The TOP Database Model — Taxonomy, Object-Orientation and Probability

Werner Kießling[1] Thomas Lukasiewicz[1] Gerhard Köstler[1]

Ulrich Güntzer[2]

[1] Lehrstuhl für Informatik II, Universität Augsburg, Universitätsstr.2, 86135 Augsburg, Germany, {kiessling | lukasiewicz | koestler}@uni-augsburg.de

[2] Wilhelm-Schickard-Institut, Universität Tübingen, Sand 13, 72076 Tübingen, Germany, guentzer@informatik.uni-tuebingen.de

Abstract

We propose to extend object-oriented databases by a language to specify *taxonomic knowledge* over so-called *t-classes*, which are compatible with, but more expressive than existent ISA-hierarchies. Separated from, but closely coupled with such taxonomic knowledge we offer *uncertain constraints* in form of a probabilistic language. The information content of such taxonomic and probabilistic knowledge can be visualized naturally by special Hasse-diagrams. A project to build a TOP-prototype using object-oriented technology like OMT and O_2, deductive technology like Datalog_S and CORAL and uncertain technology from our DUCK experience is under way.

1 Introduction

The evolution of database technology from the relational model into object-oriented databases (OODBs) and deductive databases has been pushed forward to a state where stable and usable systems are becoming widely available; also an integration of both paradigms into so-called DOOD-systems can be foreseen. Such systems can deal with various kinds of objects and query mechanisms, however, except for attempts to deal with null-values, only the specification and processing of certain (true/false) information is supported so far. But uncertainty pervades the real world and it seems mandatory for future advanced data models to capture it explicitly and appropriately. Being a topic of interest in AI for quite some period of time, database researchers are picking up interest in the field recently. There is e.g. work in the relational context by Barbará et al. [BGMP92] and for deductive databases by Ng and Subramanian [NS92b], Lakshmanan and Sadri [LS94]. Our own previous work comprises a major project with the so-called DUCK-system for reasoning under a conditional probability model (see e.g. [GKT91], [KTG92], [TKG94]).

Before extending current OODBs towards uncertainty, the following aspects must be considered: Since uncertain knowledge comes in a variety of flavors in the real world, which of its facets should we provide? (See [Som90] for a discussion.) Like other database researchers we decided on the probabilistic model. Next consider inheritance along ISA-hierarchies, being characteristic of current OODBs. This allows limited forms of classification into subclass-relationships, but not reaching the expressiveness of taxonomic classification found in terminological systems (see e.g. [Bra91]). On the other hand it is natural to understand taxonomic knowledge as special case of probabilistic knowledge. Consider the sentences "all dogs are domestic animals" and "at least 70% of all domestic animals are dogs" as examples of taxonomic and probabilistic knowledge respectively.

Guided by these considerations, we propose an extension of OODBs towards the so-called *TOP database model*:

$$\text{TOP} = \text{Taxonomy} + \text{Object-Orientation} + \text{Probability}$$

In the rest of this paper we introduce the TOP model mostly by examples. More details and novel solutions relating to the technical issues of taxonomic and uncertain deduction can be found in [LKKG94].

2 Taxonomic knowledge

Taxonomic knowledge and reasoning is a widely explored field. One of its uses is in terminological reasoning to answer typical questions like "is a class of objects subset of or disjoint to another class of objects". Considering object-oriented databases, it immediately comes to mind that ISA-hierarchies express similar constraints on the extents of classes. However, except for the basic acyclic ISA-graphs supported, more general features to express taxonomic knowledge are missing.

2.1 Syntax and semantics of taxonomic knowledge

We start out with the definition of an expressive language for taxonomic classes, called t-classes.

Definition 2.1 (Syntax of t-class terms) We consider an alphabet $\mathcal{A} := \{\boldsymbol{\emptyset}, \boldsymbol{\mathcal{O}}, B_1, \ldots, B_k\}$ of constants. $\boldsymbol{\emptyset}$ is called *empty t-class term*, $\boldsymbol{\mathcal{O}}$ is called *universal t-class term*. $\mathcal{B}^+ := \{B_1, \ldots, B_k\}$ denotes the set of *positive t-class terms*. $\mathcal{B} := \mathcal{B}^+ \cup \{\overline{B}_1, \ldots, \overline{B}_k\}$ denotes the set of *basic t-class terms*. The set $\mathcal{C}$ of all *conjunctive t-class terms* is the minimal set with $\mathcal{A} \subseteq \mathcal{C}$ and $C, D \in \mathcal{C} \Longrightarrow (CD) \in \mathcal{C}$. The set $\mathcal{G}$ of all *general t-class terms* is the minimal set with $\mathcal{A} \subseteq \mathcal{G}$ and $C, D \in \mathcal{G} \Longrightarrow (CD), (C \cup D), (\overline{C}) \in \mathcal{G}$. For convenience we apply the usual preference rules to omit unnecessary parentheses.

Definition 2.2 (Semantics of t-class terms) Let $\mathcal{D}$ be the (not necessarily finite) set of admissable objects of an OODB schema $\mathcal{S}$. An *interpretation* $\mathcal{J} := (\mathcal{O}, J)$ of the set of general t-class terms $\mathcal{G}$ consists of:

a) A finite set of objects $\mathcal{O} \subseteq \mathcal{D}$ of an OODB over $\mathcal{S}$.

b) A mapping $J : \mathcal{B}^+ \longrightarrow 2^{\mathcal{O}}$. A positive t-class B_i is interpreted as a class instance $J(B_i) = \{o_1, \ldots, o_l\}$ for objects $o_i \in \mathcal{O}$. J is extended to $\mathcal{G}$ by: $J(\boldsymbol{\mathcal{O}}) := \mathcal{O}$, $J(\boldsymbol{\emptyset}) := \emptyset$, $J(CD) := J(C) \cap J(D)$, $J(C \cup D) := J(C) \cup J(D)$ and $J(\overline{C}) := \mathcal{O} \backslash J(C)$. In the sequel, we identify $\boldsymbol{\mathcal{O}}$ with $\mathcal{O}$, $\boldsymbol{\emptyset}$ with $\emptyset$ and B_i with $J(B_i)$.

General t-class terms allow us to formulate all propositional expressions over the alphabet $\mathcal{A}$. Classification among general t-classes can now be achieved by formulating taxonomic knowledge in the following language.

Definition 2.3 (Syntax of taxonomic formulas) Let $D, C, D_1, \ldots, D_m \in \mathcal{G}$. The set of all taxonomic formulas $\mathcal{TF}$ comprises *subclass formulas* $C \subseteq D$, *equality formulas* $C = D$, *disjointness formulas* $D_1 \parallel \ldots \parallel D_m$ and *partition formulas* $D_1 \parallel \ldots \parallel D_m = D$.

Definition 2.4 (Semantics of taxonomic formulas) An interpretation $\mathcal{J} = (\mathcal{O}, J)$ of $\mathcal{G}$ is extended to an interpretation of $\mathcal{TF}$ as follows:

a) $J(C \subseteq D)$, iff $J(C) \subseteq J(D)$,

b) $J(C = D)$, iff $J(C \subseteq D)$ and $J(D \subseteq C)$,

c) $J(D_1 \parallel \ldots \parallel D_m)$, iff $J(D_i) \cap J(D_j) = \emptyset$ for all $i, j \in [1 : m]$ with $i < j$,

d) $J(D_1 \parallel \ldots \parallel D_m = D)$, iff $J(D_1 \parallel \ldots \parallel D_m)$ and $J(D_1 \cup \ldots \cup D_m = D)$.

The notions of models, satisfiability and logical consequence ($\models$) are defined as usual.

Definition 2.5 (Taxonomic knowledge-base) A *taxonomic knowledge-base* consists of a set of taxonomic formulas. Let V be an infinite set of variables, let $F \in \mathcal{TF}$ and $A, B \in \mathcal{C} \cup V$. The expressions $?F$, $?A \subseteq B$ and $?AB = \emptyset$ are called a *taxonomic queries*.

2.2 Integrating taxonomic knowledge and object-oriented databases

Among the various OODB-features, our interest here focuses on ISA-hierarchies, which specify super-/subclass relationships with inheritance. We can easily verify that the relationship B_i *isa* B_j for two classes B_i and B_j of an OODB schema implies a subclass formula $B_i \subseteq B_j$ for two positive t-classes B_i and B_j. On the other hand, many general t-classes and associated taxonomic formulas have no counterpart in existing OODBs, for instance the negated t-class $\overline{B}_i$.
To make full taxonomic reasoning available as an extension of existing OODB technology, we propose the following two-step procedure:

Step 1: ISA-hierarchies of a conventional OODB schema are translated into $\mathcal{TF}$ according to Fig. 1. The diagrams are given in OMT-notation (see [RBP+91]), neglecting attributes and methods. The translation into $\mathcal{TF}$ provides a natural extensional semantics of the OMT-notation, which originally just deals with inheritance relationships without considering the extensions of classes at all.

Step 2 (optional): Full $\mathcal{TF}$ is made available as a means to specify *additional* taxonomic knowledge. This has no impact on the inheritance schema fixed before, it solely effects class extensions.

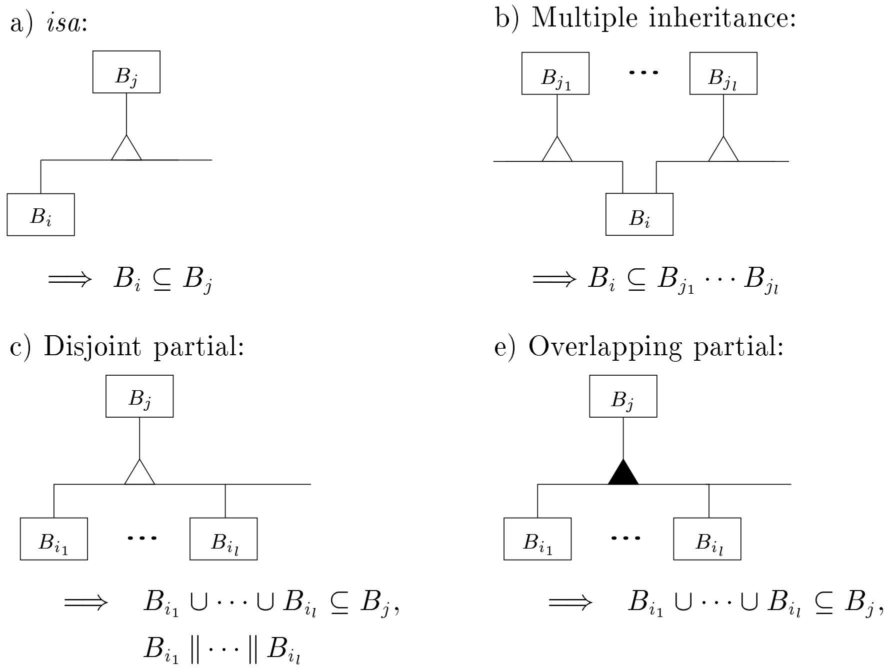

Figure 1: Some OMT-constructs translated into $\mathcal{TF}$-formulas

Example 2.6 *Step 1:* Let's assume an OODB capturing the knowledge that all dogs and cats are domestic animals, that no dog is a cat and that we want to talk about barking animals (see Fig. 2, left side).

Step 2: The additional knowledge that all cats do not bark can be expressed by the taxonomic formula $\mathsf{cats} \subseteq \overline{\mathsf{bark}}$ (see Fig. 2, right side). This leaves the class hierarchy unchanged, which is often desirable to avoid schema evolution problems. □

Example 2.7 For the rest of this paper, we want to consider a slightly different example. Let the alphabet $\mathcal{A}$ be defined by $\mathcal{A} = \{\ \emptyset, \mathcal{O}, \mathsf{dogs}, \mathsf{cats}, \mathsf{domestic}, \mathsf{bark}\ \}$ and let the taxonomic knowledge-base $\mathcal{T}_1$ be given by:

$$\mathcal{T}_1 = \{\mathsf{dogs} \cup \mathsf{cats} \subseteq \mathsf{domestic},\ \mathsf{cats} \parallel \mathsf{dogs},\ \mathsf{bark} \subseteq \mathsf{dogs}\}. \qquad \square$$

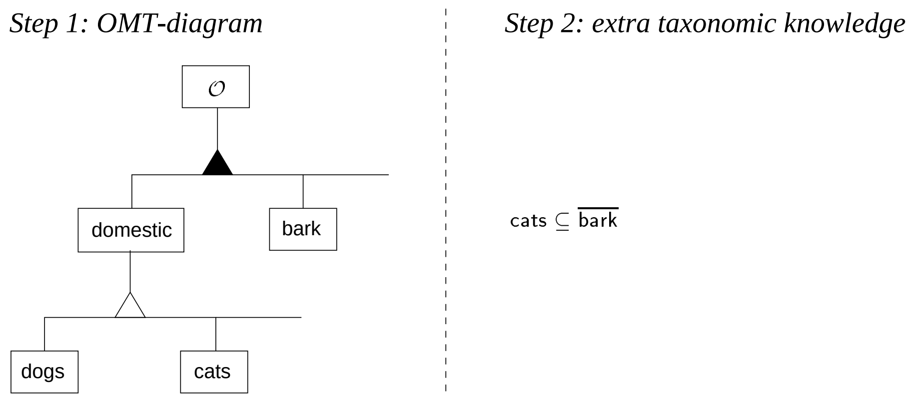

Figure 2: TOP-modeling of an animal world

2.3 Visualization of taxonomic knowledge

In the sequel, let $\mathcal{T}$ be a satisfiable taxonomic knowledge-base. Different conjunctive t-class terms may have the same interpretation due to the axioms of set theory (e.g. $AA = A$, $A\overline{A} = A\emptyset = \emptyset$) and due to taxonomic knowledge (e.g. $A \subseteq B \Longrightarrow A = AB$). Therefore we introduce an equivalence relation $\sim_\mathcal{T}$ on $\mathcal{C}$:[1] $C_1 \sim_\mathcal{T} C_2 :\Leftrightarrow \mathcal{T} \models C_1 = C_2$. Let $\mathcal{C}_\mathcal{T} := \mathcal{C}_{/\sim_\mathcal{T}}$ and $C_\mathcal{T} := [C]_{\sim_\mathcal{T}}$ for all $C \in \mathcal{C}$. The partial order $\subseteq$ on $\mathcal{C}$ is canonically extended to a partial order $\subseteq_\mathcal{T}$ on $\mathcal{C}_\mathcal{T}$ by $A_\mathcal{T} \subseteq_\mathcal{T} B_\mathcal{T} :\Leftrightarrow (AB)_\mathcal{T} = A_\mathcal{T}$.
The finite partially ordered set $(\mathcal{C}_\mathcal{T}, \subseteq_\mathcal{T})$ is a complete lattice (see [Wil82]). It can graphically be represented by a Hasse-diagram.[2] We use the constants of the alphabet $\mathcal{A}$ as a label to the corresponding nodes in the Hasse-diagrams. In this way we can get a representative for all non-labeled nodes: nodes corresponding to $\mathcal{C}_\mathcal{T}$ contain the conjunction of all basic t-class terms attached to upper nodes.

Example 2.8 Fig. 3 shows the Hasse-diagram of $\mathcal{C}_{\mathcal{T}_1}$ of Ex. 2.7. Each node represents an element of $\mathcal{C}_{\mathcal{T}_1}$, e.g. the node labeled with $\mathsf{domestic}$ represents $\mathsf{domestic}_{\mathcal{T}_1} \in \mathcal{C}_{\mathcal{T}_1}$. The node directly lying under the nodes labeled with $\mathsf{domestic}$ and $\overline{\mathsf{cats}}$ represents $(\mathsf{domestic}\,\overline{\mathsf{cats}})_{\mathcal{T}_1} \in \mathcal{C}_{\mathcal{T}_1}$. The taxonomic knowledge is represented by all the edges between the nodes: since the node

[1] For the purpose of this paper it is sufficient to define $\sim_\mathcal{T}$ on $\mathcal{C}$ instead of $\mathcal{G}$ (as done in [LKKG94]).

[2] See [LKKG94] for algorithmic details for constructing Hasse-diagrams.

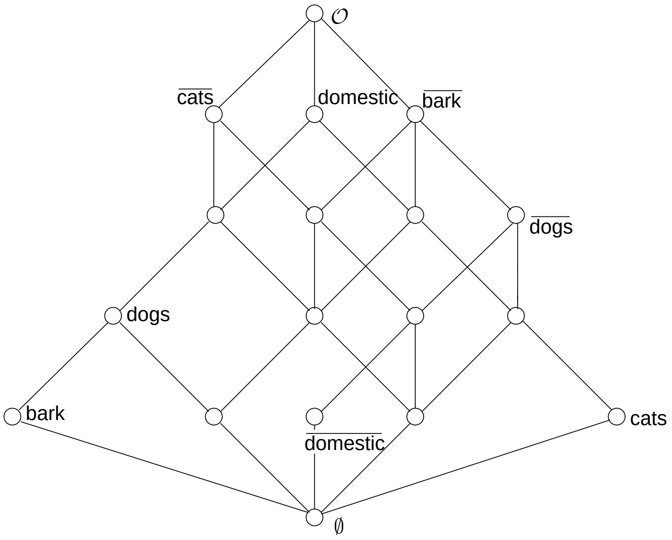

Figure 3: Visualization of the taxonomic knowledge of Ex. 2.7

labeled with dogs is subnode of the node labeled with $\mathsf{domestic}$, we have $\mathsf{dogs}_{\mathcal{T}_1} \subseteq_{\mathcal{T}_1} \mathsf{domestic}_{\mathcal{T}_1}$ and thus $\mathcal{T}_1 \models \mathsf{dogs} \subseteq \mathsf{domestic}$. Since the intersection of the two nodes labeled with dogs and cats is $\emptyset$, we have $\mathcal{T}_1 \models \mathsf{dogs}\,\mathsf{cats} = \emptyset$. The taxonomic query $?\mathsf{domestic} = \mathsf{cats} \parallel \mathsf{dogs}$ yields the answer *No*, since there may be other domestic animals than dogs or cats (in Fig. 3 we have $\overline{\mathsf{domestic}}_{\mathcal{T}_1} \neq (\overline{\mathsf{cats}}\,\overline{\mathsf{dogs}})_{\mathcal{T}_1}$). The query $?\mathsf{cats} \subseteq \overline{\mathsf{bark}}$ yields the answer *Yes*, since all barking animals are dogs and no dog is a cat (in Fig. 3 we have $\mathsf{cats}_{\mathcal{T}_1} \subseteq \overline{\mathsf{bark}}_{\mathcal{T}_1}$). □

3 Probabilistic knowledge

Uncertain knowledge comes in a variety of flavors. We pursue a probabilistic approach which continues our previous work with the DUCK-model.

3.1 Syntax and semantics of uncertain knowledge

Definition 3.1 (Syntax of probabilistic formulas) Let A, B, $C \in \mathcal{C}$ and $x_1, x_2 \in [0,1]$. The set of all probabilistic formulas $\mathcal{PF}$ comprises *pos-*

itive probability statements $pos(A)$ and *uncertain rules* $A \xrightarrow{x_1,x_2} B$.[3] We abbreviate $A \xrightarrow{x,x} B$ by $A \xrightarrow{x} B$.

Definition 3.2 (Semantics of probabilistic formulas) An interpretation $(\mathcal{O}, J, P)$ of $\mathcal{PF}$ consists of an interpretation $(\mathcal{O}, J)$ of $\mathcal{G}$ and a probability measure $P : J(\mathcal{G}) \longrightarrow [0,1]$ for the measure space $(\mathcal{O}, J(\mathcal{G}))$.

a) $J(pos(A))$, iff $P(J(A)) > 0$,

b) $J(A \xrightarrow{x_1,x_2} B)$, iff $P(J(A)) = 0$ or $x_1 \leq P(J(B)|J(A)) \leq x_2$.[4]

The notions of models, satisfiability and logical consequence are defined as usual.

Definition 3.3 (Probabilistic knowledge-base) A taxonomic knowledge-base, extended by a non-empty set of probabilistic formulas, is called a *probabilistic knowledge-base.* Let V be an infinite set of variables. For $A, B \in \mathcal{C} \cup V$, $x_1, x_2 \in [0,1] \cup V$, the expression $?A \xrightarrow{x_1,x_2} B$ is called a *probabilistic rule query.*

Note that in Def. 3.2 we do not commit ourselves a priori to a specific interpretation of probability. Probabilistic formulas may be derived by statistical means or may express subjective beliefs of an agent.[5] Furthermore they can express user-defined (application-dependent) uncertain constraints for the frame of the "real world" to be modeled in an OODB.

3.2 Restrictions on uncertain knowledge

For practical purpose it is sufficient to restrict the interpretations $(\mathcal{O}, J, P)$ of $\mathcal{PF}$ such that $P(J(A)) = 0 \iff J(A) = \emptyset$ for all $A \in \mathcal{C}$. Under this realistic assumption the following correspondence between taxonomic and probabilistic knowledge arises:

$$A \xrightarrow{1} B \text{ iff } A \subseteq B \text{ and } A \xrightarrow{0} B \text{ iff } AB = \emptyset \text{ for } A, B \in \mathcal{C}.$$

Furthermore the extensions of all classes of an OODB are assumed to be different from $\emptyset$ and $\mathcal{O}$. Hence we can assume pos(B) for all $B \in \mathcal{B}$, since $P(J(B)) = 0$ entails $J(B) = \emptyset$ and $J(\overline{B}) = \mathcal{O}$.

[3] In [LKKG94] we additionally consider correlation rules, comparative rules and conditional independencies.

[4] $P(J(B)|J(A))$ is the conditional probability of $J(B)$ under $J(A)$.

[5] The approach of [NS92a] is a representative of the statistical view.

3.3 Visualization of uncertain knowledge

The Hasse-diagrams of $\mathcal{C}_\mathcal{T}$ can be extended by labeled edges and marks to the nodes to represent also uncertain rules and positive probability statements. Since $A \xrightarrow{x_1,x_2} B \Longleftrightarrow A \xrightarrow{x_1,x_2} AB$ and $(AB)_\mathcal{T} \subseteq_\mathcal{T} A_\mathcal{T}$, we can visualize $A \xrightarrow{x_1,x_2} B$ by adding a (dashed, downward) arc from node $A_\mathcal{T}$ to $(AB)_\mathcal{T}$. A positive probability statement $\mathrm{pos}(A)$ is visualized by a node $A_\mathcal{T}$ filled black.

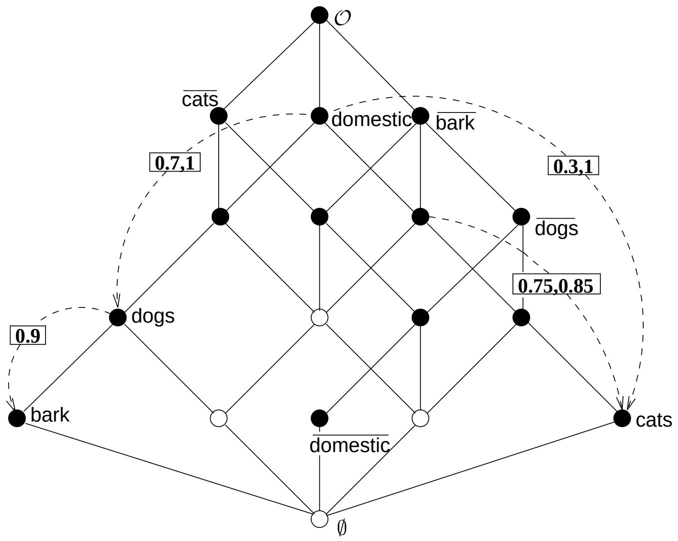

Figure 4: Visualization of the uncertain knowledge of Ex. 3.4

Example 3.4 Gaining the uncertain knowledge that at least 70% of all domestic animals are dogs, that at least 30% of all domestic animals are cats, that 90% of all dogs bark and that between 75% and 85% of all non-barking domestic animals are cats, we can add the set of uncertain rules

$$\mathcal{P} = \{\mathsf{domestic} \xrightarrow{0.7,1} \mathsf{dogs}, \mathsf{domestic} \xrightarrow{0.3,1} \mathsf{cats}, \mathsf{dogs} \xrightarrow{0.9} \mathsf{bark}, \mathsf{domestic}\,\overline{\mathsf{bark}} \xrightarrow{0.75,0.85} \mathsf{cats}\}$$

to our taxonomic knowledge-base $\mathcal{T}_1$ from Ex. 2.7. The extended Hasse-diagram of $\mathcal{C}_{\mathcal{T}_1}$ (compare Ex. 2.8) is given by Fig. 4.[6] The probabilistic rule

[6]From $\mathrm{pos}(B)$ for all $B \in \mathcal{B}$, we can derive a lot of other positive probability statements (see [LKKG94]).

query ?$\mathsf{domestic} \xrightarrow{x_1,x_2} \mathsf{bark}$ yields the answer $x_1 = x_2 = 0.63$. □

3.4 Uncertain deduction

As opposed to [GKT91], uncertain deduction is done on the complete lattice $(\mathcal{C}_\mathcal{T}, \subseteq_\mathcal{T})$, which achieves to reduce the search space enormously. Novel results concerning efficient datastructures for $\mathcal{C}_\mathcal{T}$ and better probabilistic inference rules (which take advantage of the interplay between taxonomic and uncertain knowledge) are presented in [LKKG94], but cannot be stated here due to space limitations.

4 From probabilistic to taxonomic knowledge

Very importantly, new taxonomic knowledge may be generated dynamically during uncertain deduction from probabilistic knowledge, which in turn may further reduce the amount of different t-classes.

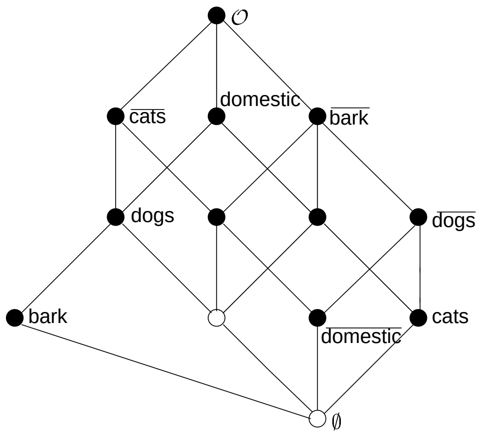

Figure 5: Dynamically reduced Hasse-diagram for Ex. 4.1

Example 4.1 By uncertain deduction we can derive $\mathsf{domestic}\,\overline{\mathsf{cats}} \subseteq \mathsf{dogs}$ from $\mathcal{T}_1 \cup \mathcal{P}$. (Note that $\mathsf{domestic}\,\overline{\mathsf{cats}} \subseteq \mathsf{dogs}$ is equivalent to $\mathsf{domestic} \subseteq \mathsf{cats} \cup \mathsf{dogs}$, which can be concluded from the knowledge that at least 70% of all domestic animals are dogs and that at least 30% of all domestic animals are cats.) Since we already have $\mathcal{T}_1 \models \mathsf{dogs} \subseteq \mathsf{domestic}\,\overline{\mathsf{cats}}$, the taxonomic formula $\mathsf{domestic}\,\overline{\mathsf{cats}} \subseteq \mathsf{dogs}$ yields a further reduction of the concept lattice.

Let $\mathcal{T}_2 = \mathcal{T}_1 \cup \{\mathsf{domestic}\,\overline{\mathsf{cats}} \subseteq \mathsf{dogs}\}$. The Hasse-diagram of $\mathcal{C}_{\mathcal{T}_2}$ is given by Fig. 5 (uncertain rules are omitted, compare Fig. 3). $\square$

5 Summary and Outlook

We have proposed an evolutionary way to extend OODBs by taxonomic and uncertain probabilistic modeling and reasoning. Since the emphasis here was on the modeling side, we could not report on the novel theoretical results and efficient implementation methods ([LKKG94]). The beneficial feedback of uncertain constraints to taxonomic knowledge should provide a strong counterargument to critics that deduction on uncertain knowledge (especially in the form of intervals) just yields too weak uncertain knowledge again ("garbage in, garbage out"). A project to build a prototype of the TOP database system is under way as follows. Applications are specified conventionally through OMT diagrams, which can be mapped semi-automatically into available OODBs like O_2. By extending OMT with taxonomic knowledge and uncertain constraints, we get the full power of the TOP-model. Uncertain deduction will be implemented in Datalog_S ([KG94], [KKTG94]) on top of CORAL ([RSS92]).

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