Towards Approximating Output-Projected Equilibria in Partially Known Multi-Context Systems*

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Abstract — *Heterogeneous nonmonotonic multi-context systems are a framework for knowledge based systems, consisting of decentralized nodes which are interlinked to allow pointwise information exchange. Semantics is defined in terms of equilibria, which are well defined if all system parts are fully known. However, in realistic scenarios, the behavior of some nodes is only known partially. For those scenarios, we introduce an approach to specify partial behavior using partially defined Boolean functions, and show how to under- and overapproximate so-called output-projected equilibria, which are witnesses for equilibria.*

I. INTRODUCTION

Heterogeneous nonmonotonic multi-context systems (MCSs) [1] are a framework for interlinked knowledge based systems, which allows to represent many current formalisms for reasoning about knowledge. MCSs are especially relevant to the Semantic Web, where decentralized nodes interact via pointwise information exchange.

In an MCS, these nodes are called contexts, and semantics is defined in terms of equilibria, which are stable states of beliefs at each context.

Output-projected equilibria are a projection of equilibria to the 'belief interface' between different contexts in a system. They are witnesses for equilibria and can therefore be utilized to incrementally evaluate MCS semantics.

In practice, semantics of some context may be known only partially, as not every knowledge base in the Semantic Web is fully disclosed, either to protect intellectual property, or because a context is a black box system with only vague specifications of its functionality.

To address this scenario, we define a representation for partial knowledge using partial defined Boolean functions, and investigate under- and overapproximation of output-projected equilibria in MCSs using this representation.

II. PRELIMINARIES

A heterogeneous nonmonotonic MCS [1] consists of *contexts*, each composed of a knowledge base with an underlying *logic*, and a set of *bridge rules* which control the information flow between contexts.

The concept of a *logic* we use here is an abstraction for many monotonic and nonmonotonic logics, e.g., classical logic, description logics, modal, default, and autoepistemic logics, circumscription, and logic programs under the answer set semantics.

Formally, a logic $L = (\mathbf{KB}_L, \mathbf{BS}_L, \mathbf{ACC}_L)$ consists of a set of well-formed knowledge bases $\mathbf{KB}_L$, a set of possible belief sets $\mathbf{BS}_L$, and an acceptability function $\mathbf{ACC}_L$. Intuitively, each knowledge base $kb \in \mathbf{KB}_L$ is a set of well-formed formulas, the belief sets are possible consequences of knowledge bases, and $\mathbf{ACC}_L$ provides the semantics, i.e., it takes some kb as argument and returns all belief sets which are consequences.

A *bridge rule* adds information to a context, depending on the belief sets which are accepted at other contexts. Given a collection $L = (L_1, \dots, L_n)$ of logics, an L_k-bridge rule r over L is of the form

$$\begin{aligned}(k : s) \leftarrow (c_1 : p_1), \dots, (c_j : p_j),\\ \mathbf{not}\ (c_{j+1} : p_{j+1}), \dots, \mathbf{not}\ (c_m : p_m). \quad (1)\end{aligned}$$

where k refers to the context receiving information s, and $(v : p)$ denotes belief p at context C_v. We denote by $hd(r)$ the formula s in the head of r.

Definition 1 *A multi-context system $M = (C_1, \dots, C_n)$ is a collection of contexts $C_i = (L_i, kb_i, br_i)$, $1 \leq i \leq n$, where L_i is a logic, $kb_i \in \mathbf{KB}_i$ a knowledge base, and br_i is a set of L_i-bridge rules over $(L_1, \dots, L_n)$.*

We denote by $IN_i = \{hd(r) \mid r \in br_i\}$ the set of bridge rule heads at context C_i. Similarly, the set OUT_i of *output beliefs* contains all beliefs p of context C_i which are contained in the body of some bridge rule in MCS M in the form of a literal '$(i : p)$' or '$\mathbf{not}\ (i : p)$'.

A *belief state* of an MCS $M = (C_1, \dots, C_n)$ is a sequence $S = (S_1, \dots, S_n)$ such that $S_i \in \mathbf{BS}_i$. We denote by $app(R, S)$ the set of all bridge rules $r \in R$ that are applicable in S, where a bridge rule (1) is applicable iff for $1 \leq i \leq j$: $p_i \in S_{c_i}$ and for $j < l \leq m$: $p_l \notin S_{c_l}$.

Equilibrium semantics selects certain belief states of an MCS as acceptable. Intuitively, an equilibrium is a belief state S, where each context C_i takes the heads of all bridge rules that are applicable in S into account, and accepts S_i. The formal definition is as follows.

Definition 2 *A belief state $S = (S_1, \dots, S_n)$ is an* equilibrium *iff, for $1 \leq i \leq n$, the following condition holds:*
$$S_i \in \mathbf{ACC}_i(kb_i \cup \{hd(r) \mid r \in app(br_i, S)\}).$$

The following method for calculating witnesses for equilibria of MCSs is given in [2].

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1. Guess for each context C_i a subset A_i of the output beliefs OUT_i.
2. Interpret each set A_i as a set of output beliefs at context C_i, and use it to evaluate applicability of all bridge rules. As a result, for each context we obtain a set $B_i \subseteq IN_i$ of bridge rule heads, which are added by rules applicable with respect to A_i.
3. Finally, we check for each context C_i, whether it accepts a belief set S_i for knowledge base $kb_i \cup B_i$ s.t. S_i exactly reproduces output beliefs A_i.

Formally, that last check is defined as follows:

$$\exists S_i \in \mathbf{ACC}_i(kb_i \cup B_i) : S_i \cap OUT_i = A_i. \quad (2)$$

Each guess $S' = (A_1, \dots, A_n)$, where that check succeeds, is a witness for an equilibrium $S \in \mathrm{EQ}(M)$. This witness S' corresponds to the projection of equilibrium S to output beliefs of M, and we call S' an *output-projected equilibrium*. Given an MCS M, we denote by $\mathrm{EQ}'(M)$ the set of all output-projected equilibria in M.

In the following, if we refer to 'equilibria' then we always mean 'output-projected equilibria'.

III. Incomplete Information

We now show how to represent part of a context's behavior by a partially defined Boolean function (pdBF). This allows to deal with MCSs where some contexts C_i is not fully specified.

Formally, a pdBF pf is a function from $\mathbb{B}^k$ to $\mathbb{B} \cup \{\star\}$, where $\mathbb{B} = \{0, 1\}$ and '$\star$' stands for undefined. A pdBF is characterized by its sets of true points $T(pf) = \{\vec{\mathrm{I}} \mid pf(\vec{\mathrm{I}}) = 1\}$ and false points $F(pf) = \{\vec{\mathrm{I}} \mid pf(\vec{\mathrm{I}}) = 0\}$.

In the following, we denote by $\vec{\mathrm{I}}$ and $\vec{\mathrm{O}}$ the Boolean vectors (bitmaps) corresponding to sets of bridge rule heads $I \subseteq IN_i$ and output beliefs $O \subseteq OUT_i$, respectively.

To specify partial behavior of some context, we represent the check formalized in (2) by a pdBF pf: arguments of pf are two Boolean vectors corresponding to A_i and B_i, pf returns 1 (resp. 0, or $\star$) if (2) for the corresponding sets is true (resp. false, or not specified).

A pdBF pf is compatible with a certain context C_i, if the true and false points of pf reflect real behavior of C_i.

Definition 3 *Formally, a pdBF pf is compatible with the semantics of context C_i in an MCS M iff $f(\vec{\mathrm{I}}, \vec{\mathrm{O}}) = 1$ implies that there exists some $O' \in \mathbf{ACC}_i(kb_i \cup I)$ s.t. $O = O' \cap OUT_i$, and moreover $f(\vec{\mathrm{I}}, \vec{\mathrm{O}}) = 0$ implies that there is no $O' \in \mathbf{ACC}_i(kb_i \cup I)$ s.t. $O = O' \cap OUT_i$.*

A Boolean function (BF) f is a pdBF with no '$\star$' values. If we know such a BF f, and it is compatible with context C_i, we can use f as an oracle for check (2) in the above algorithm, so we can calculate equilibria without knowing kb_i or $\mathbf{ACC}_i$. We denote by $\mathrm{EQ}'(M[i/f])$ the set of output-projected equilibria of M, obtained using the above algorithm, with f used for the check (2) (instead of using kb_i and $\mathbf{ACC}_i$).

IV. Approximation

Given a pdBF pf compatible with some context C_i, we know that true points correspond to 'yes' answers of check (2) and false points correspond to 'no' answers.

If, for some context C_i, we only know a compatible pdBF pf, we can use it to approximate output-projected equilibria as follows.

We denote by $\underline{pf}$ the BF where we replace all unknown values of pdBF pf by 0. This represents the assumption, that C_i accepts only input/output combinations which are known true points in pf. Therefore, each equilibrium in $\mathrm{EQ}'(M[i/\underline{pf}])$ is an actual equilibrium of M, but we might miss some equilibria due to unknown points.

Similarly, we denote by $\overline{pf}$ the BF where we set all unknown values of pf to 1. Then the set $\mathrm{EQ}'(M[i/\overline{pf}])$ contains all actual equilibria in M, plus possibly some non-actual equilibria, because of wrongly assumed true points.

Formally we obtain the following result.

Proposition 1 *Given an MCS M and a pdBF pf compatible with context C_i of M, the following holds:*

$$\mathrm{EQ}'(M[i/\underline{pf}]) \subseteq \mathrm{EQ}'(M) \subseteq \mathrm{EQ}'(M[i/\overline{pf}]) \quad (3)$$

This result easily generalizes to the case of multiple partially known contexts.

Furthermore, if $S' \in \mathrm{EQ}'(M[i/\underline{pf}]) \cap \mathrm{EQ}'(M[i/\overline{pf}])$, then this S' is an actual equilibrium of M.

V. Discussion

In summary, our results allow to approximate output-projected equilibria of MCSs without requiring full knowledge about all parts of the system. For some cases we can even obtain certain information about actual output-projected equilibria.

To the best of our knowledge, this issue has not been addressed before. A related approach are notions of approximate entailment as in [3], however these notions are motivated by computational efficiency, and they do not consider lack of information about knowledge bases or semantics, as we do.

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References

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