Lower Bound Founded Logic of Here-and-There

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A distinguishing feature of Answer Set Programming (*ASP*; [2]) is that all atoms belonging to a stable model must be founded. That is, atoms must be justifiable in a non-circular way. This can be made precise by the logic of Here-and-There (*HT*; [6]), whose equilibrium models correspond to stable models [8]. One way looking at foundedness is regarding Boolean truth values as ordered; true greater than false. Then, each Boolean variable takes the smallest truth value that can be proven for it. This idea was generalized by Aziz in [1] to ordered domains. As before, the idea is that a variable gets assigned to the smallest value that can be justified. We refer to this idea by *foundedness*. Note that *ASP* follows the rationality principle, which says that one shall only believe in things one is forced to. In the propositional case this principle amounts to foundedness, whereas for rules like $x \geq 42$ there are at least two ways of understanding. First, one may believe in any value greater or equal than 42 for x. Second, one may believe in value 42 for x if one is not forced to believe more than this, which corresponds to our definition of foundedness.

The literature of *ASP* contains several approaches dealing with atoms containing variables over non-Boolean domains, among them [3], [7] and [4], but these approaches do not address foundedness in our sense. For instance, Constraint *ASP* (*CASP*) approaches like [3] allow atoms with variables over non-Boolean domains in the body of a rule only. Thus, these atoms and the values of non-Boolean variables cannot be founded in terms of *ASP*.

Approaches like [7] and [4] allow any kind of atoms in heads and bodies. Thus atoms with variables over non-Boolean domains are founded but their variables are not necessarily assigned to the smallest value that can be justified. Now one could think about using minimization, for instance on top of the approach of [4], to achieve foundedness. The following examples illustrate that minimizing assigned values does not restore foundedness. Consider the rules

$$x \geq 0 \qquad y \geq 0 \qquad x \geq 42 \leftarrow y < 42 \tag{1}$$

The approach of [4] leads to solutions that assign values greater or equal than 42 to x and values greater or equal than 0 to y or vice versa, respectively. Thus, the two solutions with minimal values assign 42 to x and 0 to y and the other way around. Note that only the first one respects foundedness, since there is no reason assigning a value greater than 0 to y. Now, consider the rules

$$x \geq 1 \qquad x \geq 42 \leftarrow \neg(x \leq 1) \tag{2}$$

We expect two solutions in terms of foundedness. One assigns the value 1 to x and the other assigns value 42 to x, since a value greater than 1 forces the derivation of value 42. The rules of (2) give us no reason deriving a value greater than 42. In contrast, the approach presented in [4] yield an intuitive understanding assigning value 1 or a value greater or equal than 42 to x. That is, the corresponding solution with the minimal value assigned to x assigns 1 to x. The second equally founded solution is not obtained.

The existing approach of Aziz [1] behaves counter intuitive. For instance, for rule $p \leftarrow \neg p$ Aziz' approach yields a solution where p holds instead of no solution as expected in terms of ASP. To this end, I developed a logical reconstruction of Aziz' idea of foundedness in the setting of the logic of HT. More precisely, I defined the logic of HT with lower bound founded variables, short HT_{LB}, along with its equilibrium models. The idea is to additionally compare the equilibrium model candidates by their values assigned to variables regarding the given order relation. To preserve some desired properties, atoms need to be satisfied 'Here' and 'There', instead of 'Here' only. I elaborated upon the formal properties of HT_{LB} regarding persistence, negation and strong equivalence. Furthermore, I elaborated on the relation of HT_{LB} to existing formalisms, and showed that HT_{LB} corresponds to a straightforward extension of Ferraris' stable model semantics [5]. I also defined a logic program fragment dealing with linear constraints over integers and analysed it in terms of concepts from logic programming.

References

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