Answer Set Programming and Extensions

Unit 2 – Program Evaluation

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Unit Outline

1. Introduction
2. Solvers
3. Intelligent Grounding
4. Solving
5. Grounding Bottleneck

ASP Program Evaluation

Efficient evaluation of answer set programs is challenging

- NP-/Σ_2^p-completeness of consistency already in the ground (propositional) case
- different reasoning tasks
  - single solution
  - solution enumeration
  - query answering (reasoning from multiple models) projection, intersection, union
  - (multi-objective) optimization and preference
  - additional constructs (weak constraints, aggregates, function symbols, etc.)
- complex data in applications (large data volumes)

Answer Set Solvers

(see also http://en.wikipedia.org/wiki/Answer_set_programming)

ASPERIX	www.info.univ-angers.fr/pub/claire/asperix/
ASSAT	assat.cs.ust.hk/
CLASP [1]	potassco.sourceforge.net/#clasp/
CMODELS	www.cs.utexas.edu/users/tag/cmodels/
DLV [2]	www.dbai.tuwien.ac.at/proj/dlv/
ASPTOOLS	research.ics.aalto.fi/software/asp/
o ME-ASP	www.mat.unical.it/ricca/me-asp/
OMIGA	www.kr.tuwien.ac.at/research/systems/omiga
SMODELS	www.tcs.hut.fi/Software/smodels/
WASP	www.mat.unical.it/ricca/wasp/
XASP	xsb.sourceforge.net/, distributed with XSB
	....

[1] + CLASPD, CLINGO, CLINGCON etc. (http://potassco.sourceforge.net/)
[2] + DLVHEX, DLV^{DB}, DLT, DLV-COMPLEX, ONTO-DLV etc.

- Many ASP solvers are available (mostly function-free programs)
- Efforts to realize tractable fragments (*downscaling*)
- clasp was first ASP solver competitive to top SAT solvers
- ASP Solver competition (at LPNMR, since 2007)

Evaluation Approaches

- Different methods and evaluation approaches:
  - resolution-based [Bonatti *et al.*, 2008]
  - forward chaining [Lefèvre *et al.*, 2015]
  - lazy grounding: [Palù *et al.*, 2009], [Dao-Tran *et al.*, 2012], [de Cat *et al.*, 2012]
  - translation-based (see below)
  - meta-interpretation [E_ *et al.*, 2003]
- Predominant:

intelligent grounding + model search (solving)

Architecture of ASP Solvers

Common: two level architecture

1. **Intelligent Grounding**

   Given a program P, generate a (subset) of $grnd(P)$ that has the same models

2. **Solving: Model Search**

   More complicated than in SAT/CSP Solving:

   - candidate generation (classical model)
   - model checking (stability, foundedness!)
     - for SAT, model checking is in logarithmic space (in fact in ALOGTIME)
     - for normal propositional programs, model checking is PTime-complete
     - for disjunctive propositional programs, model checking is co-NP-complete

Intelligent Grounding

- Grounding is a hard problem

$$bit(0).\ \ bit(1). \tag{1}$$

$$p(X1, ..., Xn) \leftarrow bit(X1), ..., bit(Xn). \tag{2}$$

  naive: 2^n rules
- In the worst case, grounding time is exponential in the input size.
- Getting the "right" rules is difficult, already for positive programs
  - rule matching is NP-hard
  - deciding rule relevance is EXPTIME-hard
- Efficient grounding is at the heart of current systems
  - dlv's grounder (built-in);
  - lparse (smodels), gringo (clasp)
- Special techniques used:
  - *"safe rules"* (dlv): each variable in a rule occurs in the body in an unnegated atom with non-built-in predicate (exception: $X = c$)
  - *domain-restriction* (smodels)
  - Deductive DB methods: semi-naive evaluation, magic sets, ...

Grounding: Basic Ideas

- Avoid useless rule instances not applicable in any answer set

Example

$$c(1,2). \quad (3)$$

$$a(X)|b(Y) \leftarrow c(X,Y). \quad (4)$$

The full instantiation of the only rule yields

$$a(1) \mid b(1) \leftarrow c(1,1). \quad (5) \qquad a(2) \mid b(2) \leftarrow c(2,2). \quad (7)$$

$$a(2) \mid b(1) \leftarrow c(2,1). \quad (6) \qquad a(1) \mid b(2) \leftarrow c(1,2). \quad (8)$$

- (5)-(7) are useless: c(1,1), c(2,1), and c(2,2) do not occur in heads

- Basic instantiation:
  - body matching of variables (assignment)
  - backtracking on assignment
  - use *evaluation order*: $r_1 : p \leftarrow B_1. \quad r_2 : q \leftarrow p, B_2.$
    - must evaluate r_1 before r_2
    - $\Rightarrow$ use syntactic *dependency graph*

Optimizations

Intelligent grounders use a big deal of optimization techniques

- **literal ordering** in the body (= join optimization in DBs)
- **backjumping algorithm** [Perri *et al.*, 2008]
- **magic sets**
  - classic **magic sets technique** (standard Datalog): emulate goal-directed query answering (top-down) in bottom up computation
    - restrict rule instances using “magic predicates”
  - **dynamic magic sets** [Alviano *et al.*, 2012] for disjunctive programs with negation, exploit magic set information also during search [Alviano and Faber, 2011]
  - useful for hard problems in consistent query answering [Manna *et al.*, 2015]
- **parallel instantiation:**
  - exploit modern multi-core/-proc architectures with load balancing and granularity control [Perri *et al.*, 2013]
  - lparse [Pontelli *et al.*, 2003]

Grounding: Function Symbols

- Function symbols: infinite grounding, if even answer sets are finite

Example

$$p(0). \tag{9}$$

$$p(X) \leftarrow p(f(X)), \text{not } q(X). \tag{10}$$

$$q(X) \leftarrow p(f(X)), \text{not } p(X). \tag{11}$$

- dlv-grounder, gringo admit recursive function symbols
- Termination for *finitely-ground (FG) programs* [Calimeri *et al.*, 2008]
  - problem: only semi-decidable
  - Turing complete (each computable function expressible)
- **Practice:** decidable syntactic / semantic restrictions (preprocessing)
  - ω-restrictedeness [Syrjänen, 2001] (lparse)
  - λ-restrictedeness [Gebser *et al.*, 2007] (gringo x, x$<$3)
  - argument-restrictedness [Lierler and Lifschitz, 2009]
  - finite domain [Calimeri *et al.*, 2008] (dlv)
  - Γ-acyclic programs [Greco *et al.*, 2012]

Finitely-Ground (FG) Programs

Idea:

- *Modularization:* split a program P into modules $P_1 \ldots P_n$, define a proper module ordering $\prec$ for bottom up evaluation
- *Simplification:*
  - instantiate a given module P_i of P by exploiting instantiations of previous modules $\rightarrow$ "intelligent" instantiation
  - add only ground rules whose heads have a chance to be true in some answer set
- All P_i are finitely instantiated along $\prec \Rightarrow P'$ as above is found

Pros:

- FG programs are very expressive: they correspond to terminating computations of Turing Machines
- FG programs have a finite set of finite answer sets

Cons:

- Undecidable (semi-decidable) in general, not recognizable

Finite Domain (FD) Programs

- Simple, decidable subclass of FG programs [Calimeri *et al.*, 2008]
- All arguments of a program P must be *finite domain*
- Use an *argument dependency graph* $AG(P)$ of P:
  - nodes = argument positions of predicates in P; eg. $edge[1]$, $path[2]$,
  - edge $p[i] \rightarrow q[j]$, if in some rule,
    - (i) $p(t_1, \ldots, t_n)$ is in the head,
    - (ii) $q(s_1, \ldots, s_m)$ occurs positive in the body, and
    - (iii) t_i and s_j share some variable.
- Program P is *finite domain* if, for each atom $p(t_1, \ldots, t_n)$ in the head of a rule $r \in P$, and for each argument $p[i]$ of p, either
  - (i) t_i is variable-free; or
  - (ii) t_i occurs as a (sub)term of an atom not under "not " in the body; or
  - (iii) each variable occurring in t_i occurs in a positive body atom $q(s_1, \ldots, s_m)$ in some s_j s.t. $p[i]$ and $q[j]$ are not on a cycle in $AG(P)$.

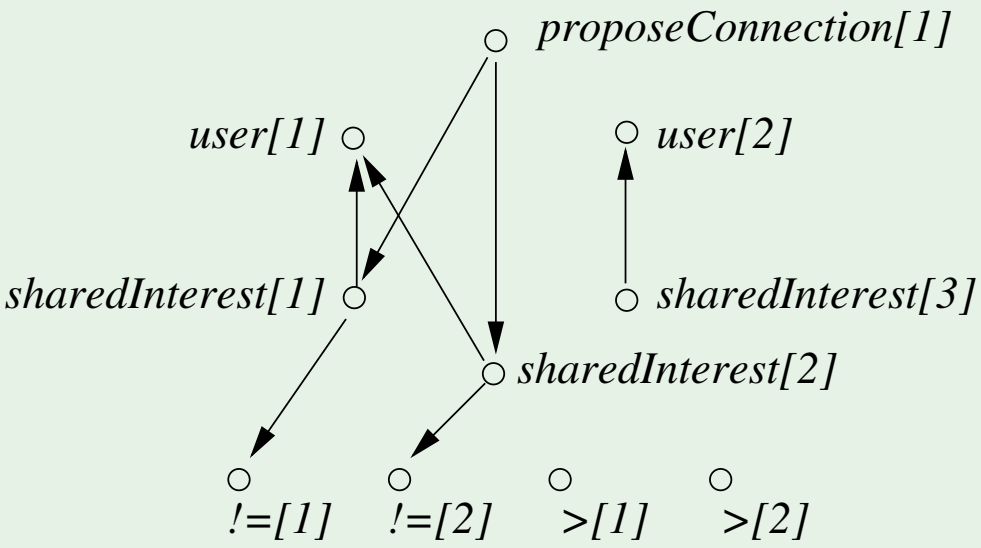
Examples

More than two shared interests

$$sharedInterests(U_1, U_2, \#intersection(S_1, S_2)) \leftarrow user(U_1, S_1), user(U_2, S_2), U_1 \neq U_2.$$

$$proposeConnection(pair(U_1, U_2)) \leftarrow sharedInterests(U_1, U_2, S), \#card(S) > 2.$$

is finite domain.

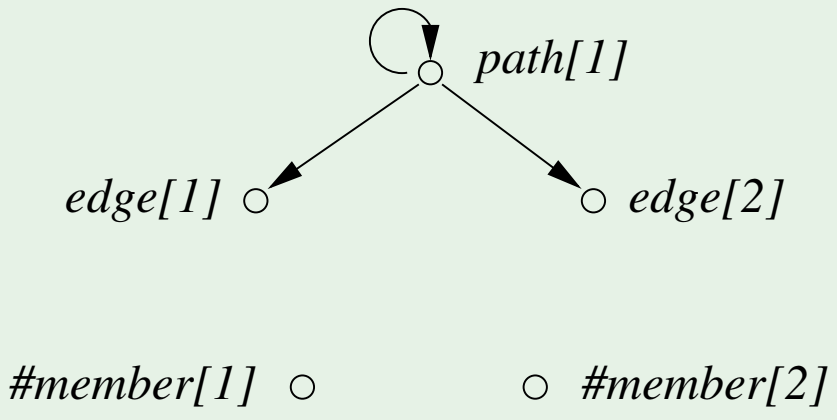


Examples (ctd.)

Paths in a graph

$$path([X,Y]) \leftarrow edge(X,Y).$$
$$path([X|[Y|W]]) \leftarrow edge(X,Y),\ path([Y|W]),\ \text{not}\ \ \#member(X,[Y|W]).$$

is not finite domain:

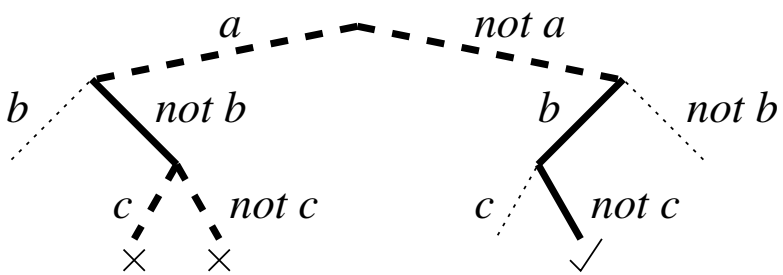


- dlv: "`Termination is not guaranteed ... Use option -nofinitecheck in order to evaluate this program anyway.`"
- still finitely-ground for finite $edge \Rightarrow$ dlv terminates
- easily modified to compute e.g. all Hamiltonian paths from s to t:

Solving: Model Search

- Applied to ground programs.
- Early solvers (e.g. smodels, dlv): native methods
  - inspired by Davis-Putnam-Logemann Loveland (DPLL) for SAT
    - 3 basic operations: decision, propagate, backtrack
  - special propagation for ASP, e.g.,
    - smodels: 5 propagators
    - dlv: *well-founded*, *must-be-true* propagation (supportedness)

$a{:-}\ not\ b.$
$b{:-}\ not\ a.$
$c{:-}\ not\ c,\ a.$



    - important: heuristics (which atom/rule is next?) [Faber *et al.*, 2001]
    - chronological backtrack-search was improved introducing backjumping and look-back heuristics [Maratea *et al.*, 2008]
    - Recent: Abstract solving [Brochenin *et al.*, 2014]
- Stability check: unfounded sets, reductions to UNSAT [Leone *et al.*, 1997b], [Koch *et al.*, 2003]

ASP Solving Approaches

- Predominant to date: modern SAT techniques
- Export of techniques for optimal answer sets to SAT
- **Genuine conflict-driven ASP solvers**
  - clasp [Gebser *et al.*, 2012], wasp [Alviano *et al.*, 2013]
- **Translation based solving:** to
  - SAT: assat [Lin and Zhao, 2004], cmodels [Lierler, 2005], lp2sat [**?**], [Giunchiglia *et al.*, 2006] (multiple SAT solver calls)
  - SMT: [Niemelä, 2009] aspmt [Bartholomew and Lee, 2014]
  - Mixed IP: [Bell *et al.*, 1994], [Liu *et al.*, 2012]; CPLEX backend
- **Cross translation:** intermediate format to ease cross translation
  - SAT modulo acyclicity [Gebser *et al.*, 2014]
    - interconnect graph based constraints with clausal constraints
    - can postpone choice of the target format to last step
- **Portfolio solvers**
  - claspfolio [Gebser *et al.*, 2011]: combines variants of clasp
  - me-asp [Maratea *et al.*, 2014]: multi-engine portfolio ASP solver

Program Decomposition

- A set S of (ground) atoms is a *splitting set* of a (ground) program P, if for each rule $r : H(r) \leftarrow B(r)$ in P either (i) $H(r) \cup B(r) \subseteq S$ or $H(r) \cap S = \emptyset$.
- $b_S(P)$ and $t_S(P)$ denotes all rules (i) and (ii), resp.

Example

E.g. $P = \left\{ \begin{array}{ll} f_1 & man(d). \\ r_1 & husband(d) \vee single(d) \leftarrow man(d). \\ r_2 & bachelor(d) \leftarrow man(d) \text{ not } husband(d). \end{array} \right\}$

$S = \{man(d), husband(d), single(d)\}$, $b_S(P) = \{f_1, r_1\}$, $t_S(P) = \{r_2\}$

Splitting Set Theorem, Lifschitz and Turner [1994]

For any splitting set S of program P, $AS(P) = \bigcup_{M \in AS(b_S(P))} AS(t_S(P) \cup M)$.

- Splitting sets allow for (de)composition and bottom up evaluation
  - $AS(b_S(P)) = \{M_1 = \{man(d), single(d)\}, M_2 = \{man(d), husband(d)\}\}$,
  - $AS(t_S(P) \cup M_1) = M_1 \cup \{bachelor(d)\}$, $AS(b_S(P) \cup M_2) = M_2$
- has also nonground versions
- solvers identify splitting sets (e.g. strongly connected components)

Characterizing Answers Sets in Classical Logic

- Answer sets of a program P are special classical models of P
  - view rule $r : a \leftarrow b_1, \ldots, b_m, \text{not } c_1, \ldots, \text{not } c_n$ (= $a \leftarrow B(r)$) as implication $b_1 \wedge \cdots \wedge b_m \wedge \neg c_1 \wedge \cdots \wedge \neg c_n \rightarrow a$ (=: $BF(r) \rightarrow a$)
- Clark [1978]: for Horn P, turn $\rightarrow$ into $\leftrightarrow$:

$$CF(P) = \{ \bigvee_{a \leftarrow B(r) \in P} BF(r) \rightarrow a \mid a \text{ is an atom } \}$$

- "Completion" $CF(P)$ captures $AS(P)$ for special normal programs

Theorem

For normal logic programs P without cyclic positive recursion (called tight programs*), $AS(P)$ is captured by $CF(P)$ (i.e., its classical models).*

Example

- $P = \{p \leftarrow q\}$: $CF(P) = \{p \leftrightarrow q,\ q \leftrightarrow \bot\}$ P is tight ✓
- $P = \{p \leftarrow q; q \leftarrow p, \text{not } r\}$: $CF(P) = \{p \leftrightarrow q,\ q \leftrightarrow p \wedge \neg r,\ r \leftrightarrow \bot\}$ P is not tight ×

Loop Formulas

$AS(P)$ is captured by adding *loop formulas* $LF(P)$ [Lin and Zhao, 2002]

Theorem

For any normal program P, $AS(P) = CF(P) \cup LF(P)$.

- Informally, $LF(P)$ contains clauses enforcing that a positive cycle C can only by true if some $r : a \leftarrow B(r)$ exists s.t. $a \in C$, $B(r)$ is satisfied and no atom from C occurs positively in $B(r)$.

Example

- $P = \{p \leftarrow q\}$: $LF(P) = \emptyset$

  $CF(P) \cup LF(P) = \{p \leftrightarrow q,\ q \leftrightarrow \bot\} \equiv \neg p \wedge \neg q$
- $P = \{p \leftarrow q,\ q \leftarrow p, \text{not } r\}$: $\quad LF(P) = \{p \wedge q \rightarrow \bot\}$

  $CF(P) \cup LF(P) = \{p \leftrightarrow q,\ q \leftrightarrow p \wedge \neg r,\ r \leftrightarrow \bot, p \wedge q \rightarrow \bot\} \equiv \neg p \wedge \neg q \wedge \neg r$

Loop Formulas /2

- Also feasible for disjunctive programs, non-propositional program [Lee and Lifschitz, 2003], [Chen *et al.*, 2006], and aggregates.
- Based on this, some ASP solvers employ SAT solvers.
- Semantically, loop formulas correspond to *unfounded-freeness* [Leone *et al.*, 1997a], [Lee, 2005]
- **Downside:** exponentially many loop formulas exist in general [Lifschitz and Razborov, 2006]
  - add loop formulas *on the fly*
- Alternative: avoid explicit loop formulas, do cautious description of reachability (*ordered completion*, [Asuncion *et al.*, 2012])

Conflict-Driven ASP Solving

- **Breakthrough in SAT**: **conflict driven clause learning (CDCL)** [Silva *et al.*, 2009]
  - represent clauses as nogoods (inadmissible assignments)
  - propagate with conflict-driven backtracking
  - clause (nogood) learning from conflicts

Example

- $(\neg a \vee b \vee c)$, $(d \vee a)$, $(d \vee \neg b) \rightsquigarrow$ nogoods $\{\mathbf{T}a, \mathbf{F}b, \mathbf{F}c\}$, $\{\mathbf{F}d, \mathbf{F}a\}$, $\{\mathbf{F}d, \mathbf{T}b\}$
- assignment $\{\mathbf{F}c\}$; set $\mathbf{F}d \Rightarrow$ derive $\mathbf{T}a$, $\mathbf{F}b$: conflict!
- learn noogod $\{\mathbf{F}c, \mathbf{F}d\}$

- **ASP: key idea** [Gebser *et al.*, 2012]
  - exploit $AS(P) = CF(P) + LF(P)$
  - use completion nogoods for $CF(P)$, loop nogoods for $LF(P)$
  - map inferences in ASP onto unit propagation on nogoods
  - enable learning

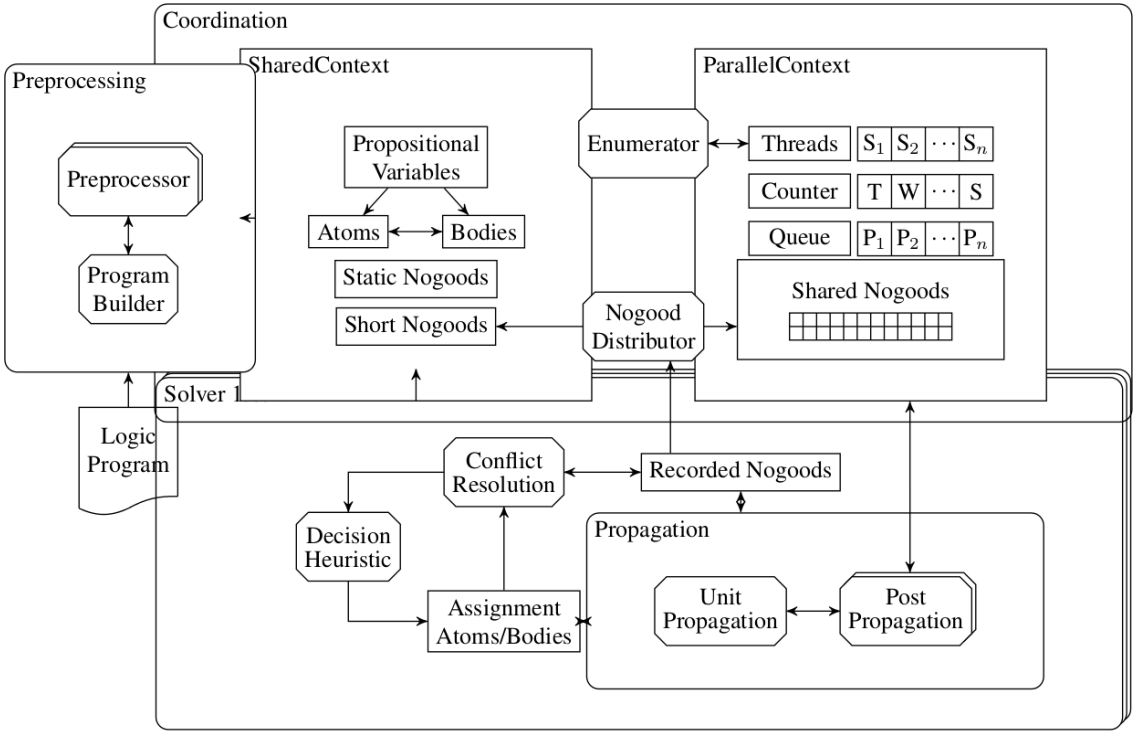
A Glimpse on clasp

- Basic CDCL algorithm

```
(I) initialize                                                    \\ initialize watches
    loop
        propagate completion, loop, and recorded nogoods \\ deterministically assign literals
(C)     if no conflict then
            if all variables assigned then
                return variable assignment                        \\ answer set found
(P)         else
                if no nogood recorded then decide                 \\ non-det. assign literal
        else
            if top-level conflict then return unsatisfiable
            else
                analyze                          \\ resolve conflict and record a conflict
(U)             backjump                 \\ undo assignments until conflict constraint is unit
```

- use unit propagation for completion nogoods (to a fixpoint)
- special loop nogood propagator
- Workflow:
  1. $P \Rightarrow P'$ simplify
  2. $P' \Rightarrow CF(P')$ (which is static)
  3. construct nogoods for $LF(P')$ *on demand*, using a dependency graph

A Glimpse on clasp /2



- multi-threading: parallel execution of clasp (search space splitting, competitive strategies)
  - learned conflict nogoods can be exchanged
- further post propagation is usable for theory-specific propagation
  - exploited e.g. by solvers clingcon, dlvhex

Preference and Optimization

- Two classes [Delgrande *et al.*, 2004]
  1. **prescriptive**: condition on how rules are/must be applied

     implementation by compilation [Delgrande *et al.*, 2003] or meta-interpretation [E_ *et al.*, 2003]
  2. **descriptive**: how answer sets relate (comparison)

     preferences among the answer sets of a program [Brewka *et al.*, 2003], [Sakama and Inoue, 2000], [Son and Pontelli, 2006]
- Class 2) has usually higher computational complexity than 1)
- Tools:
  - ASPRIN system [Brewka *et al.*, 2015]: general and flexible framework to compute optimal answer sets relative to preferences among them
  - plp tool [Delgrande *et al.*, 2000]: compiler for preferences `http://www.cs.uni-potsdam.de/~torsten/plp/`

Grounding Bottleneck

- Recall: Grounding is expensive in general
- **Practical Grounding Bottleneck** [E_ *et al.*, 2007b]
  - Unless EXPTIME = PSPACE, grounding + solving needs unavoidably exponential space in general
  - Certain programs are evaluable in polynomial space, while the top grounders still produce an exponentially large ground programs
- Example: programs P with predicate arities bounded by a constant

Theorem [E *et al.*, 2007a]

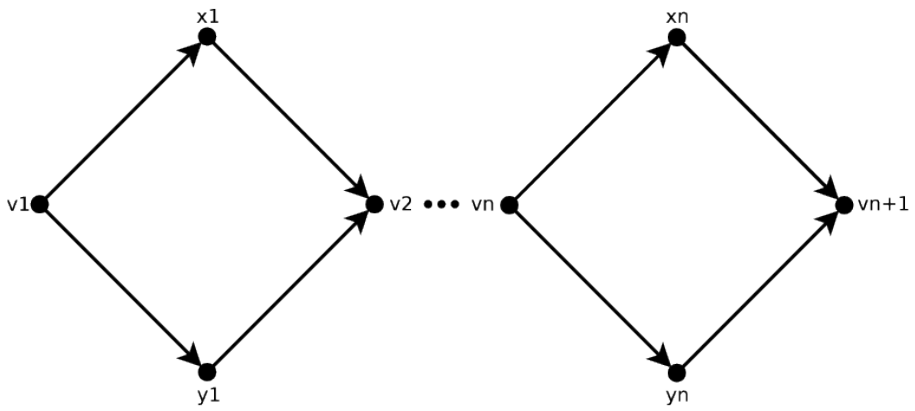
In the datalog case, deciding whether a bounded-arity program P has some answer set is (i) Σ_2^p-complete if P is normal, and (ii) Σ_2^p-complete if P is disjunctive.

- For some families $\mathbf{P}_n = P_1, P_2, \ldots$ of bounded arity programs, the grounders produce exponentially growing output

Examples

- reachability over paths of length k in a directed graph G, given by $e(v_1, v_2)$:

$$p_k(X_1, X_k) \leftarrow e(X_1, X_2), \ldots, e(X_{k-1}, X_k).$$
$$reachable(X, Y) \leftarrow p_k(X, Y).$$
$$reachable(X, Y) \leftarrow reachable(X, Z), p_k(Z, Y).$$



  - for $k = 2n + 1$ there are 2^n paths from v_1 to $v_{n+1} \Rightarrow p_k(v_1, v_{n+1})$ has 2^n derivations
  - if G is nondeterministically chosen by other rules (possibly depending on p_k) grounders don't optimize

Examples /2

- Monadic choice:

$$
\begin{array}{ll}
b(0).\ \ b(1). & (1) \\
p(X) \leftarrow b(X), \text{not}\ \ q(X). & (2) \\
q(X) \leftarrow b(X), \text{not}\ \ p(X). & (3) \\
r(X_1) \leftarrow p(X_1), \ldots, p(X_k), b(X_1), \ldots b(X_k). & (4)
\end{array}
$$

  - as above, exponential grounding result
  - modular evaluation (1),(2) before (3) can be spoiled e.g. with adding $q(X) \leftarrow r(X)$

Nonground ASP: Special Techniques

Techniques to address grounding explosion

- **lazy grounding**
- for **bounded predicate arities**
  - few generating rules property [E_ *et al.*, 2010]
  - answer set size and number of supporting rules is polynomial
  - can run in polynomial space
- **rule decomposition** [Morak and Woltran, 2012], [Bichler *et al.*, 2016]
  - split (long) rule $r : H \leftarrow B$ using decomposition of its hypergraph $H_r = (V_r, E_r)$
  - V_r = variables in B, E_r = literals (labeled with predicate)
  - the decomposition effect relies on the *treewidth of* H_r, $tw(H_r)$ (“treelikeness”)
  - avoids explosion for $tw(H_r) \leq k$, constant k (thus bounded arities)
  - tool (for full ASP-syntax) `http://dbai.tuwien.ac.at/research/project/lpopt/`

Example

- Single rule $r: \quad h(X,W) \leftarrow e(X,Y), e(Y,Z), \text{not } e(Z,W), e(W,X)$
  - $V_r = \{X, Y, X, W\}$,
  - $E_r = \{e_1{=}\{X,W\}, e_2{=}\{X,Y\}, e_3{=}\{Y,Z\}, e_4{=}\{Z,W\}, e_5{=}\{W,X\}\}$
- Tree Decomposition: $\mathcal{T} = (T, \chi)$, $T = (\{n_1, n_2\}, \{n_1 \rightarrow n_2\})$

$$\begin{array}{ll} n_1 \bullet & \chi(n_1) = \{X,Y,W\}, \quad \text{covers } h(X,W), e(X,Y), e(W,X) \\ \downarrow & \\ n_2 \bullet & \chi(n_2) = \{Y,Z,W\}, \quad \text{covers } e(Y,Z), \text{not } e(Z,W) \end{array}$$

- Decomposition algorithm yields (fresh predicate t_1, dom_W):

$$\begin{array}{l} h(X,W) \leftarrow e(X,Y), e(X,W), t_1(Y,W). \\ dom_W(W) \leftarrow e(W,X). \\ t_1(Y,W) \leftarrow e(Y,Z), \text{not } e(Z,W), dom_W(W). \end{array}$$

- Runs for bounded treewidth, i.e. $\max_{n \in T} |\chi(n_i)| - 1 = O(1)$ in polynomial space.

Application: Propositional ASP and QSAT

Bichler *et al.* [2016] (ICLP 2016)

- Transform prop. disjunctive ASP P into normal ASP w/ bounded arities
  - represent P as facts
  - few fixed rules (independent of P): guess M, check $M \models P$
  - few long rules: check foundedness of M
- Customized Encoding: 2-QSAT ($\forall\exists$-QBF satisfiability)
- Similarly, encode Σ_3^p problems into disjunctive ASP w/ bounded arities
  - 3-QSAT ($\exists\forall\exists$-QBF satisfiability)
  - stable cautious abductive reasoning [E_ *et al.*, 1997]
- For 2-QSAT, results are quite competitive to state-of-the-art solver DepQBFSolver [Lonsing and Egly, 2015]; for 3-QSAT problems, complementary
- Also good results for abductive reasoning

QSAT Encodings

QBF $\Phi = \forall x_1, \ldots, x_m \exists y_1, \ldots, y_n (c_1 \wedge \cdots \wedge c_k)$, where $c_i = \ell_{i,1} \vee \ell_{i,2} \vee \ell_{i,3}$

- "classical" encoding:

$$
\begin{array}{ll}
 & var(x_i). \text{ for all } x_i \\
 & var(y_j). \quad exists(y_j). \quad \text{for all } y_j \\
 & occ_h(c_i, z, 0). \text{ s.t. } \ell_{i,h} = \neg z, \qquad occ_h(c_i, z, 1). \text{ s.t. } \ell_{i,h} = z \\
(1) & ass(X, 1) \vee ass(X, 0) \leftarrow var(X). \\
(2) & ass(Y, 0) \leftarrow sat, exists(Y). \\
(3) & ass(Y, 1) \leftarrow sat, exists(Y). \\
(4) & sat \leftarrow (occ_i(C, X_i, A_i), ass(X_i, 1 - A_i))_{i=1}^{3}. \\
(5) & \leftarrow \text{not } sat.
\end{array}
$$

  - Informally, (1) guesses an assignment for all vars
  - by (5), we must derive sat
  - (2), (3), (4): if some assignment to the y_j violates a clause, do saturate

  Note: some answer set exists iff Φ evaluates to false

- Maratea *et al.* [2008] used a propositional version of the classic encoding; dlv was competitive with state-of-the-art QBF solvers then

QSAT Encodings /2

QBF $\Phi = \forall x_1, \ldots, x_m \exists y_1, \ldots, y_n (c_1 \wedge \cdots \wedge c_k)$, where $c_i = \ell_{i,1} \vee \ell_{i,2} \vee \ell_{i,3}$

- Bounded arity encoding

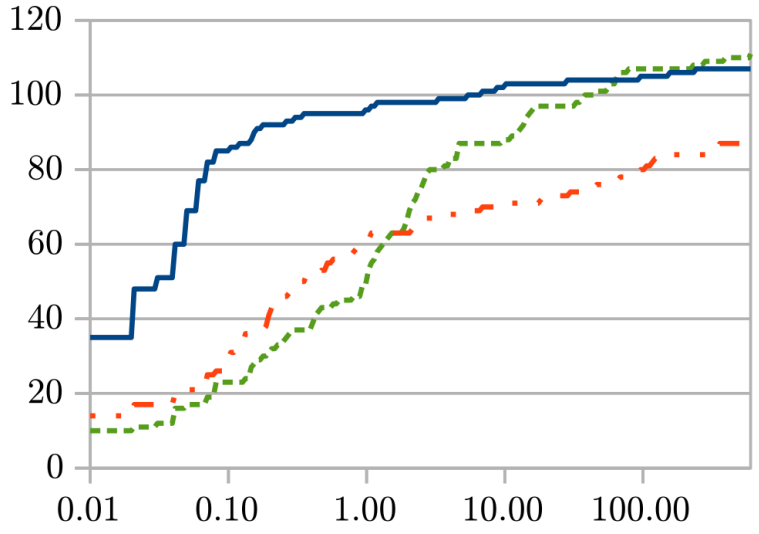
(1)	$t(x_i) \vee f(x_i) \leftarrow \ .$	for all x_i
(2)	$c_i(\vec{t}) \leftarrow t(x_j).$	for all $\vec{t} \in \vec{Y}(c_i),\ x_j \in \{\ell_{i,1}, \ell_{i,2}, \ell_{i,3}\}$
(3)	$c_i(\vec{t}) \leftarrow f(x_j).$	for all $\vec{t} \in \vec{Y}(c_i),\ \neg x_j \in \{\ell_{i,1}, \ell_{i,2}, \ell_{i,3}\}$
(4)	$c_i(\vec{t}).\ \ \vec{t} \in \vec{Y}(c_i) \setminus \{\bar{c}_i\}$	
(5)	$\leftarrow c_1(\eta(c_1)), \ldots, c_k(\eta(c_k)).$	

  - $\vec{Y}(c_i) = \{0,1\}^{h_i}$ are all assignments of 0 and 1 to the literals $\ell_{i,j_1}, \ldots, \ell_{i,j_{h_i}}$ over $y_1, \ldots, y_n$ in c_i ($h_i \leq 3$)
  - $\bar{c}_i$ is the single such assignment that makes them false
  - $\eta(c_i)$ results from $\ell_{i,j_1}, \ldots, \ell_{i,j_{h_i}}$ by replacing each y_j and $\neg y_j$ with Y_j

    Example: for $c = \neg y_1 \vee x_2 \vee y_3$, we have $\ell_{i,j_1}, \ldots, \ell_{i,j_{h_i}}$ = $\neg y_1, y_3$ and thus $\vec{Y}(c) = \{0,1\}^2$, $\bar{c} = (1,0)$, and $\eta(c) = Y_1, Y_3$
  - (1) can be replaced with "shifting" $t(x_i) \leftarrow \text{not } f(x_i).$ and $f(x_i) \leftarrow \text{not } t(x_i).$

- Lifted to QBFs $\Phi = \exists x_1, \ldots, x_l \forall x_{l+1}, \ldots, x_m \exists y_1, \ldots, y_n (c_1 \wedge \cdots \wedge c_k)$

Evaluation Results



x = time (secs), y = # solved instances

(1) classic encoding: dotted-dasher red; (2) long rule encoding:green; (3) DepQBFSolver: blue

- Publicly available 2-QBF ($\forall\exists$) competition instances
- For (1), (2), preprocessing by DepQBFSolver, (for (2), decomposition), then clingo
- For (3), DepQBFSolver
- 200 instances, 2 secs: (2) solved 111, (1) 88, and (3) 107
- Specific benchmark “stmt”: (3) $>$ (2) $>$ (1)
- 3-QBF: 151 inst. from the Eval-2012 data set of latest QBF competition
  - (3) solved 47; (2) solved ony 18, but 10 not solved by (3)

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