TEAM PROGRAMMING IN GOLOG UNDER PARTIAL OBSERVABILITY

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Abstract. We present and explore the agent programming language TEAMGOLOG, which is a novel approach to programming a team of cooperative agents under partial observability. Every agent is associated with a partial control program in Golog, which is completed by the TEAMGOLOG interpreter in an optimal way by assuming a decision-theoretic semantics. The approach is based on the key concepts of a synchronization state and a communication state, which allow the agents to passively resp. actively coordinate their behavior, while keeping their belief states, observations, and activities invisible to the other agents. We show the practical usefulness of the TEAMGOLOG approach in a rescue simulated domain. We describe the algorithms behind the TEAMGOLOG interpreter and provide a prototype implementation. We also show through experimental results that the TEAMGOLOG approach outperforms a standard greedy one in the rescue simulated domain.
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1 Introduction

During the recent years, the development of controllers for autonomous agents has become increasingly important in AI. One way of designing such controllers is the programming approach, where a control program is specified through a language based on high-level actions as primitives. Another way is the planning approach, where goals or reward functions are specified and the agent is given a planning ability to achieve a goal or to maximize a reward function. An integration of both approaches has recently been proposed through the seminal language DTGolog [3], which integrates explicit agent programming in Golog [20] with decision-theoretic planning in (fully observable) MDPs [18]. It allows for partially specifying a control program in a high-level language as well as for optimally filling in missing details through decision-theoretic planning, and it can thus be seen as a decision-theoretic extension to Golog, where choices left to the agent are made by maximizing expected utility. From a different perspective, it can also be seen as a formalism that gives advice to a decision-theoretic planner, since it naturally constrains the search space.

DTGolog has several other nice features, since it is closely related to first-order extensions of decision-theoretic planning (see especially [2, 22, 14]), which allow for (i) compactly representing decision-theoretic planning problems without explicitly referring to atomic states and state transitions, (ii) exploiting such compact representations for efficiently solving large-scale problems, and (iii) nice properties such as modularity (parts of the specification can be easily added, removed, or modified) and elaboration tolerance (solutions can be easily reused for similar problems with few or no extra cost).

However, DTGolog is designed only for the single-agent framework. That is, the model of the world essentially consists of a single agent that we control by a DTGolog program and the environment summarized in “nature”. But there are many applications where we encounter multiple agents that cooperate with each other. For example, in robotic rescue, mobile agents may be used in the emergency area to acquire new detailed information (such as the locations of injured people) or to perform certain rescue operations. In general, acquiring information as well as performing rescue operations involves several and different rescue elements (agents and/or teams of agents), which cannot effectively handle the rescue situation on their own. Only the cooperative work among all the rescue elements may solve it. Since most of the rescue tasks involve a certain level of risk for humans (depending on the type of rescue situation), mobile agents can play a major role in rescue situations, especially teams of cooperative heterogeneous mobile agents.

Another crucial aspect of real-world environments is that they are typically only partially observable, due to noisy and inaccurate sensors, or because some relevant parts of the environment simply cannot be sensed. For example, especially in the robotic rescue domain described above, every agent has generally only a very partial view on the environment.

The practical importance of controlling a system of cooperative agents under partial observability by a generalization of DTGolog has already been recognized in recent works by Ferrein [7] and Finzi and Łukasiewicz [8]. A drawback of these two works, however, is that they are implicitly centralized by the assumption of a global world model resp. the assumption that every agent knows the belief states, observations, and actions of all the other agents (and so [7, 8] have no explicit communication between the agents), which is very often not possible or not desirable in realistic applications.

In this paper, we present the agent programming language TEAMGOLOG, which is a novel generalization of DTGolog for controlling a system of cooperative agents under partial observability, which does not have such centralization assumptions. It is thus guided by the idea of truly distributed acting in multi-agent systems with a minimal interaction between the agents. The main contributions are as follows:

- We introduce the agent programming language TEAMGOLOG for controlling a system of cooperative (middle-size) agents under partial observability. We define a decision-theoretic semantics of TEAM-
GOLOG, which are inspired by decentralized partially observable MDPs (Dec-POMDPs) [16, 13].

- We introduce the concepts of a synchronization state and a communication state, which are used to coordinate the agents, taking inspiration from artificial social systems [21]: The behavior of each agent is encoded in advance in its domain theory and program, and depends on the online trace of synchronization and communication states.

- We define a TEAMGOLOG interpreter, including underlying algorithms, and provide a prototype implementation. We also provide a number of theoretical results around the TEAMGOLOG interpreter. In particular, we show that the interpreter generates optimal policies.

- We show the practical usefulness of the TEAMGOLOG approach in a rescue simulated domain. We also provide experimental results, which show that the TEAMGOLOG approach outperforms a standard greedy one in the rescue simulated domain.

The rest of this paper is organized as follows. In Section 2, we recall the situation calculus and Golog. Section 3 introduces the syntax of TEAMGOLOG and its underlying domain theory, belief states, and decision-theoretic planning model. In Section 4, we formally define a TEAMGOLOG interpreter and provide theoretical results around the interpreter. Sections 5 and 6 describe a rescue scenario and experimental results, respectively. In Section 7, we discuss related work. Section 8 summarizes our main results and gives an outlook on future research.

2 The Situation Calculus and Golog

The situation calculus [15, 20] is a first-order language for representing dynamic domains. Its main ingredients are actions, situations, and fluents. An action is a first-order term of the form \( a(\vec{u}) \), where \( a \) is an action name, and \( \vec{u} \) are its arguments. For example, \( \text{moveTo}(r, x, y) \) may represent the action of moving an agent \( r \) to the position \((x, y)\). A situation is a first-order term encoding a sequence of actions. It is either a constant symbol or of the form \( \text{do}(a, s) \), where \( a \) is an action and \( s \) is a situation. The constant symbol \( S_0 \) is the initial situation and represents the empty sequence, while \( \text{do}(a, s) \) encodes the sequence obtained from executing \( a \) after the sequence of \( s \). For example, \( \text{do}(<\text{moveTo}(r, 1, 2), \text{do}(<\text{moveTo}(r, 3, 4), S_0)>) \) stands for executing \( \text{moveTo}(r, 1, 2) \) after executing \( \text{moveTo}(r, 3, 4) \) in \( S_0 \). A fluent represents a world or agent property that may change when executing an action. It is a predicate symbol whose most right argument is a situation. For example, \( \text{at}(r, x, y, s) \) may express that the agent \( r \) is at the position \((x, y)\) in the situation \( s \). In the situation calculus, a dynamic domain is encoded as a basic action theory \( AT = (\Sigma, D_{S_0}, D_{sfa}, D_{una}, D_{ap}) \), where:

- \( \Sigma \) is the set of foundational axioms for situations.
- \( D_{una} \) is the set of unique name axioms for actions, encoding that different action terms stand for different actions.
- \( D_{S_0} \) is a set of first-order formulas describing the initial state of the domain (represented by \( S_0 \)). For example, \( \text{at}(r, 1, 2, S_0) \) may express that the agent \( r \) is initially at the position \((1, 2)\).
- \( D_{sfa} \) is the set of successor state axioms [20]. For each fluent \( F(\vec{x}, s) \), it contains an axiom \( F(\vec{x}, \text{do}(a, s)) \equiv \Phi_F(\vec{x}, a, s) \), where \( \Phi_F(\vec{x}, a, s) \) is a formula with free variables among \( \vec{x}, a, \) and \( s \). These
axioms specify the truth of the fluent \( F \) in the next situation \( do(a, s) \) in terms of the current situation \( s \), and are a solution to the frame problem (for deterministic actions). For example,
\[
at(o, x, y, do(a, s)) \equiv a = \text{moveTo}(o, x, y) \lor at(o, x, y, s) \land \neg \exists x', y' (a = \text{moveTo}(o, x', y'))
\]
may express that the object \( o \) is at the position \((x, y)\) in the situation \( do(a, s) \) iff it is moved there in the situation \( s \), or already there and not moved away in \( s \).

- \( D_{ap} \) is the set of action precondition axioms. For each action \( a \), it contains an axiom \( \text{Poss}(a(\vec{x}), s) \equiv \Pi(\vec{x}, s) \), which characterizes the preconditions of \( a \). For example, \( \text{Poss}(\text{moveTo}(o, x, y), s) \equiv \neg \exists o' (at(o', x, y, s)) \) may express that it is possible to move the object \( o \) to the position \((x, y)\) in the situation \( s \) iff no other object \( o' \) is at \((x, y)\) in \( s \).

Golog is an agent programming language that is based on the situation calculus. It allows for constructing complex actions from the primitive actions defined in a basic action theory \( AT \), where standard (and not so standard) Algol-like constructs can be used, in particular, (i) action sequences: \( p_1; p_2 \); (ii) tests: \( \phi? \); (iii) nondeterministic action choices: \( p_1|p_2 \); (iv) nondeterministic choices of action argument: \( \pi x (p(x)) \); and (v) conditionals, while-loops, and procedures.

## 3 Team Golog under Partial Observability

We now introduce the agent programming language TEAMGOLOG, which is a generalization of Golog for programming teams of cooperative agent under partial observability.

Our approach is based on the key concepts of a synchronization state and a communication state, which allow the agents to passively resp. actively coordinate their behavior, while keeping their belief states, observations, and activities invisible to the other agents. Here, the synchronization state is fully observable by all the agents, but outside their control. The communication state is a multi-dimensional state, containing one dimension for each agent, which is also fully observable by all the agents. But every agent may change its part of the communication state whenever necessary, which encodes its explicit communication to all the other agents.

Since both the synchronization state \( S \) and the communication state \( C \) are fully observable by all the agents, they can be used to condition and coordinate the behavior of the agents. At the same time, each agent can keep its belief state, observations, and actions invisible to the other agents. We thus realize a maximally distributed acting of the agents. The TEAMGOLOG program of each agent encodes the agent’s behavior conditioned on \( S \) and \( C \), and thus on the current situation. Hence, TEAMGOLOG programs bear close similarity to social laws in artificial social systems [21]. The basic idea behind such systems is to formulate a mechanism, called social law, that minimizes the need for both centralized control and online resolution of conflicts.

There are many real-world situations where we encounter such a form of coordination. For example, the traffic law “right has precedence over left” regulates the order in which cars can pass a street cross. In most cases, this law is sufficient to make the cars pass the street cross without any further interaction between the car drivers. Only in exceptional cases, such as the one where on each street a car is approaching the cross or when a car has a technical defect, some additional communication between the car drivers is necessary. Similarly, a soccer team can fix in advance the behavior of its team members in certain game situations (such as defense or attack), thus minimizing the explicit communication between the members during the game (which may be observed by the adversary). In these two examples, the synchronization state encodes
the strategy fixed by the team in their training units and before the game. The correct behavior of the car drivers resp. soccer players is encoded by traffic laws resp. the communication state encodes the explicit communication. In the rest of this section, we first define a variant of Dec-POMDPs, which underlies the decision-theoretic semantics of TEAMGOLOG programs. We then define the domain theory and the syntax of TEAMGOLOG programs.

3.1 Weakly Correlated Dec-POMDPs

We consider the following variant of Dec-POMDPs for \( n \geq 2 \) agents, which essentially consist of a transition function between global states (where every global state consists of a communication state for each agent and a synchronization state) and a POMDP for each agent and each global state, where every agent can also send a message to the others by changing its communication state. A weakly correlated Dec-POMDP \((I, S, (C_i)_{i \in I}, P, (S_i)_{i \in I}, (A_i)_{i \in I}, (O_i)_{i \in I}, (P_i)_{i \in I}, (R_i)_{i \in I})\) consists of a set of \( n \geq 2 \) agents \( I = \{1, \ldots, n\} \), a nonempty finite set of synchronization states \( S \), a nonempty finite set of communication states \( C_i \) for every agent \( i \in I \), a transition function \( P : C \times S \times S_i \times A_i \rightarrow PD(C \times S) \), which associates with every global state \((c, s) \in (C, S)\), local state \( s_i \in S_i \), and action \( a_i \in A_i \) a probability distribution over \( C \times S \), and for every agent \( i \in I \): (i) a nonempty finite set of local states \( S_i \), a nonempty finite set of actions \( A_i \), (ii) a nonempty finite set of observations \( O_i \), (iii) a transition function \( P_i : C \times S \times S_i \times A_i \rightarrow PD(C_i \times S_i \times O_i) \), which associates with every global state \((c, s) \in (C, S)\), local state \( s_i \in S_i \), and action \( a_i \in A_i \) a probability distribution over \( C_i \times S_i \times O_i \), and (iv) a reward function \( R_i : C \times S \times S_i \times A_i \rightarrow \mathbb{R} \), which associates with every global state \((c, s) \in (C, S)\), local state \( s_i \in S_i \), and action \( a_i \in A_i \) a reward \( R_i(c, s, s_i, a_i) \) to agent \( i \).

The \( q \)- and \( v \)-functions for agent \( i \in I \) of a finite-horizon value iteration are defined in Fig. 1 for \( n > 0 \) and \( m \geq 0 \), where \( P_{c_i}^j(\cdot | c, s) \) is the conditioning of \( P(\cdot | c, s) \) on \( c_i' \), and \( c_{i \rightarrow j} \) denotes \( c' \) without \( c_i' \). That is, an optimal action of agent \( i \) in the global state \((c, s)\) and the local state \( s_i \) when there are \( n \) steps to go is given by \( \text{argmin}_{a_i \in A_i} Q_i^m(c, s, s_i, a_i) \). Notice that these are the standard definitions of \( q \)- and \( v \)-functions, adapted to our framework of local and global states.

\[
Q_i^0(c, s, s_i, a_i) = R_i(c, s, s_i, a_i)
\]
\[
Q_i^n(c, s, s_i, a_i) = R_i(c, s, s_i, a_i) + \sum_{c' \in C} \sum_{s' \in S} \sum_{s_i' \in S_i} \sum_{a_i \in A_i} P_i(c_i', s_i', a_i | c, s, s_i, a_i) \cdot P_i(c_{i \rightarrow i} | c, s) \cdot V_{i \rightarrow i}^{m-1}(c_{i \rightarrow i}, s_i', s_i')
\]
\[
V_i^m(c, s, s_i) = \min_{a_i \in A_i} Q_i^m(c, s, s_i, a_i),
\]

Figure 1: \( q \)- and \( v \)-functions.

3.2 Domain Theory

TEAMGOLOG programs are interpreted relative to a domain theory, which extends a basic action theory by stochastic actions, reward functions, and utility functions. Formally, a domain theory \( DT_i = (AT_i, ST_i, OT_i) \) consists of \( n \geq 2 \) agents \( I = \{1, \ldots, n\} \), and for each agent \( i \in I \): a basic action theory \( AT_i \), a stochastic theory \( ST_i \), and an optimization theory \( OT_i \), where the latter two are defined below.

The finite nonempty set of primitive actions \( A \) is partitioned into nonempty sets of primitive actions \( A_1, \ldots, A_n \) of agents \( 1, \ldots, n \), respectively. We assume a finite nonempty set of observations \( O \), which is
partitioned into nonempty sets of observations $O_1, \ldots, O_n$ of agents 1, \ldots, n, respectively.

A stochastic theory $ST_i$ for agent $i \in I$ is a set of axioms that define stochastic actions for agent $i$. We represent stochastic actions through a finite set of deterministic actions, as usual [9, 3]. When a stochastic action is executed, then with a certain probability, “nature” executes exactly one of its deterministic actions and produces exactly one possible observation. As underlying decision-theoretic semantics, we assume the weakly correlated Dec-POMDPs of Section 3.1, along with the relational fluents that associate with every situation $s$ a communication state $c_j$ of agent $j \in I$, a synchronization state $z$, and a local state $s_i$ of agent $i$, respectively. The communication and synchronization properties are visible by all the agents, the others are private and hidden. We use the predicate $\text{stochastic}(a, s, n, o, \mu)$ to encode that when executing the stochastic action $a$ in the situation $s$, “nature” chooses the deterministic action $n$ producing the observation $o$ with the probability $\mu$. Here, for every stochastic action $a$ and situation $s$, the set of all $(n, o, \mu)$ such that $\text{stochastic}(a, s, n, o, \mu)$ is a probability function on the set of all deterministic components $n$ and observations $o$ of $a$ in $s$. We also use the notation $\text{prob}(a, s, n, o)$ to denote the probability $\mu$ such that $\text{stochastic}(a, s, n, o, \mu)$. We assume that $a$ and all its nature choices $n$ have the same preconditions. A stochastic action $a$ is indirectly represented by providing a successor state axiom for every associated nature choice $n$. The stochastic action $a$ is executable in a situation $s$ with observation $o$, iff $\text{prob}(a, s, n, o) > 0$ for some $n$. The optimization theory $OT_i$ for agent $i \in I$ specifies a reward and a utility function for agent $i$. The former associates with every situation $s$ and action $a$, a reward to agent $i \in I$, denoted $\text{reward}(i, a, s)$. The utility function maps every reward and success probability to a real-valued utility $\text{utility}(v, pr)$. We assume $\text{utility}(v, 1) = v$ and $\text{utility}(v, 0) = 0$ for all $v$. An example is $\text{utility}(v, pr) = v \cdot pr$. The utility function suitably mediates between the agent reward and the failure of actions due to unsatisfied preconditions.

**Example 3.1 (Rescue Domain)** We consider a rescue domain where several autonomous mobile agents have to localize some victims in the environment and report their positions to a remote operator. We assume a team of three heterogeneous agents $a_1$, $a_2$, and $a_3$ endowed with shape recognition (SH), infrared (IF), and CO2 sensors, respectively. A victim position is communicated to the operator once sensed and analyzed by all the three sensing devices. Each agent $a_i$ can execute one of the actions $\text{goTo}_i(pos)$, $\text{analyze}_i(pos, type_i)$, and $\text{reportToOp}_i(pos)$. The action theory $AT_i$ is described by the fluents $\text{at}_i(pos, s)$, $\text{analyzed}_i(pos, type_i, s)$, and $\text{reported}_i(x, s)$, which are accessible only by agent $a_i$.

The successor state axioms for these fluents are defined as follows:

\[
\begin{align*}
\text{at}_i(pos, do(a, s)) &\equiv a = \text{goTo}_i(pos) \lor \text{at}_i(pos, s) \land \neg \exists pos' (a = \text{goTo}_i(pos')) , \\
\text{analyzed}_i(pos, type_i, do(a, s)) &\equiv a = \text{analyze}_i(pos, type_i) \lor \text{analyzed}_i(pos, type_i, s) , \\
\text{reported}_i(x, do(a, s)) &\equiv a = \text{reportToOp}_i(pos) \lor \text{reported}_i(x, s) ,
\end{align*}
\]

and the precondition axioms, one for each action, are given by

\[
\begin{align*}
\text{Poss}(\text{analyzed}_i(pos, type_i), s) &\equiv \text{at}_i(pos, s) , \\
\text{Poss}(\text{goTo}_i(pos), s) &\equiv \neg \text{at}_i(pos, s) , \\
\text{Poss}(\text{reportToOp}_i(pos), s) &\equiv \text{at}_i(pos, s) ,
\end{align*}
\]

As for the global state, the communication state is defined by the fluent $cs_i(data, s)$, where $i$ is the agent, and $data$ is the shared info, for example, $cs_1(\text{atVictim}((2, 2), IF), s)$ means that $a_1$ detected a victim in position $(2, 2)$ through the IF sensor. Other global data are $\text{repVictim}(p)$ (victim reported in position $p$) and
notice that the action goTo detVictim finally, fullyAnalyzed victims taking into account the decisions of the other agents. However, given the penalty on the conflicts, the agents are encouraged to distribute their analysis on different hence, in a greedy policy, the agent should go towards the closest non-analyzed victim and analyze it.

where noVictim possible, for example, because the wireless connection is up.
The synchronization states is described by gsConnect(s) stating that the global state communication is possible, for example, because the wireless connection is up.

gsConnect(do(a, s)) \equiv gsConnect(s) \land \neg a = disconnect \lor a = reconnect.

In STi, we define the stochastic versions of the actions in ATi, for example, goToS_i(pos) and analyzeS_i(pos, type_i). Each of these can fail resulting in an empty action, for example,

\begin{align*}
prob(goToS_i(pos), s, goTo_i(pos), obs(succ)) &= 0.9, \\
prob(goToS_i(pos), s, nop, obs(fail)) &= 0.7, \\
prob(analyzeS_i(pos, type_i), s, analyze_i(pos, type_i), obs(succ)) &= 0.9, \\
prob(analyzeS_i(pos, type_i), s, nop, obs(fail)) &= 1.
\end{align*}

In OTi, we provide a high reward for a fully analyzed victim correctly reported to the operator, a low reward for the analysis of a detected victim, and a (distance-dependent) cost is associated with the action goTo. Since two agents can obstacle each other when operating in the same location, we penalize the agents analyzing the same victim at the same time. More precisely, we employ the following reward:

\begin{align*}
reward(i, a, s) = r & \stackrel{\text{def}}{=} \exists p, t (a = analyze_i(p, t) \land \\
(detVictim(p, s) \land (\neg conflicts_i(a, s) \land r = 50) \lor \\
conflicts_i(a, s) \land r = 10) \lor \neg detVictim(p, s) \land r = -10) \lor \\
a = reportToOp_i(p) \land fullyAnalyzed(p, s) \land r = 200 \lor \\
a = goTo_i(p) \land \exists p' (at_i(p', s) \land r = - \text{dist}(p', p)))
\end{align*}

where conflicts_i(s) is true if another agent communicates the analysis of the same location in the global state, i.e.,

\begin{align*}
conflicts_i(s) = \stackrel{\text{def}}{=} \exists p (\exists t (cs_i(comm(p, t), s) \land \bigwedge_{j \in I} \exists t' (cs_j(comm(p, t'), s))),
\end{align*}

where cs_k(comm_k(p, t), s) means “just communicated to the global state”, i.e.,

\begin{align*}
\text{cs}_k(comm_k(p, t), do(a, s)) \equiv a = analyze_i(p, t);
\end{align*}

detVictim(p, s) is true if at least one agent has discovered a victim in p, i.e.,

\begin{align*}
detVictim(p, s) = \stackrel{\text{def}}{=} \bigvee_{i \in I} \exists cs_i(atVictim(p, t), s);
\end{align*}

finally, fullyAnalyzed(p, s) means that all the analysis has been performed, i.e.,

\begin{align*}
\text{fullyAnalyzed}(p, s) = \stackrel{\text{def}}{=} \bigvee_{i,j,k \in I} cs_i(atVictim(p, SH), s) \land \\
\text{cs}_j(atVictim(p, IF), s) \land \text{cs}_k(atVictim(p, CO_2), s).
\end{align*}

Notice that the action goTo_i(p) has a cost depending on the distance between starting point and destination, hence, in a greedy policy, the agent should go towards the closest non-analyzed victim and analyze it. However, given the penalty on the conflicts, the agents are encouraged to distribute their analysis on different victims taking into account the decisions of the other agents.
3.3 Belief States

We next introduce belief states over situations for single agents, and define the semantics of actions in terms of transitions between belief states. A belief state $b$ of agent $i \in I$ is a set of pairs $(s, \mu)$ consisting of an ordinary situation $s$ and a real $\mu \in (0, 1]$ such that (i) all $\mu$ sum up to 1, and (ii) all situations $s$ in $b$ are associated with the same joint communication state and the same synchronization state. Informally, every $b$ represents the local belief of agent $i \in I$ expressed as a probability distribution over its local states, along with unique joint communication and synchronization states. The probability of a fluent formula $\phi(s)$ (uniform in $s$) in the belief state $b$, denoted $\phi(b)$, is the sum of all $\mu$ such that $\phi(s)$ is true and $(s, \mu) \in b$. In particular, $\text{Poss}(a, b)$, where $a$ is an action, is defined as the sum of all $\mu$ such that $\text{Poss}(a, s)$ is true and $(s, \mu) \in b$, and $\text{reward}(i, a, b)$ is defined in a similar way.

Given a deterministic action $a$ and a belief state $b$ of agent $i \in I$, the successor belief state after executing $a$ in $b$, denoted $\text{do}(a, b)$, is the belief state

$$b' = \{(do(a, s), \mu/\text{Poss}(a, b)) \mid (s, \mu) \in b, \text{Poss}(a, s)\}.$$

Furthermore, given a stochastic action $a$, an observation $o$ of $a$, and a belief state $b$ of agent $i \in I$, the successor belief state after executing $a$ in $b$ and observing $o$, denoted $\text{do}(a, o, b)$, is the belief state $b'$, where $b'$ is obtained from all pairs $(do(n, s), \mu \cdot \mu')$ such that $(s, \mu) \in b, \text{Poss}(a, s)$, and $\mu' = \text{prob}(a, s, n, o) > 0$ by normalizing the probabilities to sum up to 1.

The probability of making the observation $o$ after executing the stochastic action $a$ in the local belief state $b$ of agent $i \in I$, denoted $\text{prob}(a, b, o)$, is defined as the sum of all $\mu \cdot \mu'$ such that $(s, \mu) \in b$ and $\mu' = \text{prob}(a, s, n, o) > 0$.

**Example 3.2 (Rescue Domain cont’d)** Suppose that agent $a_1$ is aware of its initial situation, and thus has the initial belief state $\{(S_0, 1)\}$. After executing the stochastic action $\text{goToS}_1(1, 1)$ and observing its success $\text{obs(succ)}$, the belief state of $a_1$ then changes to $\{(S_0, 0.1), (\text{do}(\text{goToS}_1(1, 1), S_0), 0.9)\}$ (here, $\text{prob}(\text{goToS}_1(\text{pos}), s, \text{goTo}_1(\text{pos}), \text{obs(succ)}) = 0.9$, and $\text{goToS}_1(\text{pos})$ is always executable).

3.4 Syntax

Given the actions specified by a domain theory $DT_i$, a program $p$ in TEAMLOGO for agent $i \in I$ has one of the following forms (where $\phi$ is a condition, $p, p_1, p_2$ are programs, and $a, a_1, \ldots, a_n$ are actions of agent $i$):

1. **Deterministic or stochastic action**: $a$. Do $a$.

2. **Nondeterministic action choice**: choice[i: $a_1 | \cdots | a_n$].
   
   Do an optimal action among $a_1, \ldots, a_n$.

3. **Test action**: $\phi$?. Test $\phi$ in the current situation.

4. **Action sequence**: $p_1; p_2$. Do $p_1$ followed by $p_2$.

5. **Nondeterministic choice of two programs**: $(p_1 | p_2)$.
   
   Do $p_1$ or $p_2$.

6. **Nondeterministic choice of an argument**: $\pi x (p(x))$.
   
   Do any $p(x)$.
7. Nondeterministic iteration: $p^*$. Do $p$ zero or more times.

8. Conditional: $\text{if } \phi \text{ then } p_1 \text{ else } p_2$.

9. While-loop: $\text{while } \phi \text{ do } p$.

10. Procedures, including recursion.

Example 3.3 (Rescue Domain cont’d) The following code represents an incomplete procedure $\text{explore}_i$ of agent $i$:

```plaintext
proc(\text{explore}_i,
\pi_\text{x}(\text{goToS}_i(x));
\text{if } \text{obs}(\text{succ}) \text{ then } [\text{analyzeS}_i(x, \text{type}_i);
\text{if } \text{obs}(\text{succ}) \land \text{fullyAnalyzed}(x) \text{ then }
\text{reportToOp}_i(\text{repVictim}(x))];
\text{explore}_i).
```

Here, agent $i$ first has to decide where to go. Once the position is reached, agent $i$ analyzes the current location deploying one of its sensing devices. If a victim is detected, then the position of the victim is communicated to the operator.

4 TEAMGOLOG Interpreter

In this section, we first specify the decision-theoretic semantics of TEAMGOLOG programs in terms of an interpreter. We then provide theoretical results about the interpreter.

4.1 Formal Specification

We now define the formal semantics of a TEAMGOLOG program $p$ for agent $i \in I$ relative to a domain theory $DT$. We associate with every TEAMGOLOG program $p$, belief state $b$, and horizon $H \geq 0$, an optimal $H$-step policy $\pi$ along with its expected utility $U$ to agent $i \in I$. Intuitively, this $H$-step policy $\pi$ is obtained from the $H$-horizon part of $p$ by replacing every nondeterministic action choice by an optimal action.

Formally, given a TEAMGOLOG program $p$ for agent $i \in I$ relative to a domain theory $DT$, a horizon $H \geq 0$, and a start belief state $b$ of agent $i$, we say that $\pi$ is an $H$-step policy of $p$ in $b$ with expected $H$-step utility $U$ to agent $i$ iff $DT \models G(p, b, H, \pi, \langle v, pr \rangle)$ and $U = \text{utility}(v, pr)$, where the macro $G(p, b, h, \pi, \langle v, pr \rangle)$ is defined by induction on the different constructs of TEAMGOLOG. The definition of $G$ for some of the constructs is given as follows (the complete definition is given in the full version of this paper):

- Null program ($p = \text{nil}$) or zero horizon ($h = 0$):

  $$G(p, b, h, \pi, \langle v, pr \rangle) =_{\text{def}} \pi = \text{stop} \land \langle v, pr \rangle = \langle 0, 1 \rangle.$$  

  Intuitively, $p$ ends when it is null or at the horizon end.
Deterministic first program action $c$:

$$G([c; p'], b, h, \pi, \langle v, pr \rangle) \overset{\text{def}}{=} (\text{Poss}(c, b) = 0 \land \pi = \text{stop} \land v = 0 \land pr = 1) \lor$$

$$\lor (\text{Poss}(c, b) > 0 \land \exists \pi', v', pr' \ (G(p', do(c, b), h-1, \pi', \langle v', pr' \rangle) \land \pi = c; \pi' \land v = v' + \text{reward}(c, b) \land pr = pr' \cdot \text{Poss}(c, b)).$$

Informally, suppose that $p = [c; p']$, where $c$ is a deterministic action. If $c$ is not executable in the belief state $b$, then $p$ has only the policy $\pi = \text{stop}$ along with the expected reward $v = 0$ and the success probability $pr = 0$. Otherwise, the optimal execution of $[c; p']$ in the belief state $b$ depends on that of $p$ in $do(c, b)$. Observe that $c$ is executable in $b$ with the probability $\text{Poss}(c, b)$, which affects the overall success probability $pr$.

Stochastic first program action with observation and $h > 0$:

$$G([a_o; p'], b, h, \pi, \langle v, pr \rangle) \overset{\text{def}}{=} (\text{Poss}(a_o, b) = 0 \land \pi = \text{stop} \land \langle v, pr \rangle = (0, 1)) \lor (\text{Poss}(a_o, b) > 0 \land$$

$$\exists (\bigwedge^l_{q=1} G(p', do(a_o, b), h-1, \pi_q, \langle v_q, pr_q \rangle) \land$$

$$\pi = a_o; \text{for } q = 1 \text{ to } l \text{ do if } o_q \text{ then } \pi_q \land$$

$$v = \text{reward}(i, a_o, b) + \sum^l_{q=1} v_q \cdot \text{prob}(a_o, b, o_q) \land$$

$$pr = \text{Poss}(a_o, b) \cdot \sum^l_{q=1} pr_q \cdot \text{prob}(a_o, b, o_q))'.$$

Here, $\exists(F)$ is obtained from $F$ by existentially quantifying all free variables in $F$. Moreover, $o_1, \ldots, o_l$ are the different pairs of a joint communication state and a synchronization state that are compatible with $a_o$, and $\text{prob}(a_o, b, o_q)$ is the probability of arriving in such $o_q$ after executing $a_o$ in $b$. Informally, suppose $p = [a_o; p']$, where $a_o$ is a stochastic action with observation. If $a_o$ is not executable in $b$, then $p$ has only the policy $\pi = \text{stop}$ along with the expected reward $v = 0$ and the success probability $pr = 0$. Otherwise, the optimal execution of $[a_o; p']$ in $b$ depends on that one of $p$ in $do(a_o, b)$.

Stochastic first program action and $h > 0$:

$$G([a; p'], b, h, \pi, \langle v, pr \rangle) \overset{\text{def}}{=}$$

$$\exists (\bigwedge^l_{q=1} G([a_o; p'], b, h, a_oq, \pi_q, \langle v_q, pr_q \rangle) \land$$

$$\pi = a_oq; \text{for } q = 1 \text{ to } l \text{ do if } o_q \text{ then } \pi_q \land$$

$$v = \sum^l_{q=1} v_q \cdot \text{prob}(a, b, o_q) \land$$

$$pr = \sum^l_{q=1} pr_q \cdot \text{prob}(a, b, o_q))'.$$

Here, $o_1, \ldots, o_l$ are the possible observations of the stochastic action $a$. The generated policy is a conditional plan in which every such observation $o_q$ is considered.

Non-deterministic first program action and $h > 0$:

$$G([\text{choice}(i: a_1, \ldots, a_n); p'], b, h, \pi, \langle v, pr \rangle) \overset{\text{def}}{=}$$

$$\exists (\bigwedge^l_{q=1} G([a_q; p'], b, h, a_q, \pi_q, \langle v_q, pr_q \rangle) \land$$

$$k = \arg\max_{q \in \{1, \ldots, n\}} \text{utility}(v_q, pr_q) \land$$

$$\pi = \pi_k \land v = v_k \land pr = pr_k).$$
Test action:

\[ G([\phi?;p'], b, h, \pi, (v, pr)) = \text{def} \ (\phi[b] = 0 \land \pi = \text{stop} \land v = 0 \land pr = 0) \lor \exists pr'(\phi[b] > 0 \land G(p', b, h, \pi, (v, pr')) \land pr = pr' \cdot \phi[b]). \]

Informally, let \( p = [\phi?;p'] \). If \( \phi \) is false in \( b \), then \( p \) has only the policy \( \pi = \text{stop} \) along with the expected reward \( v = 0 \) and the success probability \( pr = 0 \). Otherwise, \( \pi \) is a policy of \( p \) with the expected reward \( v \) and success probability \( pr' \cdot \phi[b] \) iff \( \pi \) is a policy of \( p' \) with the expected reward \( v \) and success probability \( pr' \).

- The macro \( G \) is naturally extended to nondeterministic choices of action arguments, nondeterministic iterations, conditionals, while-loops, and procedures.

### 4.2 Theoretical Results

The following result shows that the TEAMGOLOG interpreter indeed generates an optimal \( H \)-step policy \( \pi \) along with its expected utility \( U \) to agent \( i \in I \) for a given TEAMGOLOG program \( p \), belief state \( b \), and horizon \( H \geq 0 \).

**Theorem 4.1** Let \( p \) be a TEAMGOLOG program for agent \( i \in I \) w.r.t. a domain theory \( DT_i \), let \( b \) be a belief state, and let \( H \geq 0 \) be a horizon. Then, the optimal \( H \)-step policy \( \pi \) of \( p \) in \( b \) along with its expected utility \( U \) to agent \( i \in I \) is given by \( DT_i \models G(p, b, H, \pi, (v, pr)) \) and \( U = \text{utility}(v, pr) \).

The next result gives an upper bound for the number of leaves in the evaluation tree, which is polynomial when the horizon is bounded by a constant. Here, \( n \) is the maximum among the maximum number of actions in nondeterministic action choices, the maximum number of observations after actions, the maximum number of arguments in nondeterministic choices of an argument, and the number of pairs consisting of a synchronization state and a communication state.

**Theorem 4.2** Let \( p \) be a TEAMGOLOG program for agent \( i \in I \) w.r.t. a domain theory \( DT_i \), let \( b \) be a belief state, and let \( H \geq 0 \) be a horizon. Then, computing the \( H \)-step policy \( \pi \) of \( p \) in \( b \) along with its expected utility \( U \) to agent \( i \in I \) via \( G \) generates \( O(n^{3H}) \) leaves in the evaluation tree.

### 5 Rescue Scenario

Consider the rescue scenario in Fig. 2. We assume that three victims have already been detected in the environment, but not completely analyzed: in position \((3, 7)\), the presence of Alice was detected by \( a_1 \) through the SH sensor; in position \((7, 7)\), agent \( a_2 \) discovered Bob through IF, and \( a_3 \) analyzed him through the CO\textsubscript{2} sensor; finally, in position \((4, 2)\), victim Carol was detected by \( a_2 \) with IF. We assume that this information is available in the global state, that is, the properties \( cs_1(\text{atVictim}(3, 7), \text{SH}, s) \), \( cs_3(\text{atVictim}(7, 7), \text{IF}, s) \), \( cs_2(\text{atVictim}(7, 7), \text{CO}_2, s) \), and \( cs_2(\text{atVictim}(4, 2), \text{IF}, s) \) hold in the communication state of the agents. As for the local state, we assume the belief states \( b_1 = \{(s_{1, 1}, 0.8), (s_{1, 2}, 0.2)\} \), \( b_2 = \{(s_{2, 1})\} \), and \( b_3 = \{(s_3, 1)\} \), with \( at_1(3, 6, s_{1, 1}) \), \( at_1(3, 5, s_{1, 2}) \), \( at_2(7, 7, s_2) \), and \( at_3(3, 7, s_3) \).

Given this situation, the task of the team of agents is to fully analyze the discovered victims and report their positions to the operator once the victim analysis is completed. This task can be encoded by the
following procedure:

\[ \text{proc}(\text{explore}_i, \pi x \in \{ (3, 7), (7, 7), (4, 2) \} (\text{goToS}_i(x)); \]
\[ \text{if} \ obs(\text{succ}) \text{ then } \text{analyzeS}_i(x, \text{type}_i); \]
\[ \text{if} \ obs(\text{succ}) \land \text{fullyAnalyzed}(x) \text{ then } \]
\[ \text{reportToOp}_i(\text{repVictim}(x))); \]
\[ \text{explore}_i, \]

where type\(_1\) = SH, type\(_2\) = IF, and type\(_3\) = CO\(_2\). Every agent a\(_i\) with i \in \{1, 2, 3\} has to separately compile the procedure explore\(_i\) using its global and local information. Assuming the horizon \(H = 5\) and the initial belief state \(b_i\), the optimal 4-step policy \(\pi_i\) for agent a\(_i\), produced by the TEAMGOLOG interpreter, is such that \(DT_i \models G([\text{explore}_i; \text{nil}], b_i, 4, \pi_i, (v_i, pr_i))\). Here, \(\pi_1, \pi_2, \text{ and } \pi_3\) are complex conditional plans branching over all possible observations and global states. For example, the beginning of \(\pi_1\) is as follows:

\[ \text{goToS}_1(7, 7); \]
\[ \text{if} \ obs(\text{succ}) \text{ then } \text{analyzeS}_1((7, 7), \text{SH}); \]
\[ \text{if} \ obs(\text{succ}) \land \text{fullyAnalyzed}(7, 7) \text{ then } \]
\[ \text{reportToOp}_1(\text{repVictim}(7, 7)); \]
\[ \text{goToS}_1(4, 2); \ldots \]

6 Empirical Results

We tested our approach in a rescue scenario that generalizes the one introduced in the previous section. In this context, we have \(N\) agents analyzing \(N\) victims; we assume each agent endowed with one sensor, IF, SH, or CO\(_2\); a victim position can be reported only if fully analyzed by these three sensors. This scenario has been realized in an abstract simulator. The simulator captures the key features of the environment and allows to execute agent actions computing associated rewards. Here, the simulator has been developed in C++, the TEAMGOLOG interpreter has been implemented in Eclipse Prolog, and the executor of TEAMGOLOG actions has been realized combing C++ and Eclipse Prolog (Eclipse Prolog embedded in C++).

A greedy control strategy has been devised as a comparison with the policies \(\pi_i\) generated by the TEAMGOLOG interpreter. In the greedy strategy, at each time step, each agent searches for the best victim to
analyze, based on the current distance to the victim. Whenever a victim has been completely analyzed, the agent can report the victim state to the operator.

The policies $\pi_i$ are generated by a TEAMGOLOG program generalizing the one presented in the previous section, i.e.,

$$\text{proc} (\text{explore}_i, \pi x \in \text{Pos} (\text{goToS}_i(x)) ;$$

$$\text{if} \ obs(\text{succ}) \ \text{then} \ [\text{analyzeS}_i(x, \text{type}_i);$$

$$\text{if} \ obs(\text{succ}) \land \text{fullyAnalyzed}(x) \ \text{then}$$

$$\text{reportToOp}_i(\text{repVictim}(x))]);$$

$$\text{explore}_i ,$$

Here, the possible destinations for $\text{goToS}_i(x)$ are chosen among $\text{Pos}$ representing locations stored in the global state of partially analyzed victims. Following Example 3.1, the communication state contains only the information about the detected victims, for example, $\text{cs}_i(\text{atVictim}((2, 2), IF), s)$, while the synchronization state is represented by the predicate $\text{gsConnection}(s)$ stating that the global communication among the agents is possible.

During our tests, we considered teams of 3, 4, 5, 7, and 10 agents with the task of analyzing an equal number of victims; for each case, we executed 30 runs in different configurations (victim and agents initial positions and properties to be analyzed). Furthermore, here we assume the global communication to be always available, i.e., $\text{gsConnection}(s)$ true for each situation. Given the uncertainty of the actions’ outcomes, we executed 20 runs for each initial configuration of victims and agents collecting the average reward values. We assumed that at least one feature for each victim was already available in the initial global state, in order to have already available the possible locations to be explored, in this way the agents can focus on the coordination task. Our aim is to assess the effects of a fast and frugal planning activity with respect to a simple greedy policy. In particular, we consider the reward gained by the agents after 3 and 4 execution cycles, where the TEAMGOLOG program horizon is set to 3 and 4 steps ahead, respectively.

Figs. 3 and 4 illustrate the average global reward gained via the TEAMGOLOG and the greedy algorithm.

Both the greedy algorithm and the policies $\pi_i$ were effective in reporting victims to the operator, however, the policies $\pi_i$ revealed to be superior with respect to the greedy strategy gaining more reward. Since each victim requires at most two sensing analysis, usually, the agents were able to complete the mission in three steps, in these cases the fourth execution step was useless and could provide additional costs due to unnecessary action executions and conflicts (given the partial observability of the domain, some of these conflicts cannot be eliminated by the TEAMGOLOG strategy, for example, introducing a $\text{nop}$). Indeed, in Fig. 3 the rewards for $H = 3$ sometimes are greater than the one for $H = 4$. The gain of TEAMGOLOG agents with respect to the greedy agents ranges from 25\% ($N = 3, H = 4$) to 93\% ($N = 4, H = 3$). In the case of few agents and victims (i.e., $N = 3$ or $N = 4$), this gain is mostly due to minimization of conflicts among team members; when more agents and victims are involved, TEAMGOLOG agents not only outperform the greedy agents managing better the conflicts, but also reporting more victims. Notice that, information available to the two strategies are the same, and that the better performance of the policies $\pi_i$ are achieved using planning over the global and local states. Note also that, these policies have been produced by the interpreter by exploiting weak global information (information about already analyzed victims) that is not sufficient to prevent conflicts, however this weak coordination mechanism allows us to obtain a significant enhancement in the agents’ performances.

During our tests, we could also observe a pretty stable behavior of the TEAMGOLOG agents when compared with the one of the greedy agents. Indeed, while the greedy agents performances can be dramatically
Figure 3: TEAMGолог vs. greedy for $H = 3$.

Figure 4: TEAMGолог vs. greedy for $H = 4$. 
Figure 5: Standard deviation discrepancy between the TEAMGOLOG and the greedy algorithm.

Figure 6: Standard deviation discrepancy between the TEAMGOLOG, the mixed, and the greedy algorithm.
influenced by the uncertain effect of actions, the variations of the global reward gathered by the TEAMGOLOG strategies are usually more contained because constrained by the program structure. For example, Fig. 5 illustrates the typical discrepancy between the standard observed in the reward values collected by the greedy and the TEAMGOLOG agents after several runs within the same initial configuration. In Fig. 5 we report the result related to the following special initial configuration: (i) each victim needs only one final analysis to be completely recognized and reported to the operators (i.e., for each victim, two analysis are already performed); (ii) the agents’ sensor equipment allows to completely analyze and report all the victims in horizon 3; the execution horizon is set to 3 steps ahead. The collected values are obtained by executing 20 times the greedy algorithm and the TEAMGOLOG strategies starting form this initial configuration.

We have also considered a situation where the communication among the agents can be lost. In order to manage this event we have equipped the agents with the following program mixing the greedy algorithm and the TEAMGOLOG programs:

\[
\text{proc}(\text{explore}^*\text{}_i),
\]

\[
\text{if } (\neg gs\text{Connection}) \text{ then greedyStrategy, } \text{elseif } \pi x \in Pos(\text{goToS}_i(x));
\]

\[
\text{if } obs(\text{succ}) \text{ then } \text{analyzeS}_i(x, \text{type}_i);
\]

\[
\text{if } obs(\text{succ}) \land \text{fullyAnalyzed}(x) \text{ then reportToOp}_i(\text{repVictim}(x)));
\]

\[
\text{explore}_i).
\]

Here, whenever the connection is lost, the agents start behaving greedily, otherwise, in the presence of connection, the behavior of \text{explore}^*\text{}_i\ is the one of \text{explore}_i. Assuming the special initial setting used for the results reported in Fig. 5, we tested the TEAMGOLOG agents endowed with the \text{explore}^*\text{}_i\ program in the presence of communication interruptions. Setting the horizon equal to 3 steps ahead and considering random interruptions uniformly distributed along the 3 steps, as expected we obtained an expected reward comprised between the greedy and the TEAMGOLOG as depicted in Fig. 6.

As for the performance, Table 1 shows the average CPU time (in seconds) for the TEAMGOLOG program interpretation in these tests, which is measured on a 1.33 GHz PowerPC with 512 MB main memory.

These results illustrate that the approach is scalable. As expected, the computational charge raises with the number of victims and agents, however also with 10 agents and 10 victims the TEAMGOLOG interpreter can generate a coordination strategy in less than 0.15 seconds. This seems to suggest that, keeping a short horizon (3 or 4 step ahead) and a simple coordination mechanisms, it is possible to obtain a fast generation of effective coordination strategies.

## 7 Related Work

Among the most closely related works are perhaps other recent extensions of DTGolog [4, 7, 6] to a multi-agent setting. In [4], Lakemeyer and his group present IPCGolog, a multi-agent Golog framework for team
playing. IPCGolog integrates different features like concurrency, exogenous actions, continuous change, and the possibility to project into the future. This framework is deployed in the soccer domain [7]. Here, multi-agent coordination is achieved without communication by assuming that the world models of the agents do not differ too much. Differently from TEAMGOLOG, the setting is fully observable. Furthermore, in TEAMGOLOG the coordination mechanisms is based on communication and synchronization states which allow the agents to keep their belief states, observations, and activities invisible to the other agents.

In [10], the authors propose a framework for agent programming extending DTGolog with qualitative preferences, which are compiled into a DTGolog program, integrating competing preferences through multi-program synchronization. Here, multi-program synchronization is used to allow the execution of a DTGolog program along with a concurrent program that encodes the qualitative preferences. Qualitative preferences are ranked over the quantitative one. Differently from our work, high-level programming is used only for a single agent setting where the control is centralized.

Other related research deals with multi-agent decision-theoretic planning using extensions of Markov decision processes (MDPs) [12, 19, 17]. In particular, decentralized partially observable MDPs (DEC-POMDPs) are multi-agent partially observable MDPs where the process is controlled by multiple distributed agents with common payoff, instead, each with possibly different information about the current state of the world. Multi-agent generalizations of partially observable MDPs are provided by [19] with communicative multi-agent team decision problems, which allow to subsume and analyze many existing models of multi-agent cooperative systems. Interestingly, both logic-based and decision-theoretic approaches can be embedded and assessed in this framework. A similar approach is proposed by [12] by introducing and investigating DEC-POMDPs with communication. This model enables the study of the trade-off between the cost of information and the value of the information acquired in the communication process and its influence on the joint utility of the agents. Interactive POMDPs [11] are based on a control paradigm that complements and generalizes the traditional (Nash) equilibrium approach. In [1], the authors present a memory-bounded dynamic programming algorithm for infinite-horizon DEC-POMDPs. This algorithm is based on a stochastic finite-state controller representing the joint policy for the agents. The coordination mechanism is a joint controller composed of a set of local controllers along with a correlation device. The correlation device is a finite-state machine that sends a signal to all of the agents on each time step. The correlation device plays a role analogous to the synchronization and coordination state presented in this paper.

8 Summary and Outlook

We have presented the agent programming language TEAMGOLOG for programming a team of cooperative agents under partial observability. The approach is based on a decision-theoretic semantics and the key concepts of a synchronization state and a communication state, which allow the agents to passively resp. actively coordinate their behavior, while keeping their belief states, observations, and activities invisible to the other agents. We have shown the practical usefulness of TEAMGOLOG in a rescue simulated domain. We have also described a TEAMGOLOG interpreter along with a prototype implementation. In experimental results, the TEAMGOLOG approach outperforms a standard greedy one in the rescue simulated domain.

An interesting topic for future research is to develop an adaptive generalization. Another topic for future work is to explore whether the approach can be generalized to multi-agent systems with competitive agents.

References


