certain* that Tom is a heart patient
  - Cannot deal with inconsistency

- **Syntax and semantics of DLs need to be extended**
Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
  - Description logics
  - Probabilistic description logics
- Possibilistic description logics and its extension
- Revising ontologies in description logics
- Mapping repair in description logics
Probabilistic Description Logics

- Syntax

- Classification of individuals: a set of classical individuals $I_C$ and a finite set of probabilistic individuals $I_P$

- Basic classification concept (basic c-concept): DL concepts that are free of probabilistic individuals from $I_P$

- c-concept:
  - Every basic concept is a c-concept
  - If $C$ and $D$ are C-concept, $\neg C$ and $C \sqcap D$ are c-concepts

- Conditional constraint: $(D \mid C)[l,u]$
  - $C$ and $D$ are c-concepts
  - Meaning: probability of $D$ given $C$ lies between $l$ and $u$
Probabilistic Description Logics

Example

<table>
<thead>
<tr>
<th>A terminological probabilistic knowledge base</th>
</tr>
</thead>
<tbody>
<tr>
<td>ax₁: (HighBloodPressure</td>
</tr>
<tr>
<td>ax₂: (¬HasHighBloodPressure</td>
</tr>
<tr>
<td>ax₃: (MalePacemakerPatient</td>
</tr>
<tr>
<td>ax₄: (Male</td>
</tr>
<tr>
<td>ax₅: PacemakerPatient ⊑ Heartpatient</td>
</tr>
</tbody>
</table>
Probabilistic Description Logics

- Syntax

- **PTBox** $PT=(T,P)$
  - $T$: classical DL knowledge base
  - $P$: is a finite set of conditional constraints

- **PABox** $P_o$
  - $o$ is a probabilistic individual in $I_P$
  - $P_o$ is a finite set of conditional constraints
    - $(D | C)[l,u] \in P_o$: if $C(o)$ holds then $D(o)$ holds with a probability between $l$ and $u$
    - $(D | \top)[l,u] \in P_o$: $D(o)$ holds with a probability between $l$ and $u$
    - $(\exists R{o'}) | C)[l,u] \in P_o$: if $C(o)$ holds then $R(o,o')$ holds with a probability between $l$ and $u$
## Probabilistic Description Logics

**Example**

<table>
<thead>
<tr>
<th>A terminological probabilistic knowledge base</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax_1$: $(\text{HighBloodPressure} \mid \text{Heartpatient}) [1,1]$</td>
</tr>
<tr>
<td>$ax_2$: $(\neg \text{HasHighBloodPressure} \mid \text{PacemakerPatient}) [1,1]$</td>
</tr>
<tr>
<td>$ax_3$: $(\text{MalePacemakerPatient} \mid \text{PacemakerPatient}) [0.4,1]$</td>
</tr>
<tr>
<td>$ax_4$: $(\text{Male} \mid \text{PacemakerPatient}) [0.4,1]$</td>
</tr>
<tr>
<td>$ax_5$: $\text{PacemakerPatient} \sqsubseteq \text{Heartpatient}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A PABox for the probabilistic individual Tom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax_6$: $(\text{PacemakerPatient} \mid \top) [0.8,1]$</td>
</tr>
</tbody>
</table>
Probabilistic Description Logics

- Syntax

- Probabilistic knowledge base $\text{KB} = (T, P, (P_o)_{o \in I_P})$
  - $P$: probabilistic knowledge about randomly chosen individuals
    - Conditional constraints in $P$ are default statements
  - $P_o$: probabilistic knowledge about the concrete individual $o$
    - Conditional constraints in every $P_o$ with $o \in I_P$ are strict statements
Probabilistic Description Logics

-Semantics

World $I$: a set of basic $c$-concepts $C \subseteq \mathcal{C}$ such that \{ $C(i) \mid C \in I$\} $\cup$ \{ $\neg C(i) \mid C \in \mathcal{C} \setminus I$\} is satisfiable

- $\mathcal{C}$ is the set of all basic $c$-concepts
- $i$ is a new individual
- $\mathcal{I}_C$ denotes the set of all worlds relative to $\mathcal{C}$

Model of a TBox: $I$ is a model of $T(I \models T)$ iff $T \cup$ \{ $C(i) \mid C \in I$\} $\cup$ \{ $\neg C(i) \mid C \in \mathcal{C} \setminus I$\} is satisfiable

Model of a $c$-concept: $I \models C$

- $I \models C$ iff $C \in I$, $C$ is a basic $c$-concept
- $I \models \neg C$ iff $I \not\models C$ does not hold
- $I \models C \cap D$ iff $I \models C$ and $I \models D$
Probabilistic Description Logics

- Semantics

- **Probabilistic interpretation** $Pr$: a probability function on $\mathcal{I}_c$
  - $\mathcal{I}_c$ denotes the set of all worlds relative to $C$
  - $Pr(C)$: sum of all $Pr(I)$ such that $I \models C$

- $Pr \models T$: $I \models T$ for every $I \in \mathcal{I}_c$ such that $Pr(I) > 0$

- $Pr \models (D | C)[l,u]$: $Pr(C) = 0$ or $Pr(D | C) \in [l,u]$
  - $Pr(D | C) = Pr(D \cap C) / Pr(C)$

- $Pr \models F$: $I \models F$ for every $F \in \mathcal{F}$
  - $\mathcal{F}$ is a set of conditional constraints
Probabilistic Description Logics

-Semantics

- $Pr \text{ verifies } (D | C)[l,u] \text{ iff } Pr(C)=1 \text{ and } Pr \models (D | C)[l,u]$
  
  - $Pr \text{ falsifies } (D | C)[l,u] \text{ iff } Pr(C)=1 \text{ and } Pr \not\models (D | C)[l,u]$

- $\mathcal{F} \text{ tolerates } (D | C)[l,u] \text{ under } T: T \cup \mathcal{F} \text{ has a model that verifies } (D | C)[l,u]$

- Consistency of a PTBox $PT=(T,P)$
  
  - $T \text{ is satisfiable}$
  
  - There exists an ordered partition $(P_0,...,P_k)$ of $P$ such that
    
    - Each $P_i$, $0 \leq i \leq k$, is the set of all $F \in P_i \cup ... \cup P_k$ tolerated under $T$ by $P_i \cup ... \cup P_k$
    
    - Idea of the partition: when in conflict, remove conditional constraints in the lower strata
    
    - The partition follows the rule of maximum specificity
    
    - Called $z$-partition of $PT$
## Probabilistic Description Logics

### Example

A terminological probabilistic knowledge base

| $P_0$ | $ax_1$: (HighBloodPressure | Heartpatient) [1,1] |
|-------|-----------------------------------------------|
|       | $ax_2$: (¬HasHighBloodPressure | PacemakerPatient) [1,1] |
|       | $ax_3$: (MalePacemakerPatient | PacemakerPatient) [0.4,1] |
|       | $ax_4$: (Male | PacemakerPatient) [0.4,1] |
|       | $ax_5$: PacemakerPatient $\sqsubseteq$ Heartpatient |

A PABox for the probabilistic individual Tom

| $P_1$ | $ax_6$: (PacemakerPatient | $\top$) [0.8,1] |
Probabilistic Description Logics—Semantics

- $Pr$ is lex-preferable to $Pr'$ iff some $i$ exists such that
  - $|\{F \in P_i | Pr \models F\}| > |\{F \in P_i | Pr' \models F\}|$ and
  - $|\{F \in P_i | Pr \models F\}| = |\{F \in P_i | Pr' \models F\}|$ for all $j < i \leq k$

- Lex-minimal model $Pr$ of $F$: no model of $F$ is lex-preferable to $Pr$

- Lex-entailment: $(D \mid C)[l,u]$ is a lex-consequence of $\mathcal{F}$ under $PT$, denoted $\mathcal{F} \models^{\text{lex}} (D \mid C)[l,u]$ iff
  - Each lex-minimal model of $T \cup \mathcal{F} \cup \{(C \mid \top)[1,1]\}$ satisfies $(D \mid C)[l,u]$

- Tight lex-entailment: $(D \mid C)[l,u]$ is a tight lex-consequence of $\mathcal{F}$ under $PT$, denoted as $\mathcal{F} \models^{\text{lex,tight}} (D \mid C)[l,u]$ iff
  - $l = \inf Pr(\psi \mid \phi)$ (resp., $u = \sup Pr(\psi \mid \phi)$) subject to all lex-minimal models $Pr$ of $L \cup \{\phi > 0\}$
Probabilistic Description Logics

Example: the following are tight lex–consequence of $PT$

<table>
<thead>
<tr>
<th>A terminological probabilistic knowledge base</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0$</td>
</tr>
<tr>
<td>$ax_1$: (HighBloodPressure $</td>
</tr>
<tr>
<td>$ax_2$: (~HasHighBloodPressure $</td>
</tr>
<tr>
<td>$ax_3$: (MalePacemakerPatient $</td>
</tr>
<tr>
<td>$P_1$</td>
</tr>
<tr>
<td>$ax_4$: (Male $</td>
</tr>
<tr>
<td>$ax_5$: PacemakerPatient $\sqsubseteq$ Heartpatient</td>
</tr>
</tbody>
</table>

- $(\text{HighBloodPressure} \mid \text{Male} \sqcap \text{Heartpatient}) [1,1]$
- $(\text{MalePacemakerPatient} \mid \text{PacemakerPatient}) [0.4,1]$

But $(\text{HasHighBloodPressure} \mid \text{PacemakerPatient}) [1,1]$ is not inferred
Log-linear Description Logics

- Knowledge base $K = (K^D, K^U)$
  - $K^D$: a classical DL knowledge base
  - $K^U$: a set of weighted axioms (like possibilistic DLs)

- Semantics
  - Based on probability distributions over consistent knowledge bases
    
    $K^D \subseteq K' \subseteq K^D \cup \{c : (c, w_c) \in K^U\}$

    
    $$Pr_K(K') = \begin{cases} 
    \frac{1}{Z} \exp\left(\sum_{c \in K' \setminus K^D} w_c\right) & \text{if } K' \text{ consistent} \\
    0 & \text{otherwise}
    \end{cases}$$

  - Marginal probability of an axiom: sum of the probabilities of the consistent knowledge bases containing it
Outline

- Probabilistic logic vs possibilistic logic
- Probabilistic description logics
- Possibilistic description logics and its extension
- Revising ontologies in description logics
- Mapping repair in description logics
Possibilistic Description Logics

A possibilistic DL knowledge base

Tool(amilcare): 1.0
Tool(cavido): 0.46
Application(cavido): 0.46
Tool ⊑ Application : 0.3

Note: Inference is non-trivial when inconsistency exists
Certainty degree is attached to inferred axiom
Possibilistic Description Logics

- Syntax

- Possibilistic axiom: \((\phi, a)\), \(\phi\) is a DL axiom
- Possibilistic DL knowledge base \(B=\{(\phi_i, a_i) : i=1, \cdots, n\}\)
- Classical DL Base \(B^*=\{\phi : (\phi, a) \in B\}\)
- \(\alpha\)-cut of \(B\): \(B_{\geq \alpha} = \{\phi \in B^* : (\phi, b) \in B, \text{ and } b \geq a\}\)
- Inconsistency degree: \(\text{Inc}(B) = \max\{a : B_{\geq a} \text{ is inconsistent}\}\)
- Example:

  \(B=\{(\text{Tool(amilcare)}, 1), (\text{Application(cavido)}, 0.46),\)

  \((\text{Tool(cavido)}, 0.46), (\text{disjoint(Tool, Application)}, 0.3)\}\)
Possibilistic Description Logics
– Semantics

- Possibility distribution \( \pi: I \rightarrow [0,1] \), \( I \) is the set of all interpretations
  - \( \pi(l) \) represents the degree of compatibility of \( I \) with available information
  - \( \pi(l_1) > \pi(l_2) \): \( l_1 \) is preferred to \( l_2 \)

- Possibility measure \( \Pi \)
  \[ \Pi(\phi) = \max\{\pi(l): l \in I, l \models \phi\} \]

- Necessity Measure \( N \)
  \[ N(\phi) = 1 - \max\{\pi(l): l \not\models \phi\} \]
Possibilistic Description Logics

Example

<table>
<thead>
<tr>
<th>A possibilistic DL knowledge base</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax_1$: (Heartpatient $\sqsubseteq$ HighBloodPressure, 1.0)</td>
</tr>
<tr>
<td>$ax_2$: (PacemakerPatient $\sqsubseteq$ $\neg$HighBloodPressure, 1.0)</td>
</tr>
<tr>
<td>$ax_3$: (HeartPatient $\sqsubseteq$ $\exists$HasHealthInsurance.PrivateHealth,0.9)</td>
</tr>
<tr>
<td>$ax_4$: (PacemakerPatient(Tom), 0.8)</td>
</tr>
</tbody>
</table>
Possibilistic Description Logics

- Inference services: instance checking

<table>
<thead>
<tr>
<th>A possibilistic DL knowledge base</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax_1$: $(\text{Heartpatient} \sqsubseteq \text{HighBloodPressure}, 1)$</td>
</tr>
<tr>
<td>$ax_2$: $(\text{PacemakerPatient} \sqsubseteq \neg \text{HighBloodPressure}, 1)$</td>
</tr>
<tr>
<td>$ax_3$: $(\text{HeartPatient} \sqsubseteq \exists \text{HasHealthInsurance.PrivateHealth}, 0.9)$</td>
</tr>
<tr>
<td>$ax_4$: $(\text{PacemakerPatient}(\text{Tom}), 0.8)$</td>
</tr>
<tr>
<td>$ax_5$: $(\text{HeartPatient}(\text{Tom}), 0.5)$</td>
</tr>
<tr>
<td>$ax_6$: $(\text{HeartPatient} \sqsubseteq \text{MalePacemakerPatient}, 0.4)$</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
Possibilistic Description Logics

- Inference services: instance checking with weight

A possibilistic DL knowledge base

\[
\begin{align*}
ax_1: & \text{ (Heartpatient } \sqsubseteq \text{ HighBloodPressure, 1)} \\
ax_2: & \text{ (PacemakerPatient } \sqsubseteq \neg \text{HighBloodPressure, 1)} \\
ax_3: & \text{ (HeartPatient } \sqsubseteq \exists \text{HasHealthInsurance.PrivateHealth,0.9)} \\
ax_4: & \text{ (PacemakerPatient(Tom), 0.8)} \\
ax_5: & \text{ (HeartPatient(Tom),0.5)} \\
ax_6: & \text{ (HeartPatient } \sqsubseteq \text{MalePacemakerPatient, 0.4)}
\end{align*}
\]

\[\neg \text{ HighBloodPressure(Tom): 0.8}\]
Reduction

Inference services

Computing inconsistency degree
Algorithms

- A black-box algorithm (Qi et al. ECSQARU2007, IJIS 2011)
  - Idea: search the weights by a binary search
    - Call a standard DL reasoner to check inconsistency
  - A system called PossDL has been implemented

- A tableaux algorithm (Qi and Pan ASWC 2008)
  - Idea: extending classical tableaux algorithm for DL ALC
    - A weight is attached to a concept name or a role name
  - No implementation is done
Generalizations of Possibilistic Description Logics

- **Linear order inference** (Qi et.al. ECSQARU2007, IJIS 2011)
  - Algorithm idea: compute the inconsistency degree and remove axioms whose weights are equal to it
  - Call a standard DL reasoner to check inconsistency
  - PossDL provides functionalities to compute consequences of linear order inference

- **Lexicographic inference** (Du and Qi RR 2008)
  - Algorithm idea: compile the DL axioms to propositional programs
  - No implementation has been done