Temporal Here-and-There with Constraints

Pedro Cabalar[1], Martin Diéguez[2], François Olivier[2], Torsten Schaub[3], and Igor Stéphan[2]

[1] University of Corunna, Spain
[2] University of Angers, France
[3] University of Potsdam, Germany

1 Introduction

The declarative language Answer Set Programming (ASP; [1]) has been constantly enriched in recent years to allow for various modeling capabilities, among them the addition of temporal operators and constraints in the language [2, 3]. However to the best of our knowledge, those formalism were never combined in the same framework. Inspired by the existent Linear Temporal Logics with concrete domains [4], we present a combination of two very popular extensions of Equilibrium Logic [5] such as Temporal Equilibrium Logic [2] (TEL) and Here-and-There with constraints [3] (HT_c). The new formalism, called Temporal Here-and-There with Constraints (THT_c) is presented in Section 2 together with its equilibrium version. Moreover, we show the new features of this formalism when describing temporal scenarios like the one presented in Example 1 below. To conclude, we discuss some future lines of research and potential uses of this logic in Section 3.

Example 1. Consider a radar located at km 400 on a road. The speed limit is 90 km/h. A car is traveling at 80 km/h, accelerates by 11.35 km/h at time 4, and then slows down by 2.301 km/h at time 6. The question is to know whether the car is going to get a fine or not.

2 Temporal Here and There with Constraints and its Equilibrium Companion

A *constraint satisfaction problem*, which consists of a triple $\langle \mathcal{X}, \mathcal{D}, \mathcal{A} \rangle$ where $\mathcal{X}$ corresponds to the set of variables; $\mathcal{D}$ stands for the *domain* and $\mathcal{A}$ corresponds to a set of *temporal constraint atoms.* In our (temporal) case, each temporal constraint atom, or more simply *constraint atom*, is represented as $C(\circ^{l_1} x_1, \cdots, \circ^{l_n} x_n)$ where $\circ^{l_1} x_1, \cdots, \circ^{l_n} x_n$ is a sequence of n terms of the form $\circ^{l_i} x_i$ where x_i is a variable and $\circ^{l_i}$ corresponds to l_i consecutive occurrences of the operator $\circ$ ("next") and C is any n-ary relation on $\mathcal{D}$. THT_c formulas are defined in terms of the following BNF grammar.

$$\varphi, \psi ::= C(\circ^{l_1} x_1, \ldots, \circ^{l_n} x_n) \mid \bot \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \to \psi \mid \circ\varphi \mid \varphi \, \mathbb{U} \, \psi \mid \varphi \, \mathbb{R} \, \psi.$$

where $C(\circ^{l_1} x_1, \ldots, \circ^{l_n} x_n)$ is a constraint atom. The first building block of our semantics is the notion of *valuation*, which is a (partial) function $v : \mathcal{X} \rightarrow \mathcal{D} \cup \{\mathbf{u}\}$ which associates a value from $\mathcal{D}$ or the special symbol $\mathbf{u} \notin \mathcal{D}$ that stands for *undefined* and it means that v does not have a value for a given variable.

A *THT$_c$ trace*, in symbols $\langle \mathbf{v_h}, \mathbf{v_t} \rangle$, is an infinite sequence of interpretations $\mathbf{v_h} = v_h^0 \cdot v_h^1 \cdots$ and $\mathbf{v_t} = v_t^0 \cdot v_t^1 \cdots$ satisfying the relation: for all $x \in \mathcal{X}$, $d \in \mathcal{D}$, and for all $i \geq 0$, if $v_h^i(x) = d$ then $v_t^i(x) = d$.

Given an THT$_c$ trace $\mathbf{M} = \langle \mathbf{v_h}, \mathbf{v_t} \rangle$ and $i \geq 0$, the satisfaction of formulas is defined recursivelly as follows:

1. $\mathbf{M}, i \models C(\circ^{l_1} x_1, \ldots, \circ^{l_n} x_n)$ iff $\{(v_*^{i+l_1}(x_1), \ldots, v_*^{i+l_n}(x_n)) \mid * \in \{h, t\}\} \subseteq C$
2. $\mathbf{M}, i \models \varphi \wedge \psi$ iff $\mathbf{M}, i \models \varphi$ and $\mathbf{M}, i \models \psi$
3. $\mathbf{M}, i \models \varphi \vee \psi$ iff $\mathbf{M}, i \models \varphi$ or $\mathbf{M}, i \models \psi$
4. $\mathbf{M}, i \models \varphi \rightarrow \psi$ iff for all $* \in \{\mathbf{v_h}, \mathbf{v_t}\}$, $\langle *, \mathbf{v_t} \rangle, i \not\models \varphi$ or $\langle *, \mathbf{v_t} \rangle, i \models \psi$
5. $\mathbf{M}, i \models \circ\varphi$ iff $\mathbf{M}, i+1 \models \varphi$
6. $\mathbf{M}, i \models \varphi \,\mathbb{U}\, \psi$ iff it exists $k \geq i$ s.t. $\mathbf{M}, k \models \psi$ and $\mathbf{M}, j \models \varphi$ for all $j \in [i..k)$.
7. $\mathbf{M}, i \models \varphi \,\mathbb{R}\, \psi$ iff for all $k \in [i..\omega)$, $\mathbf{M}, k \models \psi$ or $\mathbf{M}, j \models \varphi$ for some $j \in [i..k)$.

We denote by THT$_c$ the logic defined by means of the aforementioned semantics. The minimal model selection criterion of equilibrium logic is extended to the case of THT$_c$ in a very natural way: we say that an THT$_c$ trace of the form $\langle \mathbf{v_t}, \mathbf{v_t} \rangle$ is a *temporal equilibrium model* of a formula φ if (1) $\langle \mathbf{v_t}, \mathbf{v_t} \rangle, 0 \models \varphi$ and (2) there is no THT$_c$ trace $\langle \mathbf{v_h}, \mathbf{v_t} \rangle$, with $\mathbf{v_h} \neq \mathbf{v_t}$ s.t. $\langle \mathbf{v_h}, \mathbf{v_t} \rangle, 0 \models \varphi$.

Example 1 can be formalized in THT$_c$ by means of the following formulas:

$p := 0.$ $\quad s := 80000.$ $\quad \circ^4 acc := 11350.$ $\quad \circ^6 acc := -2301.$
$\Box(rdlimit := 90000).$ $\Box(rdpos := 400000).$ $\Box(\circ s := s + acc).$ $\Box(\circ p := p + s).$
$\Box(\circ s := s \leftarrow \neg(\circ s \neq s)).$
$\Box(fine \leftarrow p < rdpos \wedge \circ p \geq rdpos \wedge \circ s > rdlimit).$

In the first line, we assign the *position* (p) and the *speed* (s) of the car at time $t = 0$. We also state that it *accelerates* (acc) at $t = 4$ and *decelerates* at $t = 6$. In the second line we declare that the *position* ($rdpos$) and *speed limit* ($rdlimit$) of the radar is set and it does not change along time. We also say that the speed evolves as a consequence of the acceleration while the position does it because of the speed. The formula in the third line corresponds to the inertia rule and the formula in the fourth line captures whether the car gets a fine or not. As can be calculated, the only trace satisfying all the formulas is the one where the car gets a fine at time point 5 since it passes the radar at 91,350 km/h.

3 Final Comments

In this extended abstract we have presented a new logic for temporal non-monotonic reasoning with constraints. We remark that our definition of constraints is general enough to be adapted to many types of constraints such as spatial [6], periodicity [7], Presburger [4] or Peano [8] constraints. However, in many cases the resulting logic will be highly undecidable. As a consequence, it will be necessary to identify decidable fragments that would lead to an implementation.

References

1. G. Brewka, T. Eiter, and M. Truszczyński: Answer set programming at a glance. Communications of the ACM 54(12), 92–103 (2011). DOI: 10.1145/2043174.2043195
2. F. Aguado, P. Cabalar, M. Diéguez, G. Pérez, T. Schaub, A. Schuhmann, and C. Vidal: Linear-Time Temporal Answer Set Programming. Theory and Practice of Logic Programming 23(1), 2–56 (2023). DOI: 10.1017/S1471068421000557
3. P. Cabalar, R. Kaminski, M. Ostrowski, and T. Schaub: An ASP Semantics for Default Reasoning with Constraints. In: R. Kambhampati. Proceedings of the Twenty-fifth International Joint Conference on Artificial Intelligence (IJCAI'16), pp. 1015–1021. IJCAI/AAAI Press (2016). DOI: 10.5555/3060621.3060762
4. S. Demri: Linear-time temporal logics with Presburger constraints: an overview. J. Appl. Non Class. Logics 16(3-4), 311–348 (2006)
5. D. Pearce: A New Logical Characterisation of Stable Models and Answer Sets. In: J. Dix, L. Pereira, and T. Przymusinski. Proceedings of the Sixth International Workshop on Non-Monotonic Extensions of Logic Programming (NMELP'96). Lecture Notes in Computer Science, pp. 57–70. Springer-Verlag (1997). DOI: 10.1007/BFb0023801
6. F. Wolter and M. Zakharyaschev: Spatio-temporal representation and reasoning based on RCC-8. In: A.G. Cohn, F. Giunchiglia, and B. Selman. KR 2000, Principles of Knowledge Representation and Reasoning Proceedings of the Seventh International Conference, Breckenridge, Colorado, USA, April 11-15, 2000, pp. 3–14. Morgan Kaufmann (2000)
7. S. Demri: LTL over integer periodicity constraints. Theor. Comput. Sci. 360(1-3), 96–123 (2006)
8. R. Kaye: Models of Peano Arithmetic. Clarendon Press (1991)