

# Atom Definability and Well-Supportedness\* \*\*

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## Extended abstract

Many extensions of stable models [1] semantics have been proposed in the literature to cope with more general syntactic fragments beyond normal logic programs. For instance, [2] introduced a type of rule  $B \rightarrow H$  where both the body  $B$  and the head  $H$  could be a so-called *nested expression*, that is, a Boolean formula allowing conjunction, disjunction and negation. The first extension to arbitrary propositional formulas, including nested implications, was actually provided with the previous definition of *Equilibrium Logic* [3] which is a conservative extension of nested expressions. Although Equilibrium Logic constitutes nowadays one of the most successful and better studied logical characterisations for *Answer Set Programming* (ASP), other approaches have been proposed trying to overcome some features on which no agreement seems to have been reached so far. For instance, one of those properties pursued by some authors is that stable models of a program should be *minimal* with respect to the set of their true atoms. Although this holds for disjunctive logic programs in all ASP semantics, neither nested expressions nor their extension, Equilibrium Logic, satisfy this property for a more general syntax. In an attempt to guarantee minimality for programs with aggregates, [4] (FLP) came out with a new semantics that was generalised to arbitrary propositional formulas in [5], keeping minimality.

Apart from minimality, another property that has been recently considered by Shen et al in [6] is the extension of Fages' *well-supportedness* [7], originally defined for normal logic programs, to rules with a more general syntax like, for instance, allowing Boolean formulas in the head or the body. Intuitively,

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a model  $M$  is said to be *well-supported* if its true atoms can be assigned a derivation ordering (via modus ponens) from the positive part of the program, while the interpretation of negated atoms is fixed with respect to  $M$ , acting like an assumption *a priori*. Fages proved that well-supported models coincide with stable models for normal logic programs, but did not specify how to extrapolate well-supportedness to other syntactic classes.

In this presentation, we provide a general definition of *well-supportedness* for programs with a head atom and a Boolean formula in the body. This definition is parametrized in two ways: (1) the type of formulas that can be used as “assumptions,” that is, whose truth is fixed with respect to some model  $M$ ; and (2), the monotonic logic that defines satisfaction of a rule body before applying the rule to derive a new conclusion. For (1), we study three cases: negated atoms, negated literals, and negated arbitrary formulas. For (2), we analyse the whole range of intermediate logics, from intuitionistic to classical logic, both included. In the presentation, we explain that a group of variants collapse either into Equilibrium Logic or Clark’s completion. To compare the different alternatives, we analyse one more property we call *atom definability*. This property asserts that if we replace occurrences of a formula  $\varphi$  in one or more rule bodies by a new auxiliary atom  $a$ , and we define this atom with an additional rule  $\varphi \rightarrow a$ , then we should get a strongly equivalent program (modulo the original alphabet). As we will see, this is important since semantics satisfying atom definability immediately provide a way to unfold programs with double negation into regular, normal logic programs. We show that, among the analysed variants, only those collapsing to Equilibrium Logic or to Clark’s completion satisfy atom definability.

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