Determining Inference Semantics for Disjunctive Logic Programs (Extended Abstract)*

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In a seminal paper, Gelfond and Lifschitz [1] introduced simple disjunctive logic programs, where in rule heads the disjunction operator "|" is used to express incomplete information, and defined the answer set semantics (called *GL-semantics* for short) based on a program transformation (called *GL-reduct*) and the minimal model requirement. Our observations reveal that the requirement of the GL-semantics, i.e., an answer set should be a minimal model of rules of the GL-reduct, may sometimes be too strong and exclude some answer sets that would be reasonably acceptable.

For example, the following simple disjunctive program:

$$\Pi: \quad a \tag{1}$$
$$a \mid b \tag{2}$$
$$b \leftarrow \neg b \tag{3}$$

has no answer set under the GL-semantics. Rule (2) models incomplete information concerning the truth values of a and b, namely that it is insufficient to establish whether a is true or b is true, but nonetheless sufficient to establish that at least one of the two is true. That is, rule (2) presents two alternatives and infers either a or b. Rule (1) establishes the truth of a. So rules (1) and (2) together either infer a (when rule (2) infers a) or $a \wedge b$ (when rule (2) infers b), which yields *two potential answer sets for Π:* $I_1 = \{a\}$ and $I_2 = \{a, b\}$. Rule (3) is a constraint stating that there is no answer set that does not contain b; this excludes I_1. As a result, applying the three rules yields the only candidate answer set I_2 for Π. As I_2 is minimal in the sense that no proper subset J of I_2 is a candidate answer set for Π, we expect I_2 to be an answer set of Π.

To address this, we present a more permissive answer set semantics, and make the following main contributions:

(1) We present a general answer set semantics for disjunctive programs, called *determining inference semantics* (*DI-semantics* for short), which interprets the operator | in rule heads differently from the classical connective $\vee$, and does not require that answer sets should be minimal models. Specifically, we introduce a head selection function *sel* to formalize the rule head operator |, i.e., for every interpretation I and every rule head $H_1 \mid \cdots \mid H_k$, $sel(H_1 \mid \cdots \mid H_k, I)$ nondeterministically selects one alternative H_i satisfied by I. Then we define answer sets as follows: (i) Given an interpretation I and

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a selection function *sel*, we transform a disjunctive program Π into a normal program Π^I_{sel}, called a *disjunctive program reduct*, such that for every rule $head(r) \leftarrow body(r)$ in Π, $sel(head(r), I) \leftarrow body(r)$ is in Π^I_{sel} if I satisfies $body(r)$; (ii) given a base answer set semantics $\mathcal{X}$ for normal programs, we define I to be a *candidate answer set* of Π w.r.t. $\mathcal{X}$ if I is an answer set of Π^I_{sel} under $\mathcal{X}$; and (iii) we define I to be an *DI-answer set* of Π w.r.t. $\mathcal{X}$, if I is a minimal candidate answer set.

(2) By replacing the base semantics $\mathcal{X}$ in the above general semantics with the GL_{nlp}-semantics [2], we induce a DI-semantics for simple disjunctive programs. We show that an answer set under the GL-semantics is an answer set under the DI-semantics, but not vice versa; the main reason is that the GL-semantics for | in rule heads amounts to the classical connective $\vee$ for such programs and further requires that answer sets must be minimal models; this may exclude some desired answer sets. To see the essential difference between the DI- and the GL-semantics, we characterize the latter using a disjunctive program reduct Π^I_{sel}. This allows us to resolve the open problem in [3] about characterizing split normal derivatives of a simple disjunctive program Π.

(3) By replacing the base semantics $\mathcal{X}$ with the well-justified semantics defined by [4], we further induce a DI-semantics for general disjunctive programs consisting of rules of the form $H_1 \mid \cdots \mid H_k \leftarrow B$, where B and every H_i are arbitrary first-order formulas. This settles the issue of extending the well-justified semantics from general normal programs with rules of the form $H_1 \leftarrow B$ to general disjunctive programs.

(4) In disjunctive programs, every rule head $H_1 \mid \cdots \mid H_k$ can be viewed as a set $\{H_1, \cdots, H_k\}$ of alternatives. Other set related constructs in the ASP literature are *choice constructs* and *set introduction rules*. Choice constructs seem most closely related to disjunctive rule heads as both of them are used to express a set of alternatives in rule heads; we clarify the difference between them. In particular, we use a generalization of the well-known strategic companies problem to show that because the information expressed by a disjunctive rule head $a_1 \mid \cdots \mid a_m$ is incomplete, we cannot use a choice construct $1\{a_1, \cdots, a_m\}u$ to replace the rule head, where the a_i's are ground atoms.

(5) Finally, we consider the complexity of deciding answer set existence and of brave/-cautious reasoning under DI-semantics in the propsitional case. The problems are NP-complete and Σ^p_2-/Π^p_2-complete, respectively, for simple disjunctive programs and one level higher up in the polynomial hierarchy in the general case.

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