

Towards Approximating Output-Projected Equilibria in Partially Known Multi-Context Systems

Multi-Context Systems: Subject of Investigation

- ▶ What is a **multi-context system**?

A collection of contexts:

$$M = (C_1, \dots, C_n)$$

- ▶ What is a **context** C_i ?

$$C_i = (L_i, \quad \text{a logic} \\ kb_i, \quad \text{the context's knowledge base} \\ br_i) \quad \text{a set of bridge rules (1)}$$

- ▶ What is a **logic** L_i ?

$$L = (KB_L, \quad \text{set of well-formed knowledge bases} \\ BS_L, \quad \text{set of possible belief sets} \\ ACC_L) \quad \text{acceptability function } KB_L \rightarrow 2^{BS_L}$$

Given a knowledge base, ACC_L answers:

Which belief sets are accepted?

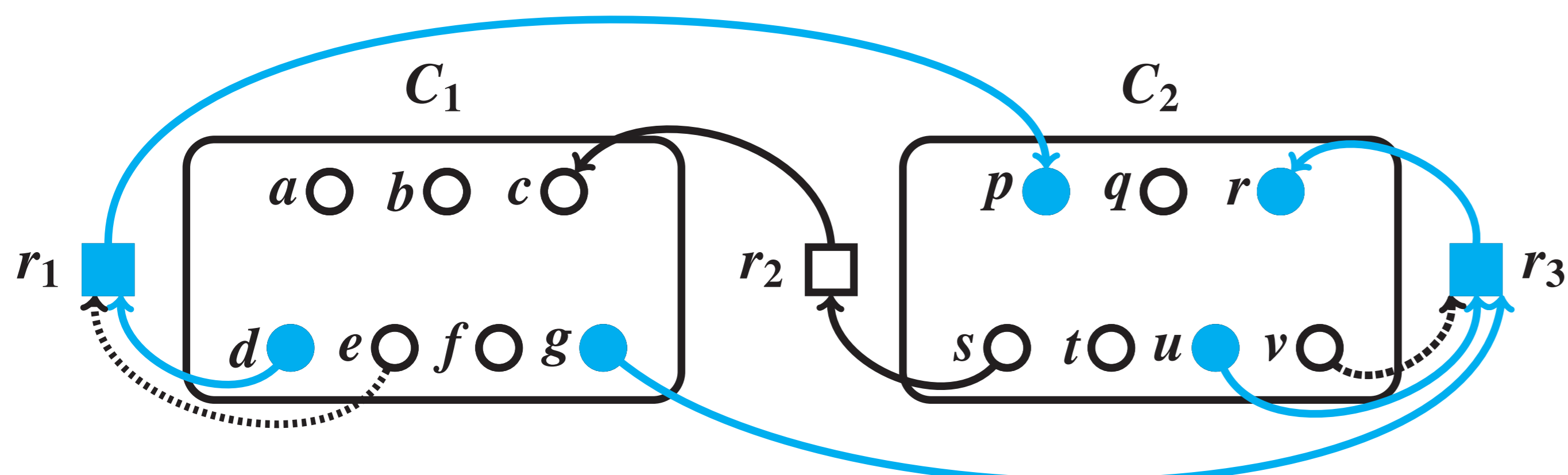
- ▶ What is a **bridge rule**?

$$(k : s) \leftarrow (c_1 : p_1), \dots, (c_j : p_j), \\ \text{not } (c_{j+1} : p_{j+1}), \dots, \text{not } (c_m : p_m). \quad (1)$$

What is it useful for?

Domain:	Contexts:	Bridge Rules:
WWW	Server or Client	Network Connections
Hospital	Patient DB, Diagnosis System, Ontology (SNOMED, NCI, ...)	Knowledge Exchange
In General	Knowledge Processing Units	Information Flow

Example



$$r_1 \quad (2 : p) \leftarrow (1 : d), \text{not}(1 : e). \\ r_2 \quad (1 : c) \leftarrow (2 : s). \\ r_3 \quad (2 : r) \leftarrow (1 : g), (2 : u), \text{not}(2 : v).$$

Equilibrium Semantics

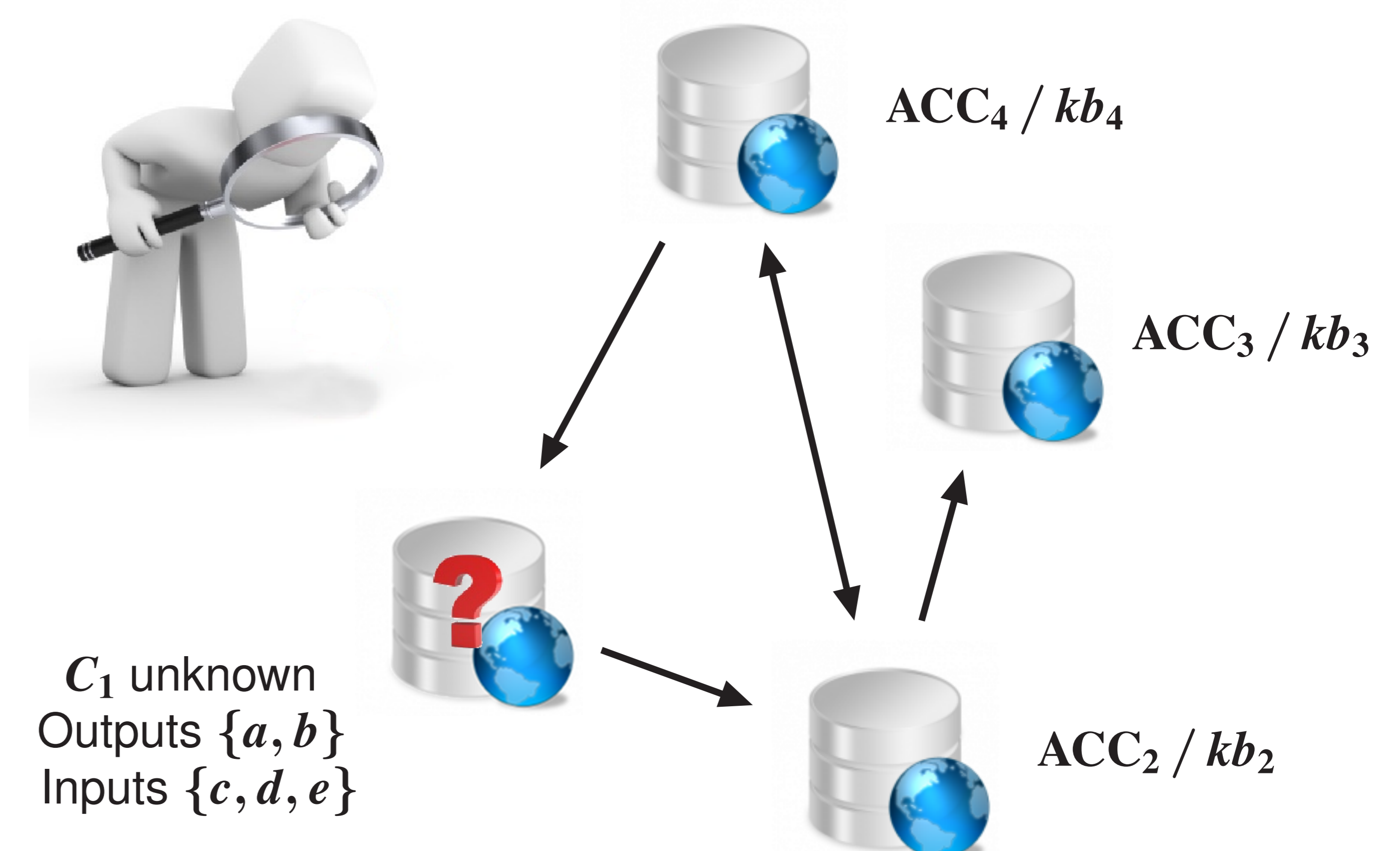
- ▶ Does C_1 accept $\{d, g\}$ without inputs?
⇒ check if $\{d, g\} \in ACC_1(kb_1)|_{\{d, e, g\}}$
- ▶ Does C_2 accept $\{u\}$ with inputs $\{p, r\}$?
⇒ check if $\{u\} \in ACC_2(kb_2 \cup \{p, r\})|_{\{s, u, v\}}$
- ▶ If both is true,
 $(\{d, g\}, \{u\})$ is an **output-projected equilibrium** of M !

References

- ▶ Gerhard Brewka and Thomas Eiter. Equilibria in heterogeneous nonmonotonic multi-context systems. In *AAAI*, pg 385–390, 2007.
- ▶ Thomas Eiter, Michael Fink, Peter Schüller, and Antonius Weinzierl. Finding explanations of inconsistency in nonmonotonic multi-context systems. In *KR*, 2010. to appear.
- ▶ Marco Schaerf and Marco Cadoli. Tractable reasoning via approximation. *Artificial Intelligence*, 74(2):249–310, 1995.

Incomplete Information

- ▶ Contexts may be partially specified
⇒ because of **proprietary content**
⇒ because of **complicated functionality**
- ▶ What can we do?



Boolean Function Representation

for C_1 :

$$\Rightarrow \text{we do not check } \{a, \bar{b}\} \in ACC_1(kb_1 \cup \{c, \bar{d}, e\})|_{\{a, b\}}$$

$$\Rightarrow \text{we check instead } f_1(1, 0, \dots, 1, 0, 1) \stackrel{?}{=} 1$$

where f_1 is a Boolean Function

Partial Knowledge Representation

We use a **partially defined Boolean function** for f_1 :

$$pf_1 : \{0, 1\}^{|IN_1|+|OUT_1|} \rightarrow \{0, 1, \star\}$$

Approximations for Output-Projected Equilibria

We assume

**what we know to be true is true,
nothing else is true.**

- ⇒ $pf_1 = pf$ with **all \star set to 0**
- ⇒ we get an **underapproximation** of output-projected equilibria (fewer than really exist)

We assume

**what we know to be false is false,
nothing else is false.**

- ⇒ $\overline{pf}_1 = pf$ with **all \star set to 1**
- ⇒ we get an **overapproximation** of output-projected equilibria (more than really exist)

Results

We obtain lower and upper bounds on reality!