Semantic Independence in DL-Programs

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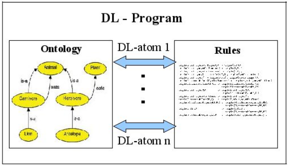
Motivation

- DL-program: ontology + rules (loose coupling combination approach);
- DL-atoms are evaluated under varying input to ontology;
- Evaluation of just one DL-atom under certain ontology input may be costly.



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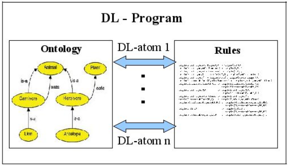

?: Which DL-atoms always have the same value regardless of (updated) ontology?

In this work: Semantic notion of independent DL-atom and its characterization (ontology is viewed as a black box).

Applications:

- optimization of DL-programs [Eiter et al, 2004];
- inconsistency diagnosis [Puehrer et al, 2010], [Fink et al, 2010];
- DL-program repair, etc.

Overview

DL-program: syntax

Signature: $\Sigma = \langle \mathcal{F}, \mathcal{P}_o, \mathcal{P}_p \rangle$, where

-$\mathcal{F}$ is a set of individuals (constants);

-$\mathcal{P}_o = \mathcal{P}_c \cup \mathcal{P}_r,\ \mathcal{P}_c(\mathcal{P}_r)$ is a set of atomic concepts (resp. roles);

-$\mathcal{P}_p$ is a set of predicate symbols of arity ≥ 0.

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DL-atom is of the form $DL[S_1 op_1 p_1, \ldots, S_m op_m p_m; Q](\mathbf{t}),\ \ m \geq 0$, where

- $S_i \in \mathcal{P}_c$ or $S_i \in \mathcal{P}_r$;
- $op_i \in \{\uplus, \cup\!\!\!\!-, \cap\!\!\!\!-\}$;
- $p_i \in \mathcal{P}_p$ (unary or binary);
- $Q(\mathbf{t})$ is a *DL-query*:
  - $C \sqsubseteq D$, $C \not\sqsubseteq D$, $\mathbf{t} = \epsilon$, where $C, D \in \mathcal{P}_c \cup \{\top, \bot\}$;
  - $C(t_1)$, $\neg C(t_1)$, $\mathbf{t} = t_1$, where $C \in \mathcal{P}_c$;
  - $R(t_1, t_2)$, $\neg R(t_1, t_2)$, $\mathbf{t} = t_1, t_2$, where $R \in \mathcal{P}_r$.

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DL-program: $KB = (\Phi, \Pi)$, Φ is a DL ontology, Π is a set of DL-rules:

$$a \leftarrow b_1, \ldots b_k, not\ b_{k+1}, \ldots, not\ b_m,$$

$m \geq k \geq 0$, a is a classical literal; b_i is a classical literal or a DL-atom.

DL-program: semantics

Consider $KB = (\Phi, \Pi)$ over $\Sigma = \langle \mathcal{F}, \mathcal{P}_o, \mathcal{P}_p \rangle$.

Interpretation I is a consistent set of ground literals over $\Sigma_p = \langle \mathcal{F}, \mathcal{P}_p \rangle$.

- for ground literal ℓ: $I \models^{\Phi} \ell$ iff $\ell \in I$;
- for ground DL-atom $a = DL[S_1 op_1 p_1, \ldots, S_m op_m p_m; Q](\mathbf{c})$:

  $$I \models^{\Phi} a$$

  iff $\Phi \cup \tau^I(a) \models Q(\mathbf{c})$, where $\tau^I(a) = \bigcup_{i=1}^{m} A_i(I)$ is a DL-update of Φ under I by a:

  - $A_i(I) = \{S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \in I\}$, for $op_i = \uplus$;
  - $A_i(I) = \{\neg S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \in I\}$, for $op_i = \cup\!\!\!-$;
  - $A_i(I) = \{\neg S_i(\mathbf{e}) \mid p_i(\mathbf{e}) \notin I\}$, for $\cap\!\!\!-$.

I is an answer set of Π iff I is a minimal model of its FLP-reduct Π^I_{FLP}.

FLP-reduct Π^I_{FLP} of Π is a set of ground DL-rules r s.t. $I \models b^+(r)$ and $I \not\models b^-(r)$.

DL-program: Example

Example

$KB = \{\Phi, \Pi\}$.
$\Phi = \{Sweet(apple)\}$;
$\Pi = \{fruit(apple)$.
$vitamin(X) \leftarrow fruit(X)$.
$healthyfood(X) \leftarrow DL[Healthy \uplus vitamin; Healthy](X).\}$

- Consider $I = \{fruit(apple), vitamin(apple), healthyfood(apple)\}$;
- $vitamin(apple) \in I$, hence $\tau^I(a) = \{Healthy(apple)\}$;
- $\Phi \cup \tau^I(a) \models Healthy(apple)$.

Independent DL-atoms

Definition

A ground DL-atom a is *independent* if for all satisfiable ontologies Φ, Φ' and all interpretations I, I' it holds that $I \models^{\Phi} a$ iff $I' \models^{\Phi'} a$.

A ground DL-atom a is a *contradiction* (resp. *tautology*), if for all satisfiable ontologies Φ and all interpretations I, it holds that $I \not\models^{\Phi} a$ (resp. $I \models^{\Phi} a$).

Contradiction:

$DL[; C \not\sqsubseteq C]()$;

. . . **?**

Tautology:

$DL[; C \sqsubseteq C]()$;

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Contradictions

When is a DL-atom contradictory in general?

Proposition

A ground DL-atom $a = DL[\lambda; Q](\boldsymbol{t})$ is contradictory iff $\lambda = \epsilon$ and $Q(\boldsymbol{t})$ is unsatisfiable, i.e. has one of the forms:

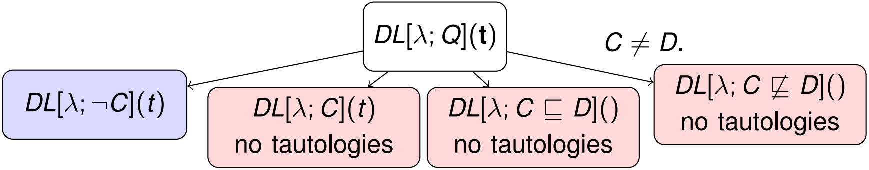
- $C \not\sqsubseteq C$;
- $C \not\sqsubseteq \top$;
- $\bot \not\sqsubseteq C$;
- $\bot \not\sqsubseteq \top$;
- $\top \sqsubseteq \bot$.

Tautologies

When is a DL-atom $a = DL[\lambda; Q](\mathbf{t})$ tautologic in general?

- Q is tautologic: $Q \in \{C \sqsubseteq \top, \bot \sqsubseteq C, C \sqsubseteq C\}$;
- λ is s.t. a is tautologic.

Concept query case distinction:



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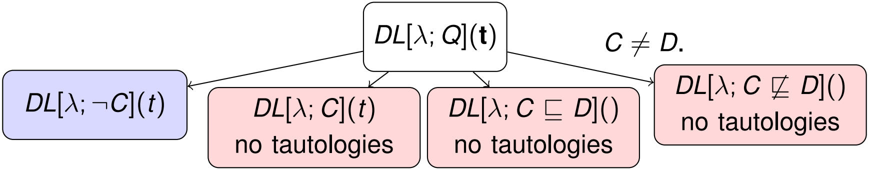

Example

$a = DL[\, C \mathbin{\cap\!\!\!\!-} p,\ C' \uplus p,\ C' \mathbin{\cap\!\!\!\!-} q,\ C \mathbin{\cup\!\!\!\!-} q; \neg C](c)$
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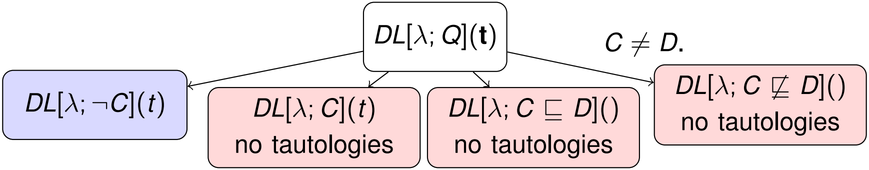

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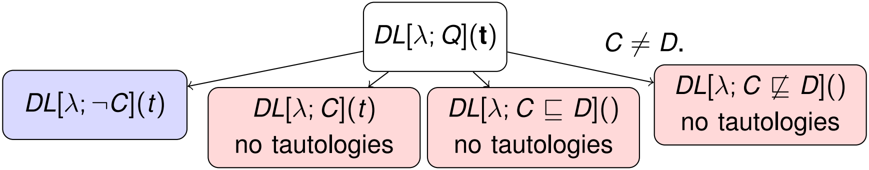

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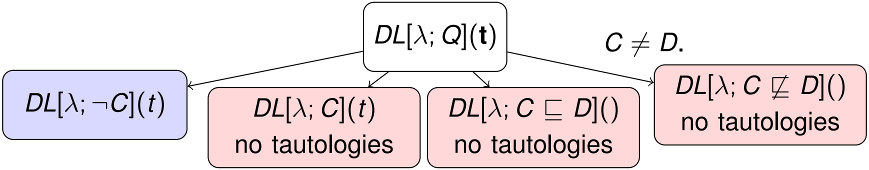

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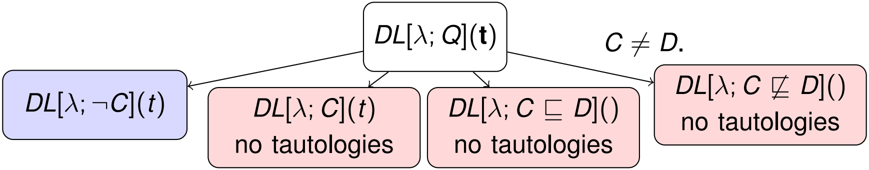

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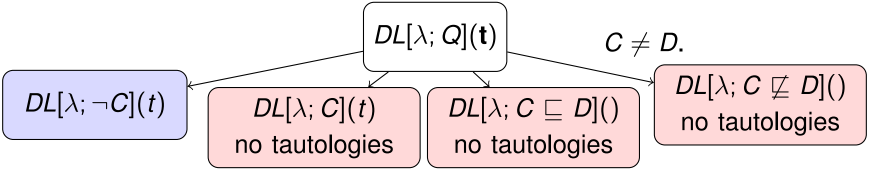

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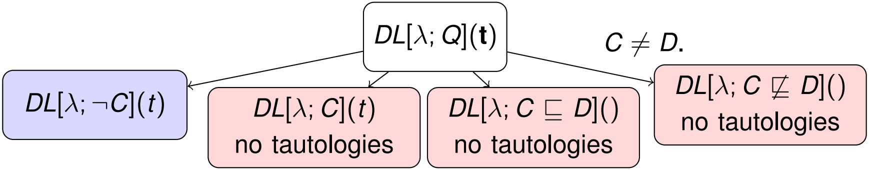

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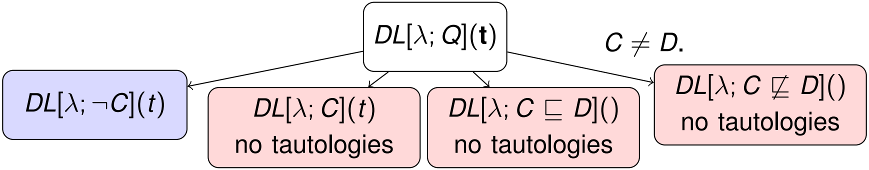

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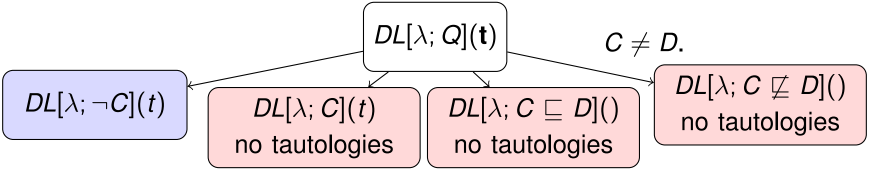

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Tautologies with Concept Query

$$DL[\lambda; \neg C](t)$$

Proposition

A ground DL-atom a with the query $\neg C(t)$ is tautologic iff it has one of the following forms

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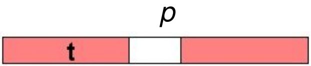

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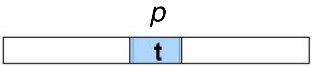

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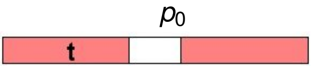

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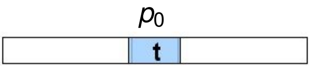

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c4. $DL[\lambda, C \cap\!\!\!\!\!- \, p_0, C^0 \uplus p_0, C^0 \cap\!\!\!\!\!- \, p'_0, C^1 \uplus p_1, C^1 \cap\!\!\!\!\!- \, p'_1, \ldots,$
$C^n \uplus p_n, C^n \uplus p'_n, \boxed{D \uplus p_{n+1}, D \cup\!\!\!\!\!- \, p'_{n+1}}; \neg C](t)$,

where for every $i = 0, \ldots, n+1$, $p_i = p'_j$ for some $j < i$ or $p_i = p_0$, and $p'_{n+1} = p'_{i_j}$ for some $j \leq n$ or $p'_{n+1} = p_0$.

Tautologies with Concept Query

$DL[\lambda; \neg C](t)$

Proposition

A ground DL-atom a with the query $\neg C(t)$ is tautologic iff it has one of the following forms

c1. $DL[\lambda, C \cap\!\!\!\!- \, p, C \cup\!\!\!\!- \, p; \neg C](t)$,

c2. $DL[\lambda, C \cap\!\!\!\!- \, p, D \uplus p, D \cup\!\!\!\!- \, p; \neg C](t)$,

c3. $DL[\lambda, C \cap\!\!\!\!- \, p_0, C^0 \uplus p_0, C^0 \cap\!\!\!\!- \, p'_0, C^1 \uplus p_1, C^1 \cap\!\!\!\!- \, p'_1, \dots,$
$C^n \uplus p_n, C^n \cap\!\!\!\!- \, p'_n, C \cup\!\!\!\!- \, p_{n+1}; \neg C](t)$,

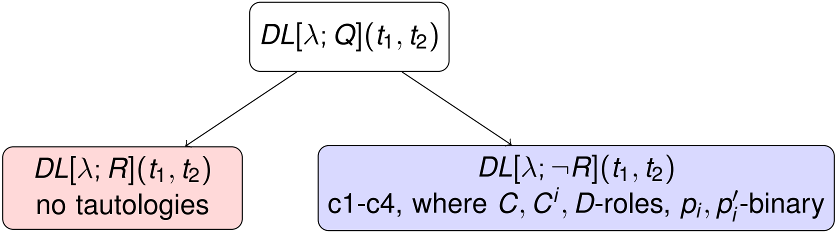
Example

$a = DL[C \cap\!\!\!\!- \, p, C' \uplus p, C' \cap\!\!\!\!- \, q, C \cup\!\!\!\!- \, q; \neg C](c)$ is the special case of c3.

Tautologies with Role Query

What if the query is a role $R(t_1, t_2)$ or negated role $\neg R(t_1, t_2)$?

Role query case distinction:



Example

(c_2) for roles is of the form $DL[\lambda, R_1 \Cap p, R_2 \uplus p; \neg R_1](t_1, t_2)$.

Axiomatization for Tautologies ($\mathcal{K}_{taut}$)

Axioms:

a0. $DL[;Q]()$,

a1. $DL[S \cap\!\!\!- p, S \cup\!\!\!- p; \neg S](\mathbf{t})$,

a2. $DL[S \cap\!\!\!- p, S' \uplus p, S' \cup\!\!\!- p; \neg S](\mathbf{t})$,

where $Q \in \{S \sqsubseteq S, S \sqsubseteq \top, \top \not\sqsubseteq \bot\}$, S, S' are distinct.

Rules of Inference:

Expansion

$$\frac{DL[\lambda; Q](\mathbf{t})}{DL[\lambda, \lambda'; Q](\mathbf{t})} \quad (e)$$

Increase

$$\frac{DL[\lambda, S \uplus p; Q](\mathbf{t})}{DL[\lambda, S \uplus q, S' \uplus p, S' \cap\!\!\!- q; Q](\mathbf{t})} \quad (in_{\uplus})$$

$$\frac{DL[\lambda, S \cup\!\!\!- p; Q](\mathbf{t})}{DL[\lambda, S \cup\!\!\!- q, S' \uplus p, S' \cap\!\!\!- q; Q](\mathbf{t})} \quad (in_{\cup\!\!\!-})$$

Inclusion Constraints

Inclusion constraint (IC): $q(Y_1, \ldots, Y_n) \leftarrow p(X_1, \ldots, X_m)$,
where $n \leq m$, Y_i are pairwise distinct from X_i;

- $p \subseteq q$, if $n = m$ and $Y_i = X_i$;
- $p \subseteq q^-$, if $n = m$ and $Y_i = X_{n-i+1}$.

$\mathcal{C}$ is a set of inclusion constraints of Π; $CL(\mathcal{C})$ is the logical closure of $\mathcal{C}$;
$inp_a(\mathcal{C})$ is a set of all $q(\boldsymbol{Y}) \leftarrow p(\boldsymbol{X})$ in $\mathcal{C}$ s.t. p, q are in λ, $a = DL[\lambda; Q](\boldsymbol{t})$;

$\mathcal{C}$ is *separable* for a if every $IC \in inp_a(CL(\mathcal{C}))$ involves predicates of same arity.

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$\mathcal{C}$ is *separable* for a if every $IC \in inp_a(CL(\mathcal{C}))$ involves predicates of same arity.

Example

$$\Pi = \{(1)\ p_2(Y, X) \leftarrow p_1(X, Y).$$
$$(2)\ p_3(Z) \leftarrow p_1(X, Y).$$
$$(3)\ r_1(X, Y) \leftarrow \underbrace{DL[S_1 \uplus p_1, S_2 \uplus p_2; S_3](X, Y)}_{a}.\}$$

$\mathcal{C} = \{p_1 \subseteq p_2^-, p_1 \subseteq p_3\}$; $CL(\mathcal{C}) = \mathcal{C}$;
$inp_a(CL(\mathcal{C})) = \{p_1 \subseteq p_2^-\}$; $\mathcal{C}$ is separable for a.

Axiomatization for Tautologies under Inclusion $\mathcal{K}_{taut}^{\subseteq}$

Axioms:

a0. $DL[;Q]()$,

a1. $DL[S \capminus p, S \uminus p; \neg S](\mathbf{t})$,

a2. $DL[S \capminus p, S' \uplus q, S' \uminus q; \neg S](\mathbf{t})$,

where $q \in \{p, p^-\}$, $Q \in \{S \sqsubseteq S, S \sqsubseteq \top, \top \not\sqsubseteq \bot\}$, S, S' are distinct.

Rules of Inference: rules of $\mathcal{K}_{taut}$ plus additional:

Inclusion

$$\frac{DL[\lambda, S \uminus p; Q](\mathbf{t}) \quad p \subseteq q}{DL[\lambda, S \uminus q; Q](\mathbf{t})} \ (i_1)$$

$$\frac{DL[\lambda, S \uplus p; Q](\mathbf{t}) \quad p \subseteq q}{DL[\lambda, S \uplus q; Q](\mathbf{t})} \ (i_2)$$

Increase

$$\frac{DL[\lambda, S \uplus p; Q](\mathbf{t})}{DL[\lambda, S \uplus q, S' \uplus p^-, S' \capminus q^-; Q](\mathbf{t})} \ (in_{\uplus}^-)$$

$$\frac{DL[\lambda, S \uminus p; Q](\mathbf{t})}{DL[\lambda, S \uminus q, S' \uplus p^-, S' \capminus q^-; Q](\mathbf{t})} \ (in_{\uminusnot}^-)$$

Example

$$\Pi = \{(1)\ so(ch, chile).$$
$$(2)\ vi(X) \leftarrow ex(X).$$
$$(3)\ sw(X) \leftarrow ex(X), not\ bi(X).$$
$$(4)\ ex(X) \leftarrow so(X, Y).$$
$$(5)\ no(X) \leftarrow DL[H \uplus vi, H \cup\!\!\!\!\!-\ sw, A \cap\!\!\!\!\!-\ ex; \neg A](X).$$

(1) Cherimoya (ch) is a Southern fruit (*so*) from Chile;

(2) All exotic fruits (*ex*) are vitaminized (*vi*);

(3) Any exotic fruit is sweet (*sw*) unless it is known to be bitter (*bi*);

(4) All Southern fruits are exotic;

(5) *H* is healthy, *A* is African, *no* is nonafrican.

Example

$$\Pi = \{(1)\ so(ch, chile).$$
$$(2)\ vi(X) \leftarrow ex(X).$$
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(4) All Southern fruits are exotic;

(5) *H* is healthy, *A* is African, *no* is nonafrican.

Example

$$
\begin{aligned}
\Pi = \{ & (1)\ so(ch, chile).\\
& (2)\ vi(X) \leftarrow ex(X).\\
& (3)\ sw(X) \leftarrow ex(X), not\ bi(X).\\
& (4)\ ex(X) \leftarrow so(X, Y).\\
& (5)\ no(X) \leftarrow DL[H \uplus vi, H \mathbin{\cup\!\!\!\!-} sw, A \mathbin{\cap\!\!\!\!-} ex; \neg A](X).
\end{aligned}
$$

(1)  (ch) is a Southern fruit (*so*) from Chile;

(2) All exotic fruits (*ex*) are vitaminized (*vi*);

(3) Any exotic fruit is sweet (*sw*) unless it is known to be bitter (*bi*);

(4) All Southern fruits are exotic;

(5) *H* is healthy, *A* is African, *no* is nonafrican.

Is $a = DL[H \uplus vi, H \mathbin{\cup\!\!\!\!-} sw, A \mathbin{\cap\!\!\!\!-} ex; \neg A](ch)$ tautologic?

Example (cont.)

Is $a = DL[H \uplus vi, H \cup\!\!\!\!- sw, A \Cap ex; \neg A](ch)$ tautologic?

$$\cfrac{\cfrac{\cfrac{DL[H \uplus ex, H \cup\!\!\!\!- ex, A \Cap ex; \neg A](ch)}{DL[H \uplus ex, H \cup\!\!\!\!- ex, A \Cap ex; \neg A](ch)} \quad ex \subseteq vi}{DL[H \uplus vi, H \cup\!\!\!\!- ex, A \Cap ex; \neg A](ch)}\,(i_2) \quad ex \subseteq sw}{DL[H \uplus vi, H \cup\!\!\!\!- sw, A \Cap ex; \neg A](ch)}\,(i_1)$$

Example (cont.)

Is $a = DL[H \uplus vi, H \cup\!\!\!\!- sw, A \cap\!\!\!\!- ex; \neg A](ch)$ tautologic? **Yes, it is!**

$$\cfrac{\cfrac{\cfrac{DL[H \uplus ex, H \cup\!\!\!\!- ex, A \cap\!\!\!\!- ex; \neg A](ch)}{DL[H \uplus ex, H \cup\!\!\!\!- ex, A \cap\!\!\!\!- ex; \neg A](ch)} \quad ex \subseteq vi}{DL[H \uplus vi, H \cup\!\!\!\!- ex, A \cap\!\!\!\!- ex; \neg A](ch)}\,(i_2) \quad ex \subseteq sw}{DL[H \uplus vi, H \cup\!\!\!\!- sw, A \cap\!\!\!\!- ex; \neg A](ch)}\,(i_1)$$

$DL[H \uplus ex, H \cup\!\!\!\!- ex, A \cap\!\!\!\!- ex; \neg A](ch)$ is an axiom **a2** of $\mathcal{K}^{\subseteq}_{taut}$.

Main Formal Results

Axiomatization for tautologies:

Theorem

The calculus $\mathcal{K}_{taut}$ ($\mathcal{K}^{\subseteq}_{taut}$) is sound and complete for tautologic ground DL-atoms a (relative to any closed set of inclusion constraints $\mathcal{C}$ separable for a).

Complexity results:

Theorem

Given a DL-atom a and a seperable set $\mathcal{C}$ of ICs for a, deciding whether a is tautologic relative to C is

- *NLogspace-complete and NLogSpace-hard even if $\mathcal{C} = \emptyset$, and is*
- *in LogSpace, and in fact first order expressible, if the DL query Q of a is not a negative concept resp. role query.*

Conclusion and Future Work

Independent DL-atoms:

- **contraditory**: simple form;
- **tautologic**: sound and complete calculus for derivation
  - general case;
  - under inclusion constraints;
- complexity results: efficiently solvable in both cases.

Future work

- Go beyond atomic concept (role) DL-queries;
- Consider further constraints;
- Take some information about ontology into account.

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