# Answer Set Programming and Extensions

## Unit 3 – Hybrid Extensions

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# Unit Outline

# Hybrid Extensions

- Language extensions before are within same semantic / computational framework
- **Need:**
  - interoperability with other logics, e.g. Description Logics
  - interfacing with programming languages, e.g. $C++$
  - access to general *external* sources of information, e.g. WordNet
- **Problems, issues:**
  - value invention: new ground terms might appear
  - impedance mismatch
  - black boxes: no (little) knowledge about external source
  - semantics: e.g. cyclic reference

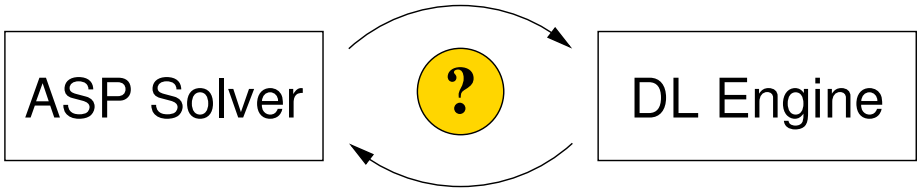
# Hybrid Extensions: Approaches

- **Embedded ASP**: akin to embedded SQL
  - XSB + stable models
  - DLV Wrapper: library to embed ASP in Java and control execution
  - Potassco: embeds ASP in Python and Lua progs w/ gringo and clasp
- **Bilateral interaction**
  - JASP [Febbraro *et al.*, 2012]: ASP $\leftrightarrow$ Java;
    ASP progs can access Java variables, answer sets are stored in Java
- **ASP + concrete theories $X$**: $X$ = ontologies (dl-programs), constraints (CASP), actions (acthex) ...
  - dlvhex/dlplugin, nlp-dl [E_ *et al.*, 2005a],[E_ *et al.*, 2008]
  - clingcon [Ostrowski and Schaub, 2012], inca [Drescher and Walsh, 2010] lc2casp [Cabalar *et al.*, 2016]
  - mingo [Liu *et al.*, 2012] dingo for ASP(SMT) [Janhunen *et al.*, 2013]
  - EZSMT [Susman and Lierler, 2016], EZCSP [Balduccini, 2009]
  - acthex [Fink *et al.*, 2013]
- **ASP + abstract "theory" $X$**: even non-logical
  - dlvhex

# ASP+Ontologies: DL-programs

- An extension of answer set programs with *queries to DL knowledge bases (DL KBs)* [E_ *et al.*, 2008]
- queries can *temporarily update* the DL KB

  *bidirectional flow of information*, with clean technical separation of DL engine and ASP solver ("loose coupling")




- Use dl-programs as "glue" for combining inferences on a DL KB.
- Experimental prototypes
  - NLP-DL (early) `https://www.mat.unical.it/ianni/swlp/`
  - dlvhex/dlplugin `http://www.kr.tuwien.ac.at/research/systems/ /dlvhex/dlplugin.html`
  - DReW `http://www.kr.tuwien.ac.at/research/systems/drew/`

# Example: Closed World Reasoning

Handle incomplete information w/ disjunction: ECWA [Gelfond *et al.*, 1986]

Let $\mathcal{K} = (O, P)$, where

- $O = \{Artist \equiv Singer \sqcup Painter,\ Artist(Nash),\ Singer(Sting)\}$
- $P = \left\{ \begin{array}{lrl} (r_1) & \neg p(X) & \leftarrow DL[\ ;Artist](X), not\, p(X). \\ (r_2) & \neg s(X) & \leftarrow DL[\ ;Artist](X), not\, s(X). \\ & p(X) & \leftarrow DL[\ ;Artist](X), \\ & & \quad DL[Painter \uplus \neg p, Singer \uplus \neg s; Painter](X). \\ & s(X) & \leftarrow DL[\ ;Artist](X), \\ & & \quad DL[Painter \uplus \neg p, Singer \uplus \neg s; Singer](X). \end{array} \right\}$
- Under ECWA, $Sting$ is not a painter, while $Nash$ is one iff he is no singer
- CWA defaults $(r_1)$, $(r_2)$: if $p$ ($s$) is not provable, conclude it's false
- $Painter \uplus \neg p, Singer \uplus \neg s$ is temporary update of $O$ with $\neg p$, $\neg s$
- answer sets: $\left\{ \begin{array}{l} I_1 = \{s(Sting), \neg p(Sting), s(Nash), \neg p(Nash)\}, \\ I_2 = \{s(Sting), \neg p(Sting), p(Nash), \neg s(Nash)\} \end{array} \right\}$
- cyclic information flow is essential!

# ASP + Constraint Programming

- Writing constraints straight in ASP faces grounding problems:

$$ok(X, Y) \leftarrow p(X, Y), X <= 10000, Y >= 50000, 2*Y < X$$

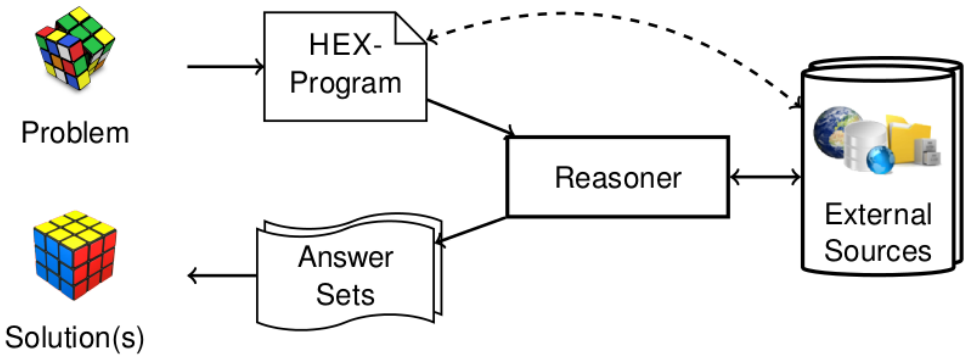
  treat constraints as *abstract predicates / objects*, avoid grounding

- Approaches:
  - ASP + difference constraints: $X - Y \leq k$ [Janhunen *et al.*, 2013]
  - ASP + linear inequalities: $\sum_{i=1}^{k} a_i * X_i \leq k$ [Mellarkod *et al.*, 2008], [Gebser *et al.*, 2009], [Liu *et al.*, 2012]
  - ASP + linear inequalities + $X$: C-HEX [De Rosis *et al.*, 2015]
  - Default constraints [Cabalar *et al.*, 2016]
  - CP + ASP: stable model version of CP [Aziz *et al.*, 2013]
  - solvers: acsolver, clingcon ezcsp, idp, mingo, dingo, lc2casp , . . .
- Often focus finite domains
- Lierler [2014]: key features of constraint ASP languages & systems
- formal link Constraint ASP $\leftrightarrow$ SMT: [Lierler and Susman, 2016]

# HEX Programs

- Motivated to meet needs of heterogeneous data access on the Web
- Generalize DL-programs, which provide ASP programs with query access to an ontology.
- **Allow to access sources of whatever type (abstract modeling)**
- **Features** (cf. [E_ *et al.*, 2005b]):
  - *Hilog-style atoms:* variables for predicate names (syntactic sugar)
    E.g., $P(X,Z) \leftarrow P(X,Y), P(Y,Z)$ (transitivity of $P$)
    $\Rightarrow$ reify atoms $p(t_1,\ldots,t_n)$ to $(p, t_1 \ldots, t_n)$
  - *External atoms:* access to external sources, e.g.,
    - ontologies (RDF, OWL, ...)
    - planners,
    - data structures (libraries, built-ins)
    - ... (user definable)

# HEX Programs: Idea




Example: import parts of external RDF triples into the program

$$p(X,Y) \leftarrow url(U), \&rdf[U](X,Y,Z).$$

- $\&rdf[U](X,Y,Z)$: *external atom* with "input" $U$ and "output" $X,Y,Z$
- Intuitively, for concrete input (a constant, or predicate name), evaluate the external source and return output
- **Issues:**
  - model external atoms
  - semantics of programs (in spirit of ASP)
  - *value invention* (constants not occurring in the program)

# Key Construct: External Atoms

$$p(X, Y) \leftarrow url(U), \&rdf[U](X, Y, Z)$$

## External Atom

An *external atom* $a$ is of the form $\&g[\vec{Y}](\vec{X})$, where $\vec{Y} = Y_1, \ldots, Y_n$ and $\vec{X} = X_1, \ldots, X_m$ are two lists of terms (*input* / *output* list), and $\&g$ is an external predicate name.

- External atoms may occur only in rule bodies
- $\&g$ has an associated function

$$f_{\&g} : 2^{HB_P} \times \mathcal{C}^{n+m} \rightarrow \{0, 1\}$$

  mapping each $(I, \vec{y}, \vec{x})$, where $\vec{y} = y_1 \ldots, y_n$, $\vec{x} = x_1, \ldots, x_m$, to $0$ or $1$, where $I$ is an interpretation and $x_i, y_j$ are from a fixed set $\mathcal{C}$ of constants.
- In practice:
  - $y_i$ are predicate names or individuals, $x_j$ are individuals
  - $HB_P$ are the Herbrand interpretations over $\mathcal{C} = \mathcal{C}_{pred} \cup \mathcal{C}_{ind}$
- Embraces e.g. aggregates $\&\min[sal](X)$, string functions $\&concat['jim',' doe'](N)$, etc

# HEX Programs: Formal Notions

## Definition (HEX program, omitting higher order)

A HEX *program* $P$ is a set of rules $r$ of the form

$$a_1 \vee \cdots \vee a_l \leftarrow b_1, \ldots, b_m, not\, c_1, \ldots, not\, c_n$$

where all $a_i$ and $b_j$ are first-order atoms, and all $c_k$ are first-order or external atoms over $\mathcal{C}$ (plus variables $\mathcal{V}$).

Semantics is defined in terms of grounding over $\mathcal{C}$ as before

## Definition (Satisfaction)

A (Herbrand) interpretation $I$ satisfies an (ground) external atom $\&g[\vec{y}](\vec{x})$, denoted $I \models \&g[\vec{y}](\vec{x})$, if $f_{\&g}(I, \vec{y}, \vec{x}) = 1$.

Satisfaction of (ground) $not\, c_k$, body $B(r) = b_1, \ldots, b_m, not\, c_1, \ldots, not\, c_n$, head $H(r) = a_1 \vee \cdots \vee a_l$, rule $r = H(r) \leftarrow B(r)$, and program $P$ are then as usual.

# HEX Programs /2

Answer sets are defined via the FLP-reduct [Faber *et al.*, 2011]

## Definition (HEX answer set)

An interpretation $I$ is an answer set of a (ground) HEX program $P$, if $I$ is a minimal model (w.r.t. $\subseteq$) of $fP^I = \{r \in P \mid I \models B\}$.

Note:

- Gelfond -Lifschitz style reduct (treating external atoms like $not$-literals) does not ensure minimality of models

  $P = \{p(a) \leftarrow \&id[p](X)\}$ where $f_{\&id}(I, p, x) = 1$ iff $I \models p(x)$ (identity)

  single answer set $\{\}$, but another GL-style answer set $\{p(a)\}$

- $not, \vee$-free HEX-programs can have multiple answer sets

  $P = \{\ p(a). \quad s_1(X) \leftarrow \&diff[p, s_2](X). \quad s_2(X) \leftarrow \&diff[p, s_1](X)\ \}$

  where $f_{\&diff}(I, p, q, x) = 1$ iff $I \models p(x)$ and $I \not\models q(x)$ (set difference)

  answer sets $\{p(a), s_1(a)\}$ and $\{p(a), s_1(a)\}$

# Implementation

DLVHEX

http://www.kr.tuwien.ac.at/research/systems/dlvhex

- **Challenges:**
  - value invention: in principle, infinitely many constants
  - black box nature: API-style interface abstract
  - minimality checking: source of complexity (from the definition)

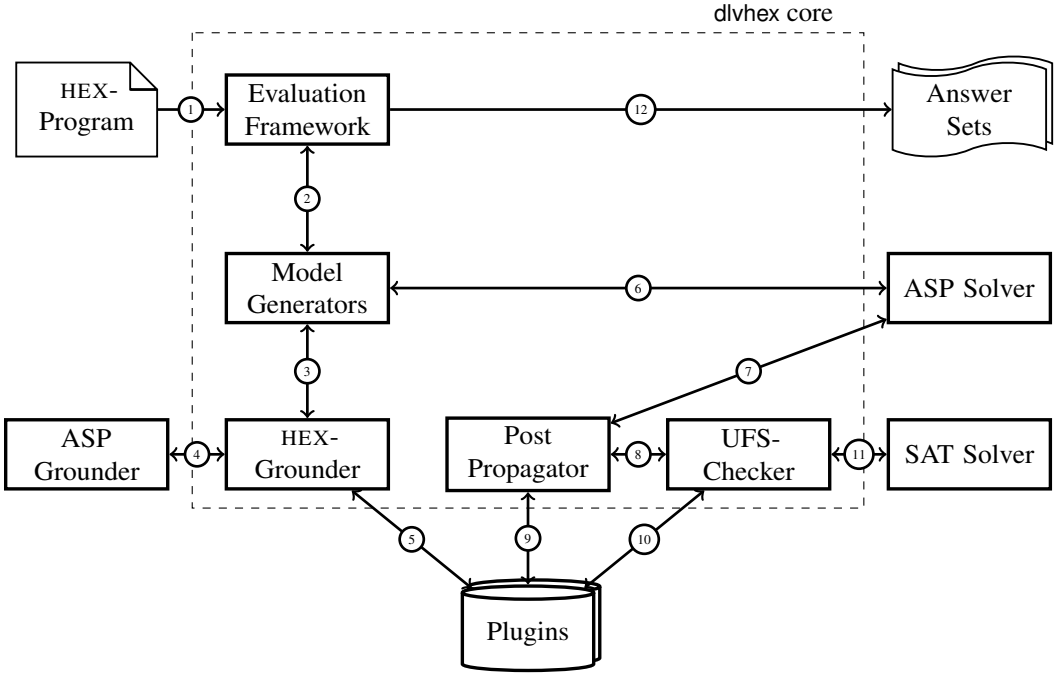
- dlvhex1:
  - **rewriting approach**
    - use replacement atoms and value guessing ($P \rightarrow \hat{P}$):

      $$\&g[\vec{y}](\vec{x}) \rightsquigarrow e_{\&g[\vec{y}]}(\vec{x}) \qquad e_{\&g[\vec{y}]}(\vec{x}) \vee ne_{\&g[\vec{y}]}(\vec{x}) \leftarrow$$

    - a-posteriori *compatibility check*: $I \models \&g[\vec{y}](\vec{x})$ iff $e_{\&g[\vec{y}]}(\vec{y}) \in I$
    - minimality checking: as $I$ is wrt. $\hat{P}$ is founded, can be often skipped
  - **plugin architecture** (C++ code), use dlv

# Implementation /2

- dlvhex2




- **conflict-driven clause learning**
- **plugin architecture** (C++, Python), gringo and clasp
- value invention: liberal domain-expansion safety (2013)
- minimality: unfounded set checking (2012)
- partial input evaluation (2016)

# Grounding and Safety

- External atoms may introduce new constants: value invention.
- $\Rightarrow$ Can lead to programs which cannot be finitely grounded.

**Example**

$$P = \left\{ \begin{array}{l} r_1 : start(s). \\ r_2 : scc(X) \leftarrow start(X). \quad r_3 : scc(Y) \leftarrow scc(X), \&edge[X](Y). \end{array} \right\}$$

- Solution: syntactic restrictions (safety)
  - allow output-variables of external atoms not occurring in ordinary positive atoms $(Y)$
  - traditionally: strong safety: no involvement in positive cycles
  - essentially, this means no recursive value invention!
  - but: overly restrictive (e.g. if external graph known to be finite)
- New approach: *liberal domain expansion (lde) safety*
  - exploit *semantic properties* of external atoms
  - aim to identify *finite groundability even under recursive value invention*
  - *a modular framework in which different properties can be combined*
  - *embraces e.g.* $\omega$*-,* $\lambda$*-, argument-restrictedness, finite domain,* $\Gamma$*-acyclicity*

# Grounding and Safety

- **Ide-safety: main ideas**
  - focus on *attribute positions* $(pred, i)$: *Ide-safe*, if the number of different terms in some grounding $P'$ that preserves $AS(P)$ is finite.
  - *program $P$ is Ide-safe*, if all its attribute positions are Ide-safe
  - exploit both syntactic and semantic criteria to find Ide-safe attributes
  - use *term bounding function (TBF)* as parameters:
    - a TBF identifies variables in rules with finitely many instantiations in $P'$
  - use a *monotone grounding operator* to create an envelope of $P'$ and to qualify TBFs;
  - define Ide-safe $(pred, i)$ in mutual induction with TBFs
  - Example (ctd): in rule $r_3$, the input/output attribute positions $(\&edge_I[X], 1)$ and $(\&edge_O[X], 1)$ are Ide-safe (for a finite graph, trivial TBFs exist)
- **Grounding Ide-safe programs:** interleave an ordinary ASP grounder with external source evaluation
- Technically involved, for details see [E_ *et al.*, 2016a]

# Model Building Framework

- For program evaluation, developed a model building framework [E_ *et al.*, 2016b]
  1. divide program $P$ into *units* $u_i$ with partial order
     - employ a **Generalized Splitting Theorem**
     - respect rule dependencies and atom dependencies
  2. compute $u_i$'s answer sets from those of all $u_j, j = i_1, \ldots i_{n_i}$, s.t. $u_i \rightarrow u_j$
     - combine answer sets $m_{i_1}, \ldots m_{i_{n_i}}$ to an "input" answer set $m_i$ of $u_i$, added as facts $F(m_i)$
     - output all answer sets of $m_i$
  3. obtain the answer sets of $P$ as those of (unique) maximal unit $u_n$
- Maintain a program graph and a model graph
  - Model combination requires some provenance condition on the $m_{i_1}, \ldots, m_{i_{n_i}}$
  - Enables parallelization and model streaming

# Model Building Framework /2

- **Example:** deciding about swimming location

$$P^{EDB}_{swim} = \left\{ \begin{array}{l} location(ind, margB), location(ind, amalB), \\ location(outd, gansD), location(outd, altD) \end{array} \right\}$$

$$P^{IDB}_{swim} = \left\{ \begin{array}{lrl} r_1: & swim(ind) \vee swim(outd) & \leftarrow . \\ r_2: & need(inoutd, C) & \leftarrow \&rq[swim](C). \\ r_3: & goto(X) \vee ngoto(X) & \leftarrow swim(P), location(P, X). \\ r_4: & go & \leftarrow goto(X). \\ r_5: & need(loc, C) & \leftarrow \&rq[goto](C). \\ c_6: & & \leftarrow goto(X), goto(Y), X \neq Y. \\ c_7: & & \leftarrow not\ go. \\ c_8: & & \leftarrow need(X, money). \end{array} \right\}$$

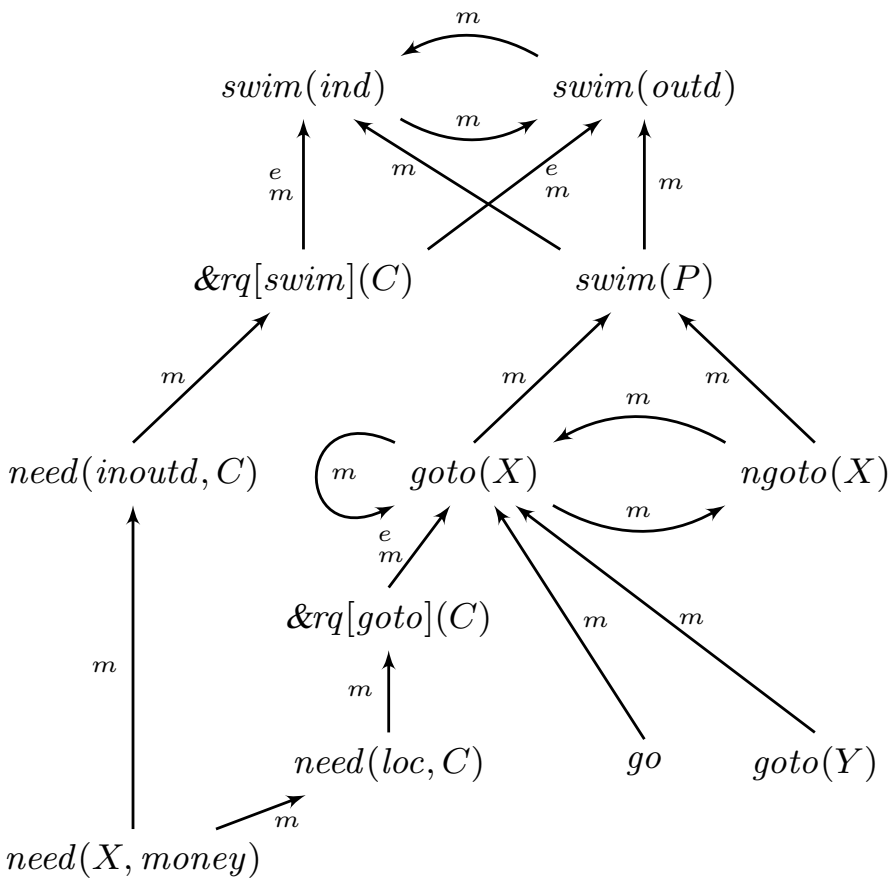
- swimming locations indoors and outdoors
- external atom $\&rq[L](A)$: informally, a given location-choice $L$ has certain required-resource claims $A$

# Model Building Framework /3

- atom dependencies: $\alpha \rightarrow \beta$ "$\alpha$ depends on $\beta$"

  $_m$ = monotonically, $_n$ = nonmonotonically, $^e$ = externally
- atom dependency graph $AD(P)$

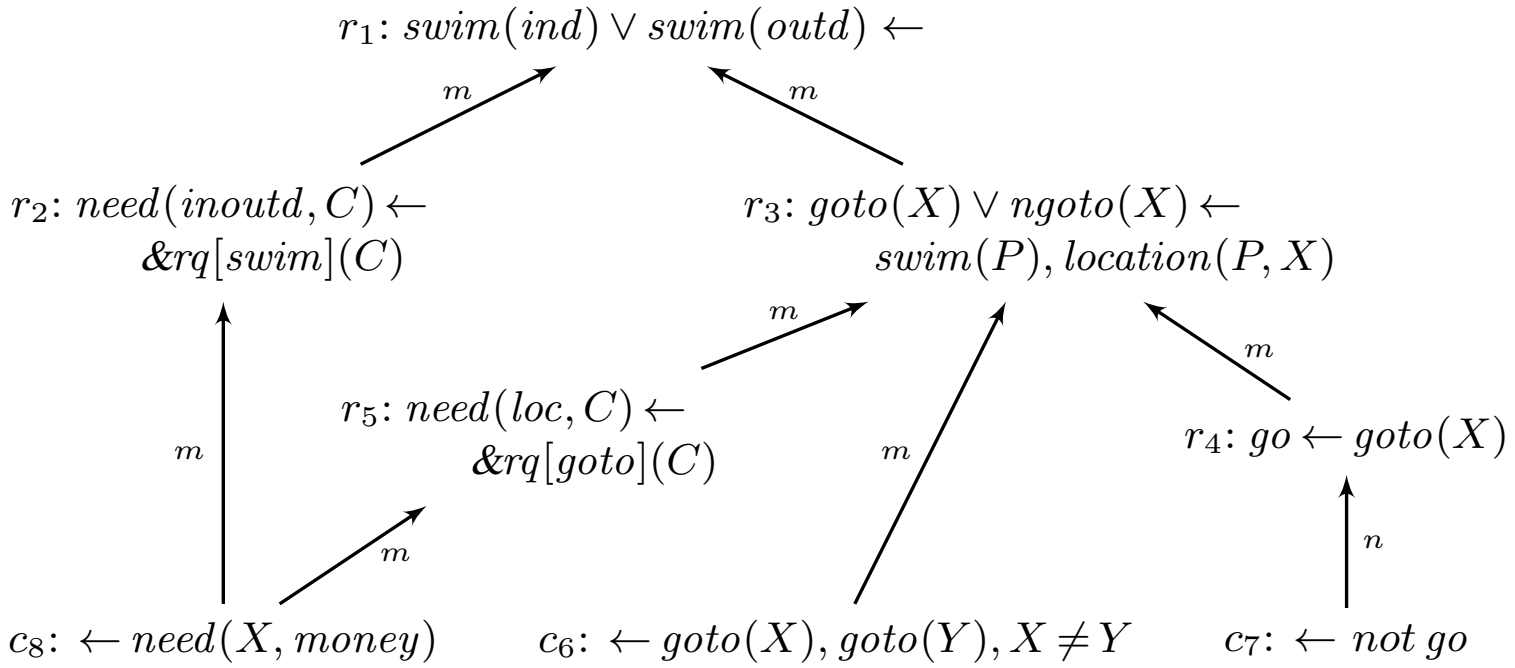
$$
\begin{array}{lrl}
r_1: & swim(ind) \vee swim(outd) & \leftarrow . \\
r_2: & need(inoutd, C) & \leftarrow \&rq[swim](C). \\
r_3: & goto(X) \vee ngoto(X) & \leftarrow swim(P), location(P, X). \\
r_4: & go & \leftarrow goto(X). \\
r_5: & need(loc, C) & \leftarrow \&rq[goto](C). \\
c_6: & & \leftarrow goto(X), goto(Y), X \neq Y. \\
c_7: & & \leftarrow not\ go. \\
c_8: & & \leftarrow need(X, money).
\end{array}
$$



# Model Building Framework /4

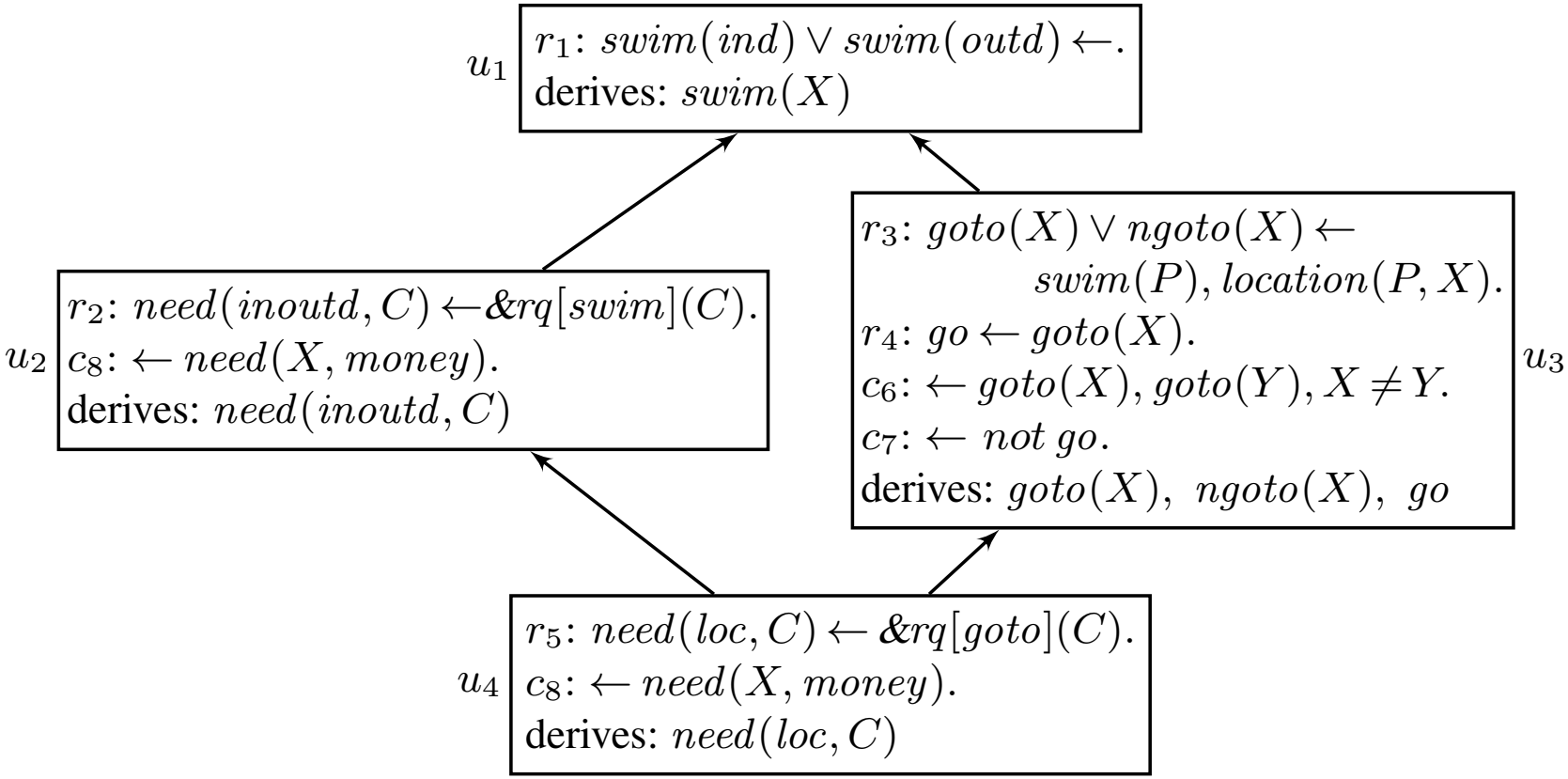
- For program decomposition, use rule dependencies $r \rightarrow s$ (more convenient)

  "application of $r$ depends on the one of $s$"
  - $_m$ = monotonically, $_n$ = nonmonotonically
  - for external atom $\alpha$ in $B(r)$, atom dependencies $\alpha \rightarrow \beta$ matter
- rule dependency graph $RG(P)$



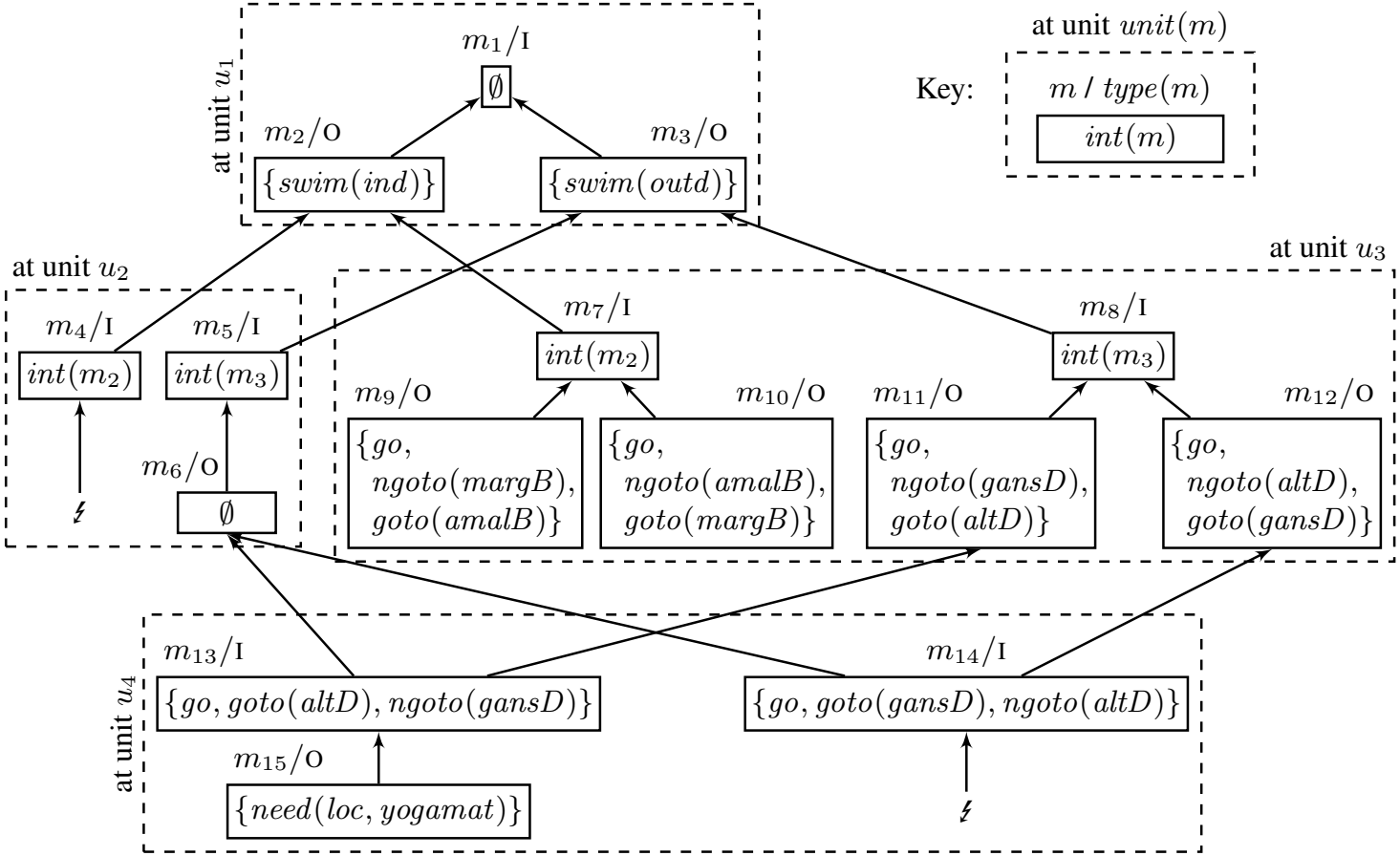
# Model Building Framework /5

- decompose $P$ via $RG(P)$ into *units*
  - closed wrt. cycles
  - may share constraints
  - respect each $r \rightarrow_n s$ always, and $r \rightarrow_m s_1, \ldots, r \rightarrow_m s_k$ at some unit



# Model Building Framework /4

- Model Evaluation: final unit $u_4$




- answer set $\{need(loc, yogamat), go, goto(altD), ngoto(gansD), swim(outd)\}$

# From Black-box to Grey-box

- **Previous Evaluation Bottleneck**
  - External sources were seen as black boxes.
  - Behavior under an interpretation did not allow for drawing conclusions about other interpretations.
  - Algorithms must be very general $\Rightarrow$ room for optimizations limited.
- **Idea**
  - Developers of external sources and/or implementer of HEX-program might have useful additional information.
  - Provide a (predefined) list of possible properties of external sources.
  - Let the developer and/or user specify which properties are satisfied.
  - Algorithms exploit them for various purposes, most importantly:
    - efficiency improvements and
    - language flexibility (reducing syntactic restrictions).
- **Important:**
  User specifies them but does **not** need to know how they are exploited!

# Specifying Properties

- **How to specify them?**
  - During development of external source using the plugin API.
  - As part of the HEX-program using property tags $\langle \cdots \rangle$.

  Example: $\&greaterThan[p, 10]()$ is true, if $\sum_{I \models p(c)} c > 10$,
  for positive integers, it is monotonic

- **Available properties (samples)**
  - **Functionality**: $\&add[X, Y](Z)\langle functional \rangle$
    Adds integers $X$ and $Y$ and is true for their sum $Z$.
    It provides exactly one output for a given input.
  - **Monotonicity in a parameter**: $\&diff[p, q](X)\langle monotonic\ p \rangle$
    Computes the difference of the extensions of $p$ and $q$.
    It is monotonic in predicate parameter $p$.
  - **Well-ordering**: $\&decrement[X](Z)\langle wellordering\ 0\ 0 \rangle$
    Decrements a given integer.
    The $0$-th output is no greater than the $0$-th input (wrt. some ordering).
  - **Three-valued semantics**:
    The external source can be evaluated under partial interpretations.
  - ...

# Exploiting Properties for Efficiency Improvement

- **Conflict-driven Solving**
  - ASP program is internally represented by nogoods
  - Additional nogoods are learned from conflicting interpretations
  - HEX-solver further learns nogoods from external sources which describe parts of their behavior to avoid future wrong guesses.

Example

- We evaluate $\&diff[p,q](X)$ under $I = \{p(a), \neg q(a), \neg p(b), q(b)\}$.
- It is true for $X = a$ (and false otherwise), i.e., $I \models \&diff[p,q](a)$.
- $\Rightarrow$ Learn nogood $N = \{p(a), \neg q(a), \neg p(b), q(b), \neg \&diff[p,q](a)\}$.

- Exploiting Properties
  - Use known properties to shrink nogoods to their essential part.

Example

- $\&diff[p,q](X)$ is monotonic in $p$
- Shrink $N$ to $N' = \{p(a), \neg q(a), q(b), \neg \&diff[p,q](a)\}$.
  (If $p(b)$ turns true, $\&diff[p,q](a)$ is still true $\Rightarrow \neg p(b)$ not needed.)

# Exploiting Properties for Language Flexibility

- Ide-Safety: use properties

**Example (ctd)**

$$P = \left\{ \begin{array}{l} r_1 : start(s). \\ r_2 : scc(X) \leftarrow start(X). \quad r_3 : scc(Y) \leftarrow scc(X), \&edge[X](Y). \end{array} \right\}$$

**Finite domain**: $\&edge[X](Y)\langle finitedomain\ 0\rangle$
Has only finitely many distinct output values; use directive *finitedomain* (0 is first argument).

**Example: set splitting**

$$P = \left\{ \begin{array}{l} r_1 : p(a). \\ r_2 : s_1(X) \leftarrow \&diff[p, s_2](X). \quad r_3 : s_2(X) \leftarrow \&diff[p, s_1](X). \end{array} \right\}$$

**Monotonicity in a parameter**: $\&diff[p, q](X)\langle monotonic\ p\rangle$
The source is monotonic in parameter $p$.

**Antimonotonicity in a parameter**: $\&diff[p, q](X)\langle antimonotonic\ q\rangle$
The source is antimonotonic in $q$ (i.e., shrinking $q$ does not shrink the output).

- Based on this, Ide-safety is concluded by the solver

# Python Programming Interface

- More convenient interface
  Previously only C++ support, but Python preferred by many developers:
  - No overhead due to makefiles, compilation, linking, etc.
  - High-level features.
  - Negligible overhead compared to plugins implemented in C++.
- Implementation of $\&edge[X](Y)$:

```
def edge(x):
  graph=((1,2),(1,3),(2,3))            # simplified implementation
  for edge in graph:                   # search for out-edges of node x
    if edge[0]==x.intValue():
      dlvhex.output((edge[1],))        # output edge target

def register():
  prop = dlvhex.ExtSourceProperties() # inform dlvhex about
  prop.addFiniteOutputDomain(0)        # finiteness of the graph
  dlvhex.addAtom("edge", (dlvhex.CONSTANT, ), 1, prop)
```

# Python Programming Interface /2

- Implementation of $\&diff[p, q](X)$

```
import dlvhex

def diff(p,q):
  for x in dlvhex.getTrueInputAtoms():          # for all true input atoms
    if x.tuple()[0] == p:                        # is it of form p(c)?
      if dlvhex.isFalse(dlvhex.storeAtom(        # is the corresponding q(c) false?
                          (q, x.tuple()[1]))):
        dlvhex.output((x.tuple()[1], ));         # then c is in the output

def register():
  prop = dlvhex.ExtSourceProperties()            # inform dlvhex about
  prop.addMonotonicInputPredicate(0)             # monotonicity/antimonotonicity
  prop.addAntimonotonicInputPredicate(1)         # in the first/second parameter
  dlvhex.addAtom("diff", (dlvhex.PREDICATE, dlvhex.PREDICATE), 1, prop)
```

# Further Improvements

## Availability

- Pre-compiled binaries for major platforms available (previously distributed only as sourcecode).
- Online demo:
  `http://www.kr.tuwien.ac.at/research/systems/dlvhex/demo.php`

## Interoperability

- Support for all features of the ASP-Core-2 standard.
- Support for input/ouput in CSV format (interoperability with tools and spreadsheet programs).

More information: [Redl, 2016], [E_ *et al.*, 2015]

# Constraint HEX-programs (C-HEX)

- **Combining HEX (thus also ordinary ASP) with CP**
  - Extension of HEX-Programs to access a constraint solver
  - Similar in spirit to clingcon
  - *Easy to combine with other external sources*
- **Evaluation techniques**
  - translation of C-HEX programs to (standard) HEX-programs
  - search space pruning techniques
    - conflict-driven learning
    - theory propagation
- **Implementation**
  - plugin for dlvhex, using CP solver GECODE

# CP Syntax and Semantics

**Syntax**

A CSP problem is a triplet $\langle V, D, C \rangle$, where:

- $V$ is a set of variables, $D$ is a domain and
- $C$ is a set of constraints of form $\langle X, R \rangle$, $X \in V^k$ and $R \subseteq D^k$.

**Semantics**

- An interpretation is a mapping $I\colon V \mapsto D$
- A solution is an interpretation $I$ such that for all $\langle \vec{X}, R \rangle \in C$ it holds that $(I(X_1), \ldots, I(X_n)) \in R$.

## Example

Consider the CSP problem $\langle V, D, C \rangle$, where:

$$V = \{x, y\}, D = \{1, \ldots, 10\},$$

$$C = \{\ c_1\colon \langle y, \{7, 8\} \rangle, c_2\colon \langle x, \{1, 2\} \rangle\ ,\ c_3\colon \langle y, \{9, 10\} \rangle\ \ \}$$

The CSP has solution, e.g., $I(x) = 1$ and $I(y) = 7$. The CSP has no solution (i.e., is inconsistent) due to $c_1$ and $c_3$.

# Constraint HEX-Programs: Formal Notions

- HEX **showcase:** adding constraints [De Rosis *et al.*, 2015]

**Definition (C-HEX program)**

A *C*-HEX *program* $P$ is a set of rules $r$ of the form

$$a_1 \vee \cdots \vee a_l \leftarrow b_1, \ldots, b_m, not\, c_1, \ldots, not\, c_n$$

- the $a_i$ are (ordinary) atoms or a constraint expressions
- the $bj$, $c_k$ are atoms, external atoms or a constraint expressions
- a *constraint expression* has the form $l \circ r$, where
  - $l, r$ are arith. expressions, atoms, terms and variables built with $\{+, -, *, /\}$
  - $\circ \in \{\equiv, \not\equiv, >, \geqslant, \leqslant, <\}$ is a comparison operator
  - domain is the integers by default

- **Intuitive Semantics:**
  - Constraint expressions are translated to constraints
  - $\Gamma(P, I)$ denotes the CSP associated to interpretation $I$ of $P$
  - *"$I$ is a constraint answer set of $P$, if it is an answer set of $P$ and the CSP denoted by $\Gamma(P, I)$ is satisfiable"*

# Constraint HEX-Programs: Example

## Example (Alice's Menu)

Alice's restaurant menus are such that

1. food should cost more than drink
2. a menu should cost no more than $20$

This can be encoded as:

$$r_1 : food(P) \leftarrow \&sql[\text{“Select price from Food”}](P)$$
$$r_2 : drink(P) \leftarrow \&sql[\text{“Select price from Drink”}](P)$$
$$r_3 : max\_price(20)$$
$$r_4 : inMenu(F,D) \vee outMenu(F,D) \leftarrow drink(D), food(F)$$
$$r_5 : \leftarrow D > F, inMenu(F,D)$$
$$r_6 : F + D \leqslant P \leftarrow inMenu(F,D), max\_price(P)$$

# Realization in HEX

- **Basic idea:**
  1. encode constraint expressions in $P$ by ordinary atoms
     - $l \circ r$ is encoded as $con(l, \text{“}\circ'', r)$
  2. guess the truth of ordinary atoms corresponding to constraint atoms
     - e.g., $con(l, \circ, r) \vee con(l, \overline{\circ}, r)$, where $\overline{\circ}$ is the negation of $\circ$
  3. check consistency with the CSP $\Gamma(P, A)$ using an external atom
     - $\leftarrow not\ \ \&check[con]()$
- **Implementation**
  - no need for a specialized grounder!
  - develop $\&check[con]()$external atom using GECODE
  - package it as a back-end of DLVHEX
  - extend nogood learning to inconsistent CSPs, theory propagation
- slower than clingcon (price of generality, as recursive external atoms)

# Clingo

- clingo family offers external atoms (cf. `http://potassco.sourceforge.net/`)
  - targets more logic / reasoners,
  - minimality or models restricted, no dynamic input/ recursion through external atoms
  - clingcon is particular "instance" (outgrow)
- clingo 4: Lua + Python interface, multishot solving [Gebser *et al.*, 2014]
- clingo 5 (upcoming) [Gebser *et al.*, 2016]
  - uses extended gringo
  - can specify theory grammars
  - define then concrete "nonground" theories that are handled by gringo

# Clingo /2

```
1  #theory difference {
2    constant    { - : 0, unary };
3    diff_term   { - : 0, binary, left };
4    linear_term { + : 2, unary; - : 2, unary;
5                  * : 1, binary, left;
6                  + : 0, binary, left; - : 0, binary, left };
7    domain_term { .. : 1, binary, left };
8    show_term   { / : 1, binary, left };
9    &dom/0 : domain_term, {=}, linear_term, any;
10   &sum/0 : linear_term, {<=,=,>=,<,>,!=}, linear_term, any;
11   &diff/0 : diff_term, {<=}, constant, any;
12   &show/0 : show_term, directive
13 }.

15 #const n=2.  #const m=1000.
16 task(1..n).  duration(T,200*T) :- task(T).
17 &dom  { 1..m } = start(T) :- task(T).
18 &dom  { 1..m } = end(T)   :- task(T).
19 &diff { end(T)-start(T) } <= D :- duration(T,D).
20 &sum  { end(T) : task(T); -start(T) : task(T) } <= m.
21 &show { start/1; end/1 }.
```

Listing 1: Logic program enhanced with difference and linear constraints (`diff.lp`)

- (1)–(13) is a theory definition with theory (t-) terms (2)-(8) and atoms (9)–(12)
- t-terms have symbol, priority, arity and associativity (if applicable)
- t-atoms have predicate/arity, names of t-terms, t-operators, and occurrence
- rules (17)–(21) use concrete instance of the generic theory terms

# Clingo /3

- **High level interfaces for theory propagation**
  - augment clasp's native propagation with conclusions from theory $T$
  - user can add custom propagation algs externally (like in dlvhex), both *stateful* and *stateless*
- Semantics: "LP modulo theories"
  - distinguish strict and non-strict (program-defined) atoms
  - **$T$-stable model concept**
    - is a regular stable model relative to extension determined by a "solution" $S$ to theory $T$

      informally, $S$ is a set of atoms compliant with $T$, where strict atoms must exactly match, true ones may be turned to false
    - augment program $P$ with facts/constraints/choice facts for $S$
  - note: minimality of models is *not* ensured, even for strict atoms, in case of cyclic reference

    Example: $P = \{p(a) \leftarrow \&id[p](X)\}$

# Clingo's CDCL Modulo Theories

```
(I)  initialize                                   // register theory propagators and initialize watches
     loop
         propagate completion, loop, and recorded nogoods          // deterministically assign literals
         if no conflict then
             if all variables assigned then
(C)              if some δ ∈ Δ_T is violated for T ∈ 𝕋 then record δ  // theory propagators check Δ_T
                 else return variable assignment                     // 𝕋-stable model found
             else
(P)              propagate theories T ∈ 𝕋 // theory propagators may record theory nogoods from Δ_T
                 if no nogood recorded then decide        // non-deterministically assign some literal
         else
             if top-level conflict then return unsatisfiable
             else
                 analyze                          // resolve conflict and record a conflict constraint
(U)              backjump                         // undo assignments until conflict constraint is unit
```

extends basic CDCL in 4 places:

1. initalizatione
2. propagation (on partial assignments):
   - "watched literals" trigger the theory propagator (user defined)
3. final check of total assignments
   - lazy theory propagation possible: delay to complete assignments
4. undo steps upon backjumping

# Evaluation

- Experiments [Gebser *et al.*, 2016]
  - Scheduling problems, using difference logic constraints
  - use different graph algorithms for satisfiability checking, stateless and stateful
  - clear profile: stateful $<$ stateless $<$ native encoding (though details are lacking)
- Purpose of clingo 5: very much for developing domain-specific solvers, "lower level"
- dlvhex, instead is more to the user-level, and targets general external sources (and their combination)

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