

Towards Ideal Semantics for Analyzing Stream Reasoning

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FAKULTÄT
FÜR INFORMATIK

Faculty of Informatics





What & Why

“Towards Ideal Semantics for Analyzing **Stream Reasoning**”

- ▶ **Stream Reasoning**



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- ▶ **Stream Reasoning**: Logical reasoning on streaming data



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- ▶ **Stream Reasoning**: Logical reasoning on streaming data
 - ▶ Streams = **tuples** (atoms) with **timestamps**
 - ▶ Essential aspect: **window** functions



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- ▶ **Semantics**

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- ▶ **Ideal**
 - ▶ Idealization: Abstract from practical (operational) issues
 - ▶ Generalization: Uniform representation

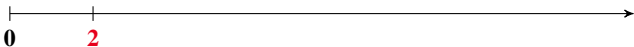
Example: Trams and buses

Arrival times at different stations p_i



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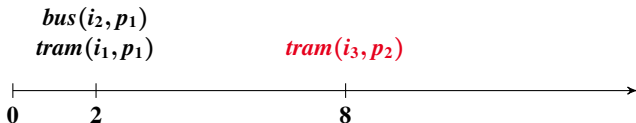
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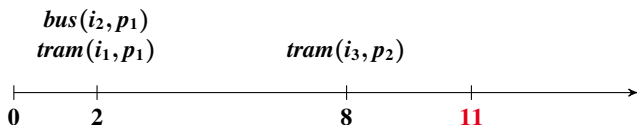
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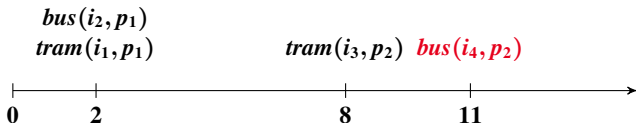
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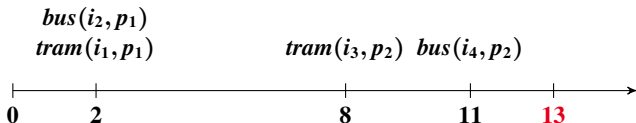
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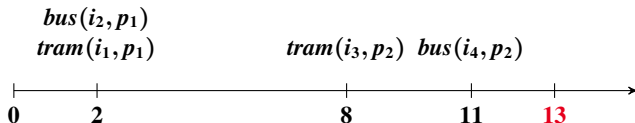
Arrival times at different stations p_i



- ▶ Stream setting, **at time 13**: Query for

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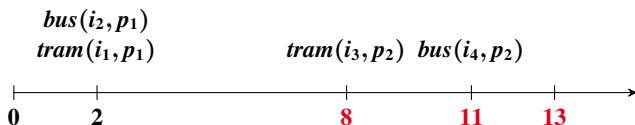
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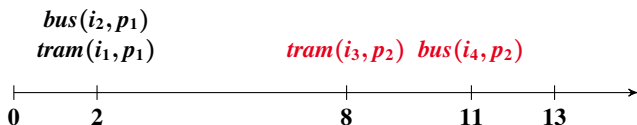
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- ▶ Stream setting, at time 13: Query for
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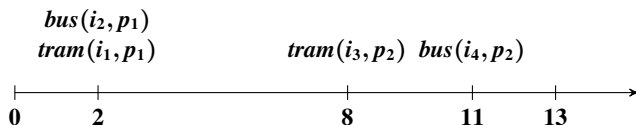
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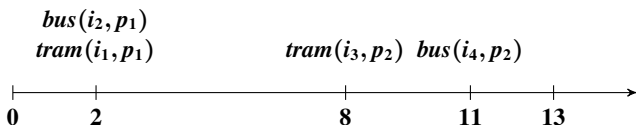
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▶ CQL

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SELECT * FROM tram [RANGE 5], bus [RANGE 5]
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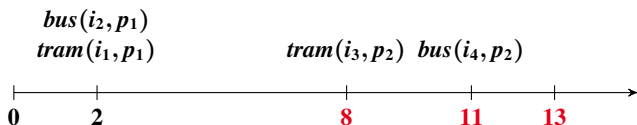
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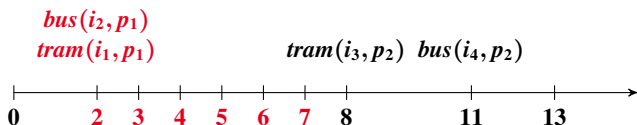


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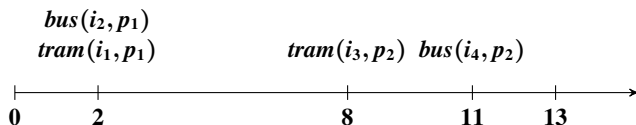


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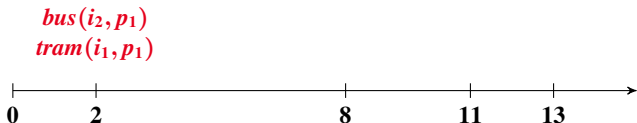
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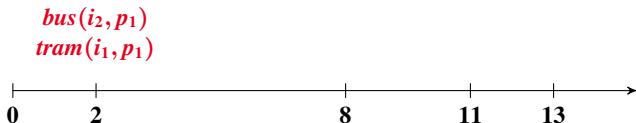
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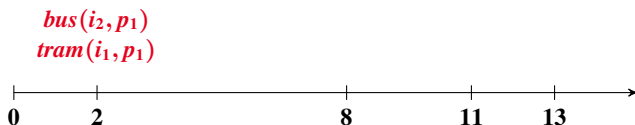
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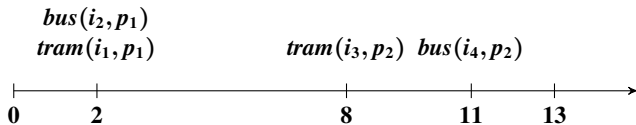
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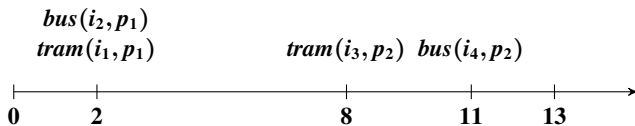
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Window Types



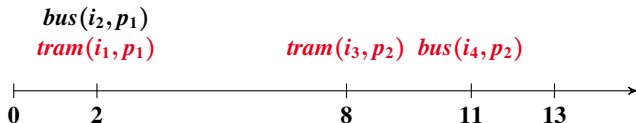
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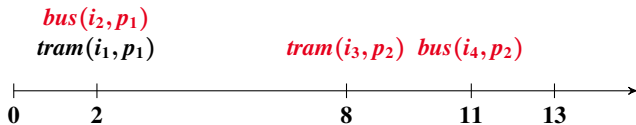
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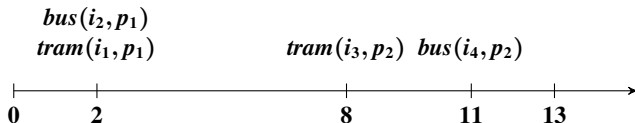
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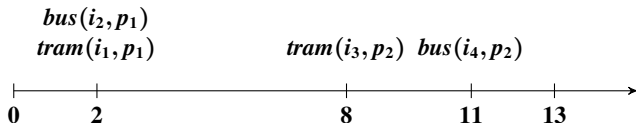
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 - ▶ Apply tuple-based window on substreams



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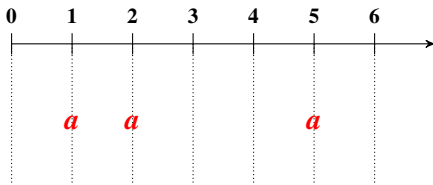
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 - ▶ $w(S, t) \mapsto S'$
 - ▶ Stream S , time point $t \in \mathbb{N}$, new stream S'

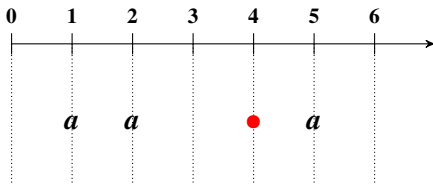
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- ▶ **Atoms a** appearing in the stream at time points **1, 2, 5**



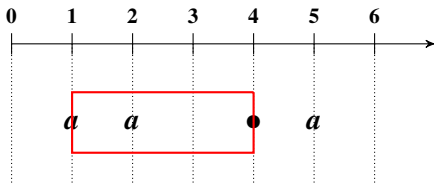
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- ▶ Atoms a appearing in the stream at time points 1, 2, 5
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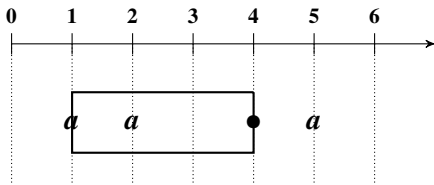
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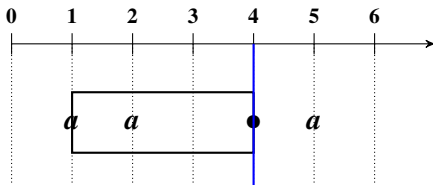
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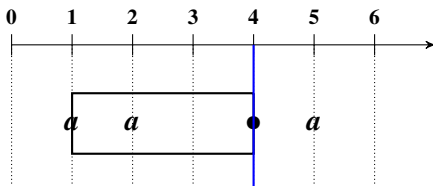
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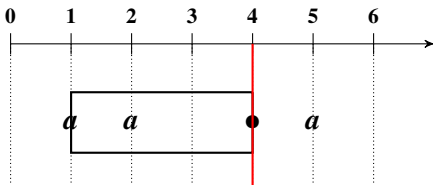
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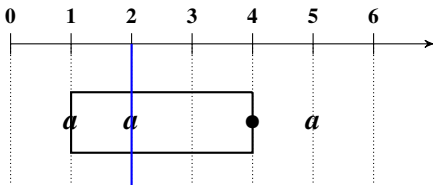
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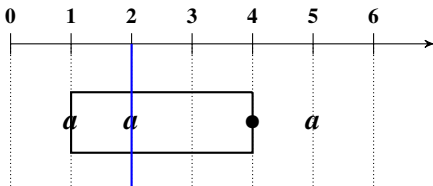
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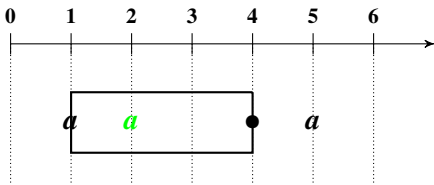
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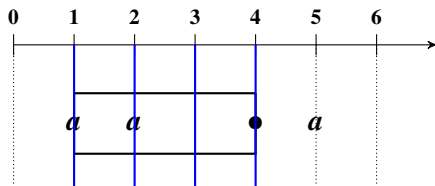
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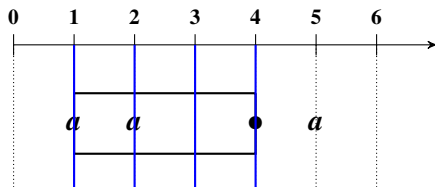
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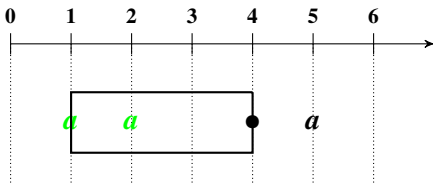
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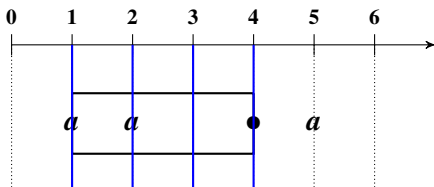
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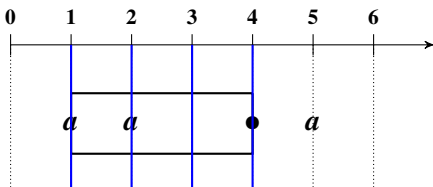
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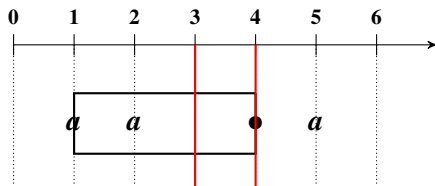
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Ideas for Time Reference

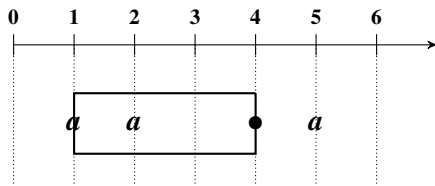
- ▶ Atoms a appearing in the stream at time points 1, 2, 5
- ▶ Query time $t = 4$. Window on interval $[1, 4]$



- ▶ Example queries: In this window, does a hold...
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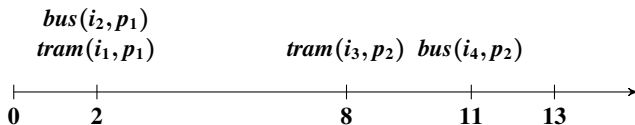
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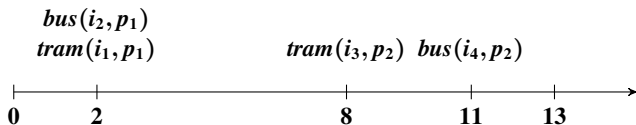
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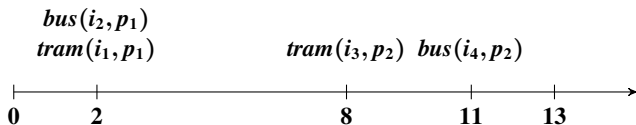
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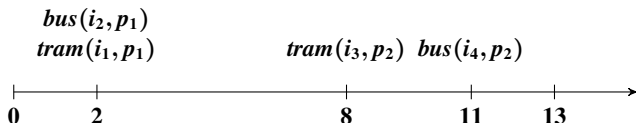
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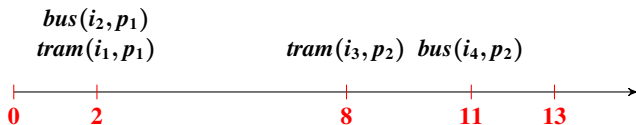
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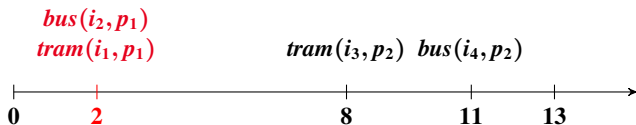
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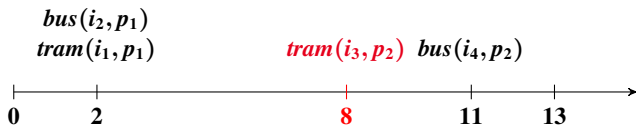
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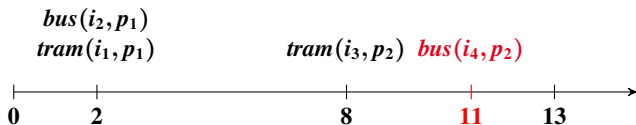
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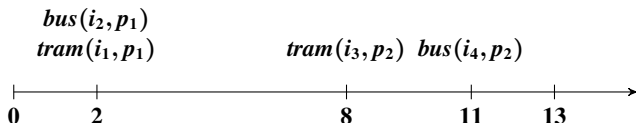
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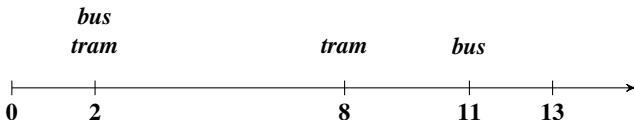
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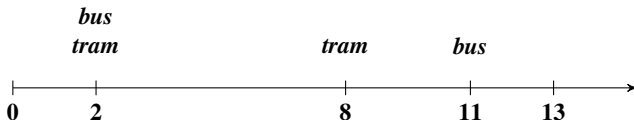
Nested Windows and Stream Choice

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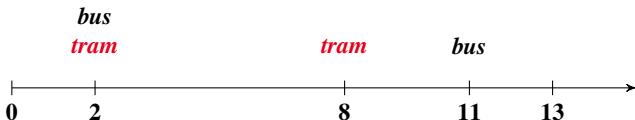
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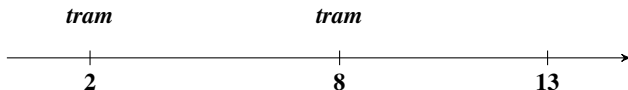
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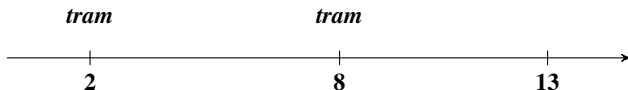
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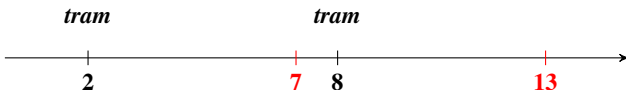
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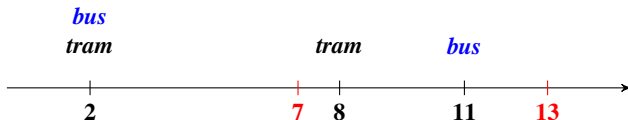
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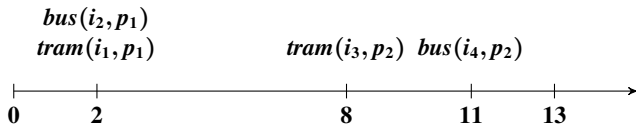
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$M, S, t \Vdash \boxplus_i\alpha$	iff	$M, S', t \Vdash \alpha$ where $S' = \hat{w}_i(S_M, S, t)$.

Queries

- ▶ Query $\alpha[t]$: “ $M, S_M, t \models \alpha$ ”?

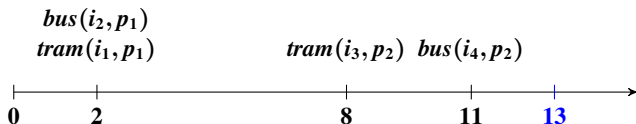
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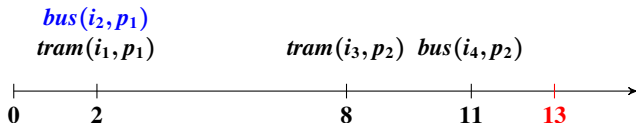
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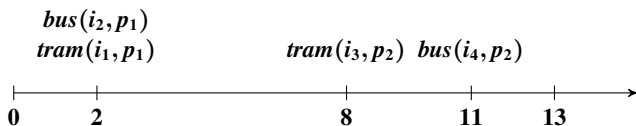
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$M, S_M, \mathbf{13} \not\models bus(i_2, p_1)$, since $bus(i_2, p_1) \notin v(\mathbf{13})$

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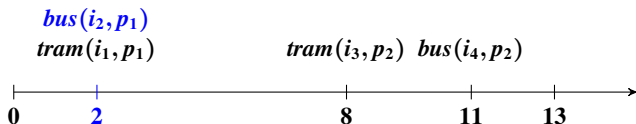
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$M, S_M, 13 \Vdash \diamond bus(i_2, p_1)$?

Queries

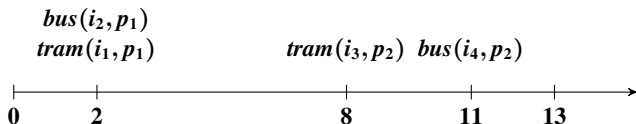
- Query $\alpha[t]$: “ $M, S_M, t \models \alpha$ ”?



$M, S_M, 13 \models \diamond bus(i_2, p_1)$, since $\exists t' \in T_{S_M}$ s.t. $bus(i_2, p_1) \in v(t')$

Queries

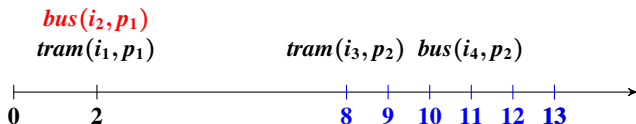
- Query $\alpha[t]$: “ $M, S_M, t \models \alpha$ ”? \boxplus_1 : last 5 min



$M, S_M, 13 \models \boxplus_1 \diamond bus(i_2, p_1)$?

Queries

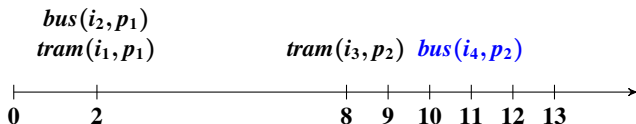
- Query $\alpha[t]$: “ $M, S_M, t \Vdash \alpha$ ”? \boxplus_1 : last 5 min



$M, S_M, 13 \not\Vdash \boxplus_1 \diamond bus(i_2, p_1)$

Queries

- Query $\alpha[t]$: “ $M, S_M, t \Vdash \alpha$ ”? \boxplus_1 : last 5 min



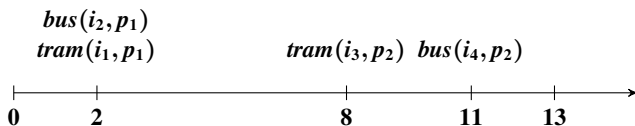
$$M, S_M, 13 \Vdash \boxplus_1 \diamond bus(i_4, p_2)$$

Non-ground Queries

- ▶ Non-ground query: Assignments s.t. substitution hold

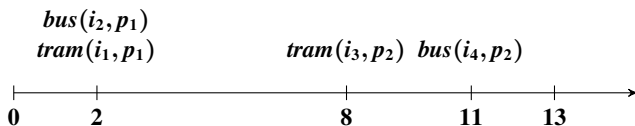
Non-ground Queries

- ▶ Non-ground query: Assignments s.t. substitution hold



Non-ground Queries

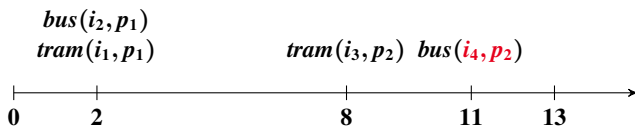
- ▶ Non-ground query: Assignments s.t. substitution hold



$M, S_M, 13 \Vdash \boxplus_1 \diamond bus(X, P)?$

Non-ground Queries

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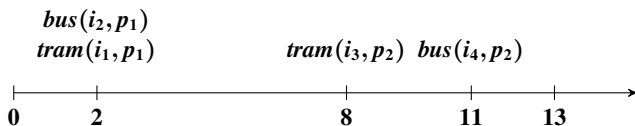


$M, S_M, 13 \Vdash \boxplus_1 \diamond bus(X, P)?$

$X \mapsto i_4, P \mapsto p_2$

Non-ground Queries

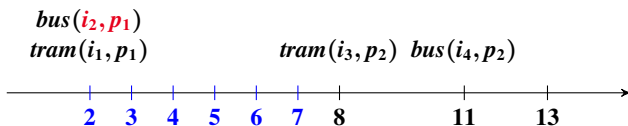
- ▶ Non-ground query: Assignments s.t. substitution hold



$$M, S, U \Vdash \boxplus_1 \diamond bus(i_2, p_1)?$$

Non-ground Queries

- ▶ Non-ground query: Assignments s.t. substitution hold

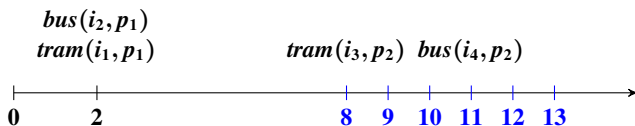


$$M, S, U \Vdash \boxplus_1 \diamond bus(i_2, p_1)?$$

$$U \mapsto 2, \dots, 7$$

Non-ground Queries

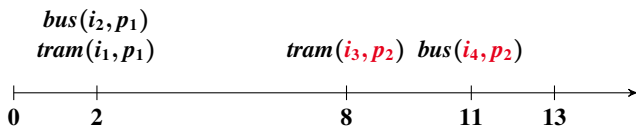
- ▶ Non-ground query: Assignments s.t. substitution hold



$$M, S_M, 13 \Vdash \boxplus_1(\diamond tram(X, P) \wedge \diamond bus(Y, P))?$$

Non-ground Queries

- ▶ Non-ground query: Assignments s.t. substitution hold

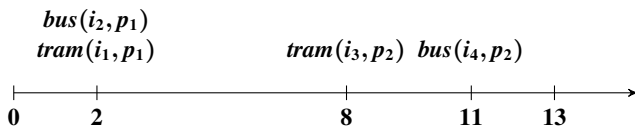


$M, S_M, 13 \Vdash \boxplus_1 (\diamond tram(X, P) \wedge \diamond bus(Y, P))?$

$X \mapsto i_3, P \mapsto p_2, Y \mapsto i_4$

Non-ground Queries

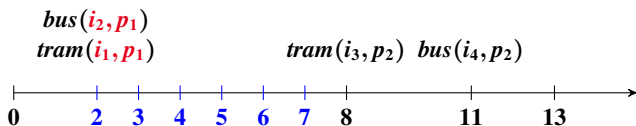
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Non-ground Queries

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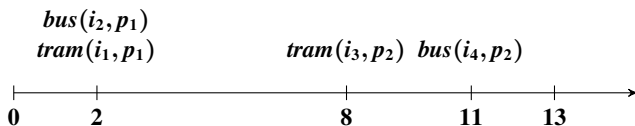


$$M, S_M, U \Vdash \boxplus_1 \diamond (\text{tram}(X, P) \wedge \text{bus}(Y, P))?$$

$$U \mapsto 2, \dots, 7 \quad \times \quad X \mapsto i_1, P \mapsto p_1, Y \mapsto i_2$$

Non-ground Queries

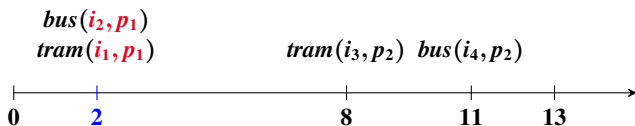
- ▶ Non-ground query: Assignments s.t. substitution hold



$$M, S_M, 13 \Vdash @_U(tram(X, P) \wedge bus(Y, P))?$$

Non-ground Queries

- ▶ Non-ground query: Assignments s.t. substitution hold

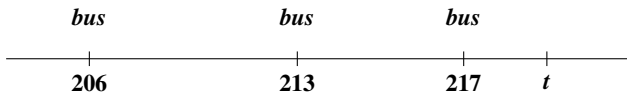


$M, S_M, 13 \Vdash @_U(tram(X, P)) \wedge bus(Y, P)?$

$U \mapsto 2, \quad X \mapsto i_1, P \mapsto p_1, Y \mapsto i_2$

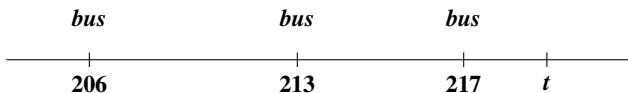
Example: Nested Window

- ▶ “In the last hour, did a bus always appear in the last 5 minutes?”



Example: Nested Window

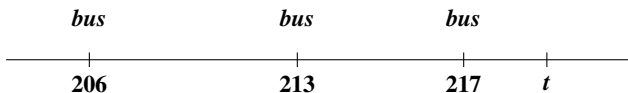
- ▶ “In the last hour, did a bus always appear in the last 5 minutes?”



- ▶ \boxplus_i : time-based window for last i minutes

Example: Nested Window

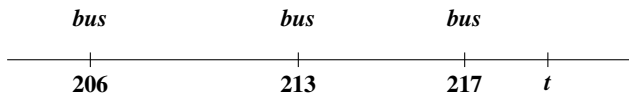
- ▶ “In the **last hour**, did a bus always appear in the last 5 minutes?”



- ▶ \boxplus_i : time-based window for last i minutes
- ▶ Query: \boxplus_{60}

Example: Nested Window

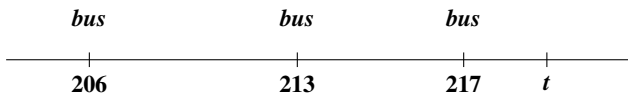
- ▶ “In the last hour, did a bus **always** appear in the last 5 minutes?”



- ▶ \boxplus_i : time-based window for last i minutes
- ▶ Query: $\boxplus_{60} \square$

Example: Nested Window

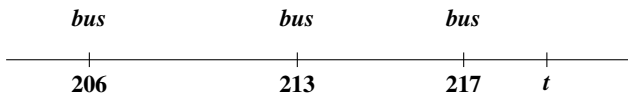
- ▶ “In the last hour, did a bus always appear in the **last 5 minutes**?”



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- ▶ Query: $\boxplus_{60} \square \boxplus_5$

Example: Nested Window

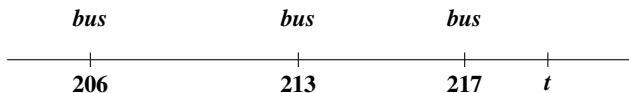
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Example: Nested Window

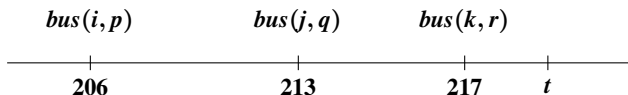
- ▶ “In the last hour, did a bus always appear in the last 5 minutes?”



- ▶ \boxplus_i : time-based window for last i minutes
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Example: Nested Window

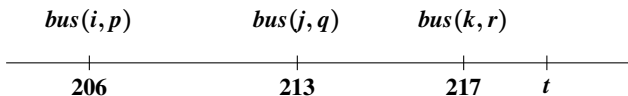
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- ▶ \boxplus_i : time-based window for last i minutes
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- ▶ Limitation: $\boxplus_{60} \square \boxplus_5 \diamond bus(X, P)$

Example: Nested Window

- ▶ “In the last hour, did a bus always appear in the last 5 minutes?”



- ▶ \boxplus_i : time-based window for last i minutes
- ▶ Query: $\boxplus_{60} \square \boxplus_5 \diamond bus$
- ▶ Limitation: $\boxplus_{60} \square \boxplus_5 \diamond bus(X, P)$
 - ▶ Result: List of fixed combinations X, P
 - ▶ Need a rule: $some_bus \leftarrow bus(X, P)$
 - ▶ Then: $\boxplus_{60} \square \boxplus_5 \diamond some_bus$

Conclusion Stream



► Past

Conclusion Stream

? \models ?



- ▶ Past: Lack of theoretical underpinning for stream reasoning

Conclusion Stream



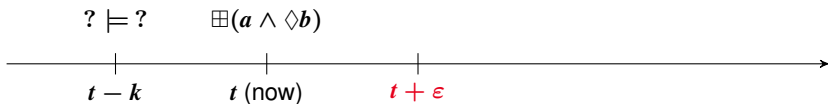
- ▶ Past: Lack of theoretical underpinning for stream reasoning
- ▶ Now

Conclusion Stream



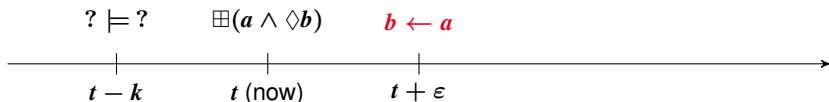
- ▶ Past: Lack of theoretical underpinning for stream reasoning
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 - ▶ flexible window operator (first class citizen)
 - ▶ time reference / time abstraction

Conclusion Stream



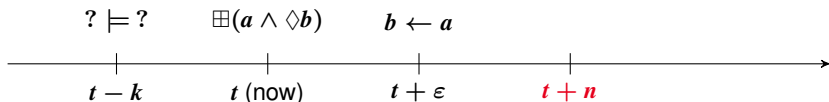
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Conclusion Stream



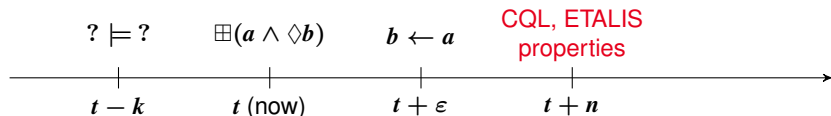
- ▶ Past: Lack of theoretical underpinning for stream reasoning
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- ▶ Soon: **Rule-based extension** (OrdRing @ ISWC, Oct.'14)

Conclusion Stream



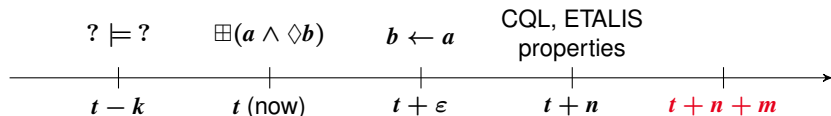
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- ▶ Soon: Rule-based extension (OrdRing @ ISWC, Oct.'14)
- ▶ Later

Conclusion Stream



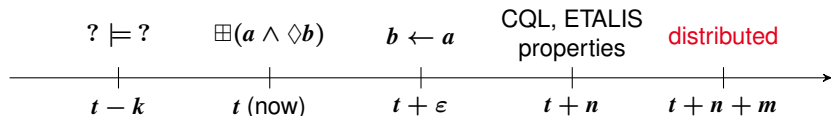
- ▶ Past: Lack of theoretical underpinning for stream reasoning
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- ▶ Later: **Language properties, capture CQL and ETALIS**

Conclusion Stream



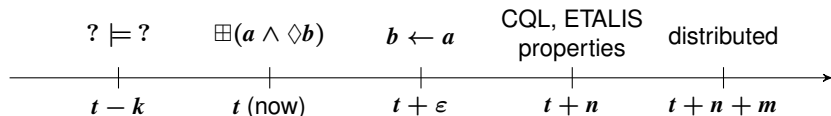
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- ▶ Later: Language properties, capture CQL and ETALIS
- ▶ **Eventually**

Conclusion Stream



- ▶ Past: Lack of theoretical underpinning for stream reasoning
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- ▶ Later: Language properties, capture CQL and ETALIS
- ▶ Eventually: **Distributed setting, heterogeneous nodes**

Conclusion Stream



- ▶ Past: Lack of theoretical underpinning for stream reasoning
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- ▶ Eventually: Distributed setting, heterogeneous nodes

To je ono.

(That's it.)

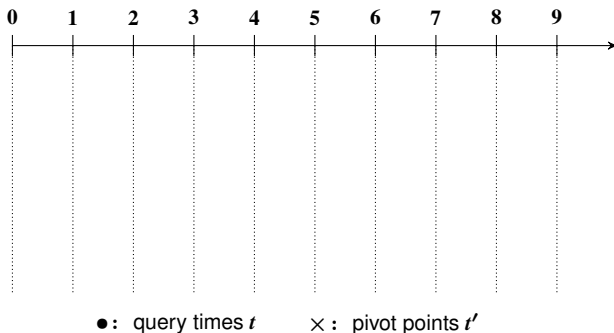
Time-based window

► Example

ℓ 2 time points into the past

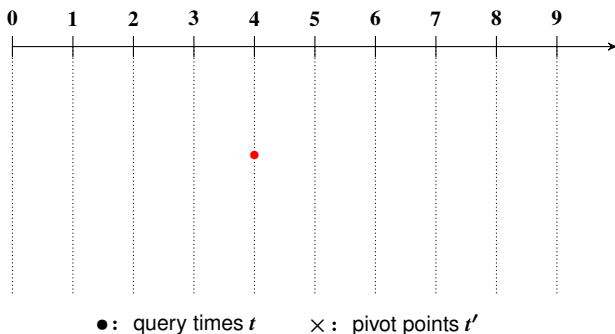
u 1 time points into the future

d 3 step size (slide parameter)



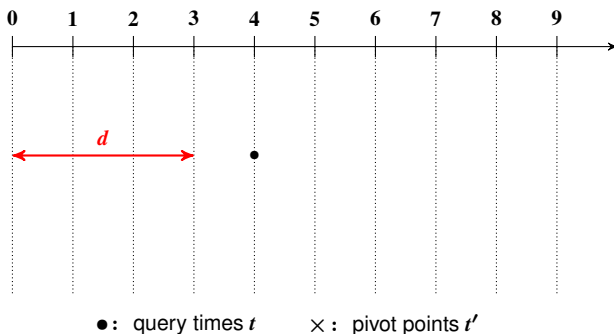
Time-based window

- ▶ Example: Query time $t = 4$
 - ℓ 2 time points into the past
 - u 1 time points into the future
 - d 3 step size (slide parameter)



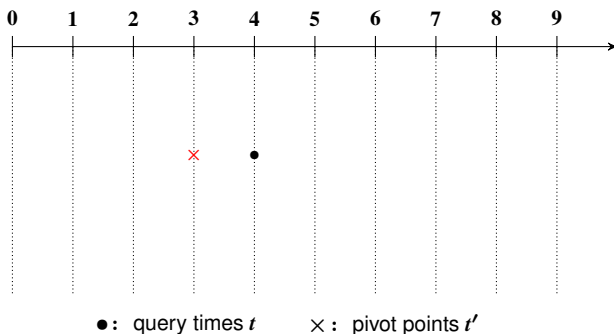
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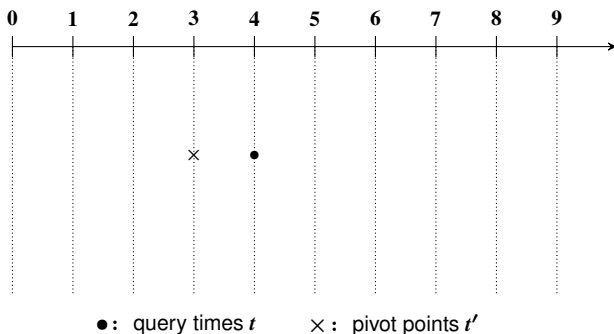
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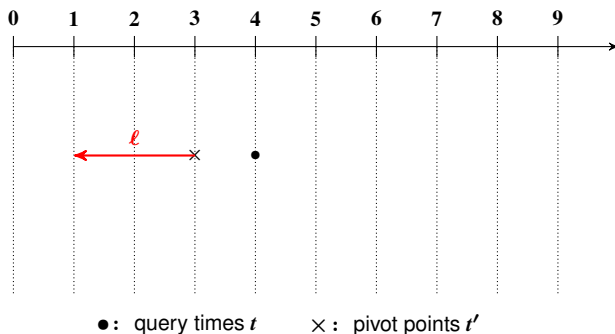
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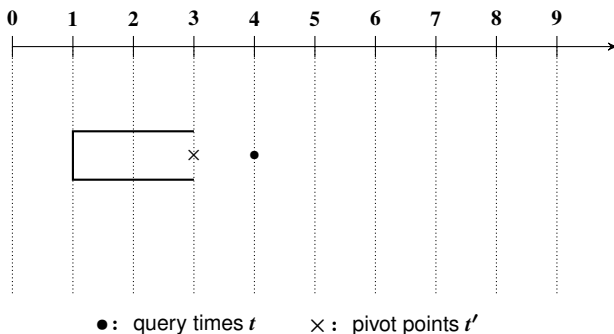
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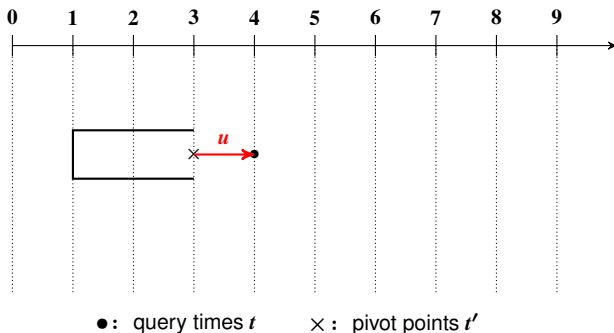
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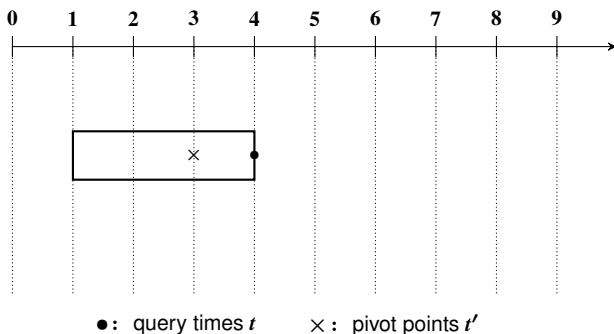
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