Determining Inference Semantics for Disjunctive Logic Programs (Extended Abstract)*

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In a seminal paper, Gelfond and Lifschitz [1] introduced simple disjunctive logic programs, where in rule heads the disjunction operator ”|” is used to express incomplete information, and defined the answer set semantics (called GL-semantics for short) based on a program transformation (called GL-reduct) and the minimal model requirement. Our observations reveal that the requirement of the GL-semantics, i.e., an answer set should be a minimal model of rules of the GL-reduct, may sometimes be too strong and exclude some answer sets that would be reasonably acceptable.

For example, the following simple disjunctive program:

\[
\Pi : \quad a \quad (1)
\]

\[
a | b \quad (2)
\]

\[
b \leftarrow \neg b \quad (3)
\]

has no answer set under the GL-semantics. Rule (2) models incomplete information concerning the truth values of \(a\) and \(b\), namely that it is insufficient to establish whether \(a\) is true or \(b\) is true, but nonetheless sufficient to establish that at least one of the two is true. That is, rule (2) presents two alternatives and infers either \(a\) or \(b\). Rule (1) establishes the truth of \(a\). So rules (1) and (2) together either infer \(a\) (when rule (2) infers \(a\)) or \(a \land b\) (when rule (2) infers \(b\)), which yields two potential answer sets for \(\Pi\): \(I_1 = \{a\}\) and \(I_2 = \{a, b\}\). Rule (3) is a constraint stating that there is no answer set that does not contain \(b\); this excludes \(I_1\). As a result, applying the three rules yields the only candidate answer set \(I_2\) for \(\Pi\). As \(I_2\) is minimal in the sense that no proper subset \(J\) of \(I_2\) is a candidate answer set for \(\Pi\), we expect \(I_2\) to be an answer set of \(\Pi\).

To address this, we present a more permissive answer set semantics, and make the following main contributions:

(1) We present a general answer set semantics for disjunctive programs, called determining inference semantics (DI-semantics for short), which interprets the operator | in rule heads differently from the classical connective \(\lor\), and does not require that answer sets should be minimal models. Specifically, we introduce a head selection function \(sel\) to formalize the rule head operator |, i.e., for every interpretation \(I\) and every rule head \(H_1 | \cdots | H_k, sel(H_1 | \cdots | H_k, I)\) nondeterministically selects one alternative \(H_i\) satisfied by \(I\). Then we define answer sets as follows: (i) Given an interpretation \(I\) and

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a selection function \( \text{sel} \), we transform a disjunctive program \( \Pi \) into a normal program \( \Pi_{\text{sel}} \), called a disjunctive program reduct, such that for every rule \( \text{head}(r) \leftarrow \text{body}(r) \) in \( \Pi \), \( \text{sel} \left( \text{head}(r), I \right) \leftarrow \text{body}(r) \) is in \( \Pi_{\text{sel}} \) if \( I \) satisfies \( \text{body}(r) \); (ii) given a base answer set semantics \( \mathcal{X} \) for normal programs, we define \( I \) to be a candidate answer set of \( \Pi \) w.r.t. \( \mathcal{X} \) if \( I \) is an answer set of \( \Pi_{\text{sel}} \) under \( \mathcal{X} \); and (iii) we define \( I \) to be an DI-answer set of \( \Pi \) w.r.t. \( \mathcal{X} \) if \( I \) is a minimal candidate answer set.

(2) By replacing the base semantics \( \mathcal{X} \) in the above general semantics with the GL\(_{\text{nlp}}\)-semantics [2], we induce a DI-semantics for simple disjunctive programs. We show that an answer set under the GL-semantics is an answer set under the DI-semantics, but not vice versa; the main reason is that the GL-semantics for \( \mid \) in rule heads amounts to the classical connective \( \lor \) for such programs and further requires that answer sets must be minimal models; this may exclude some desired answer sets. To see the essential difference between the DI- and the GL-semantics, we characterize the latter using a disjunctive program reduct \( \Pi_{\text{sel}} \). This allows us to resolve the open problem in [3] about characterizing split normal derivatives of a simple disjunctive program \( \Pi \).

(3) By replacing the base semantics \( \mathcal{X} \) with the well-justified semantics defined by [4], we further induce a DI-semantics for general disjunctive programs consisting of rules of the form \( H_1 \mid \cdots \mid H_k \leftarrow B \), where \( B \) and every \( H_i \) are arbitrary first-order formulas. This settles the issue of extending the well-justified semantics from general normal programs with rules of the form \( H_i \leftarrow B \) to general disjunctive programs.

(4) In disjunctive programs, every rule head \( H_1 \mid \cdots \mid H_k \) can be viewed as a set \( \{H_1, \cdots, H_k\} \) of alternatives. Other set related constructs in the ASP literature are choice constructs and set introduction rules. Choice constructs seem most closely related to disjunctive rule heads as both of them are used to express a set of alternatives in rule heads; we clarify the difference between them. In particular, we use a generalization of the well-known strategic companies problem to show that because the information expressed by a disjunctive rule head \( a_1 \mid \cdots \mid a_m \) is incomplete, we cannot use a choice construct \( \{a_1, \cdots, a_m\} \) to replace the rule head, where the \( a_i \)'s are ground atoms.

(5) Finally, we consider the complexity of deciding answer set existence and of brave/ cautious reasoning under DI-semantics in the propositional case. The problems are NP-complete and \( \Sigma^p_2/\Pi^p_2 \)-complete, respectively, for simple disjunctive programs and one level higher up in the polynomial hierarchy in the general case.

References