

# Equivalent Stream Reasoning Programs

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- $\Rightarrow$  Do techniques from ASP carry over to LARS?

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- Answer sets characterized by **equilibrium models**  $(X, X)$ , where no smaller  $(X', X)$  is an SE-model

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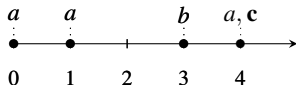
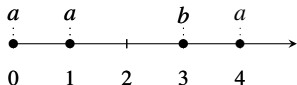
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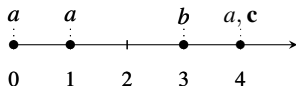
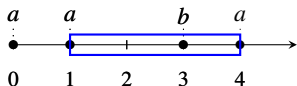
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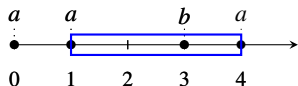
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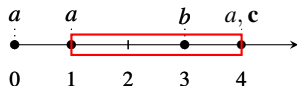
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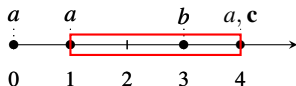
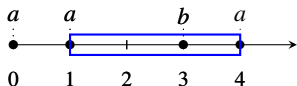
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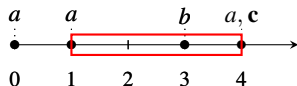
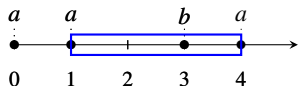
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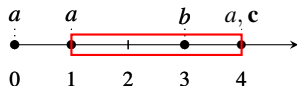
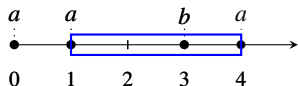
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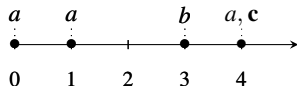
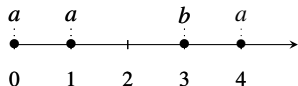
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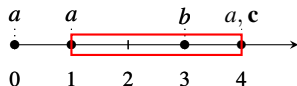
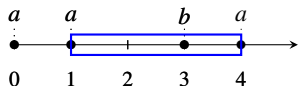
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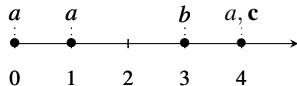
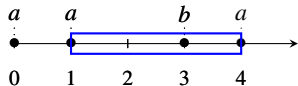
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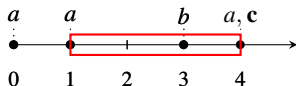
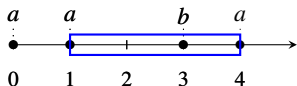
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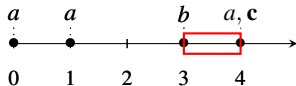
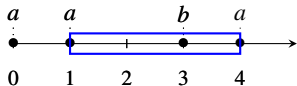
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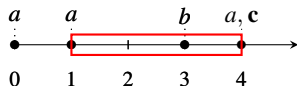
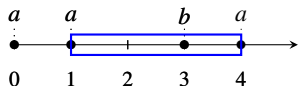
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$$(i) \mathbf{L} \subseteq \mathbf{R} \quad (ii) R, t \models P \quad (iii) L, t \models P^{R,t}$$

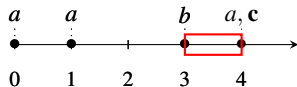
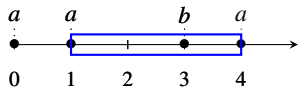
- **Time-based** window functions  $w$  are **monotone**:

$$L \subseteq R \text{ implies } w(L, t) \subseteq w(R, t)$$



$$w_{time}^3(L, 4) \subseteq w_{time}^3(R, 4)$$

- **Tuple-based** windows are not:



$$w_{tuple}^3(L, 4) \not\subseteq w_{tuple}^3(R, 4)$$

# Results

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- **Complexity** of deciding eq.: similar to ASP (mostly coNP-c.)