FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

Towards Approximating Output-Projected Equilibria in Partially Known Multi-Context Systems

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Research Area

- Information and Communication Technology (ICT/IKT)
- Artificial Intelligence
- Knowledge Based Systems
    - Reasoning Systems

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  ⇒ Representing Knowledge

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  - ⇒ Representing Knowledge
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- Expert Systems in Hospitals
- Supporting Systems
  - Medical Diagnosis
  - Filling in Tax Forms
  - Selecting Components for a Computer

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What does it mean?

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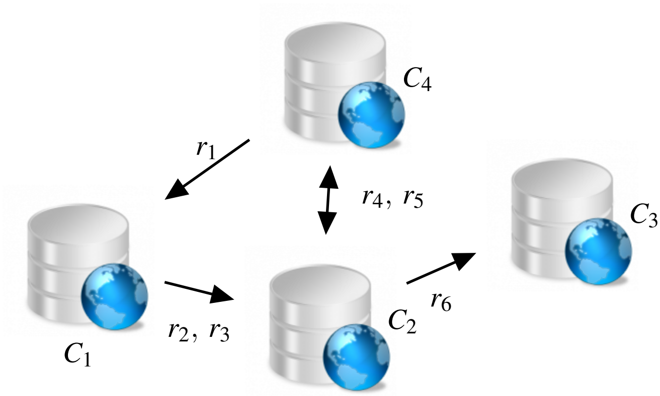
Multi-Context Systems

Knowledge Based System = Context ($C_1, \ldots, C_4$)
Links = Bridge Rules ($r_1, \ldots, r_6$)



introduced by [Giunchiglia & Serafini, 1994], extended by [Brewka & Eiter, 2007]

Bridge Rules

1. look at beliefs of source context
2. calculate if rule becomes active
3. if rule is active, add something to target context

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Example:

$$\begin{aligned}(\mathbf{door} : \mathit{openFor}(\mathit{alice})) \leftarrow\ & (\mathbf{permissionMgr} : \mathit{inGroup}(\mathit{alice}, \mathit{admin})),\\ & (\mathbf{groupDB} : \mathit{TrustedGroup}(\mathit{admin})).\end{aligned}$$

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"If *alice* is in group *admin* of trusted users, then open the **door** for her."

Formally...

- a multi-context system is a collection of contexts:

  $M = (C_1, \dots, C_n)$

- a bridge rule is of the form

$$(k : s) \leftarrow (c_1 : p_1), \dots, (c_j : p_j), \\ \mathbf{not}\ (c_{j+1} : p_{j+1}), \dots, \mathbf{not}\ (c_m : p_m). \quad (1)$$

- a context C_i consists of

$C_i = (L_i,$	a logic (abstraction)
$kb_i,$	the context's knowledge base
$br_i)$	a set of bridge rules (1)

- a logic L is

$L = (\mathbf{KB}_L,$	set of well-formed knowledge bases
$\mathbf{BS}_L,$	set of possible belief sets
$\mathbf{ACC}_L)$	acceptability function $\mathbf{KB}_L \rightarrow 2^{\mathbf{BS}_L}$

- $\mathbf{ACC}_L$ provides semantics: which belief sets are accepted?

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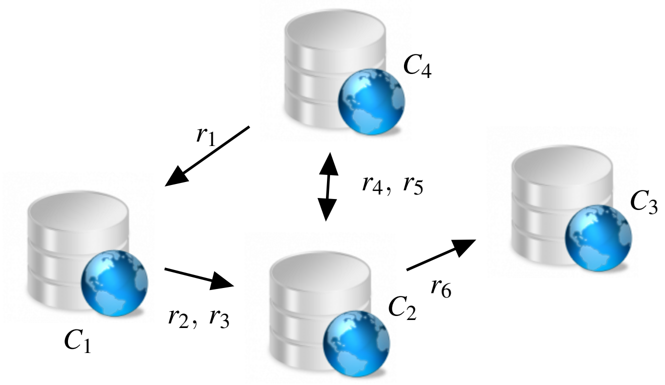
Equilibria

- Equilibrium $S = (S_1, S_2, S_3, S_4)$ is a stable state in the system
- Each context C_i believes a set of beliefs S_i

  $\Rightarrow$ defines semantics of the overall system



There might be no equilibrium!

Output-Projected Equilibria

- are parts of equilibria

⇒ witnesses for equilibria

⇒ witnesses for consistency

⇒ Output-projected equilibria are useful

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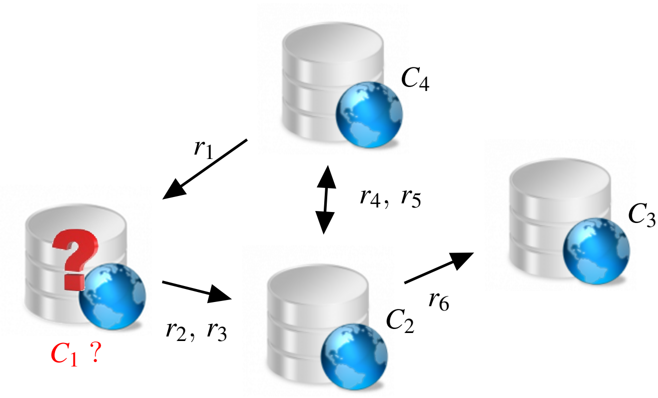
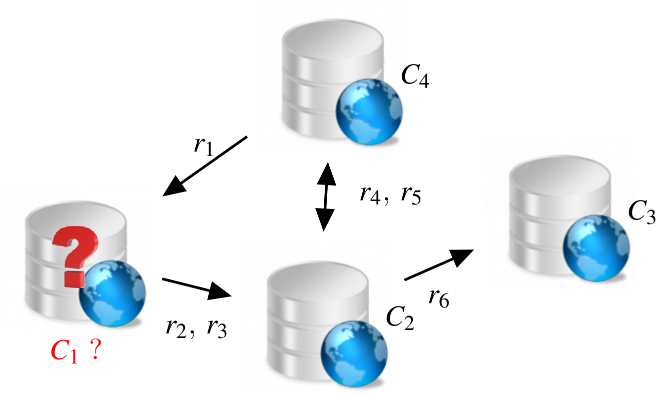
Partial Knowledge



Partial Knowledge

- What if a part of the system is not fully known? (vague specification, proprietary system, secret)
- Is the whole system useless and meaningless?



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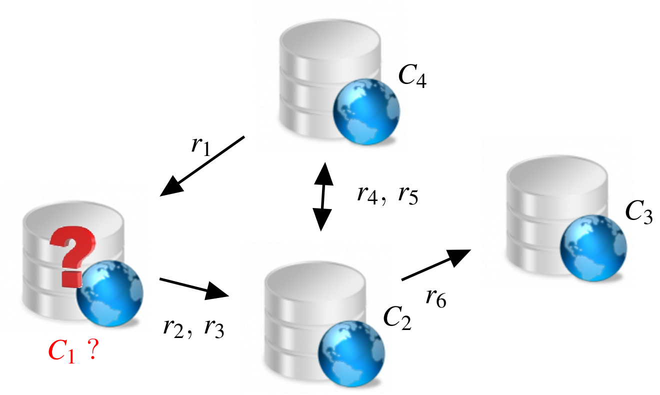

We might know the beliefs of C_1 for some inputs...

Formally…

(for C_1)

$\Rightarrow$ we do not check $\{a,\overline{b}\} \in \mathbf{ACC}_1(kb_1 \cup \{c,\overline{d},e\})|_{\{a,b\}}$

$$\updownarrow \updownarrow \qquad\qquad \updownarrow \updownarrow \updownarrow$$

$\Rightarrow$ instead we check $f_1(1,0,\ldots\ldots\ldots\ldots 1,0,1) \stackrel{?}{=} 1$

where f_1 is a Boolean Function

for partial information we use a partially defined Boolean Function

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Approximation

- MCS: M
- Set of Equilibria: $EQ(M)$
- Set of Output-Projected Equilibria: $EQ'(M)$
- Partially known MCS: $M[i/f_i]$

  $\Rightarrow$ we cannot calculate $EQ'(M[i/f_i])$

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- Approximation of Output-Projected Equilibria:

  Overapproximation: set all unknown points to $1 \Rightarrow EQ'(M[i/\overline{f_i}])$

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Theorem

$EQ'(M[i/\underline{f_i}])$	$\subseteq$	$EQ'(M)$	$\subseteq$	$EQ'(M[i/\overline{f_i}])$
(underapprox.)	$\subseteq$	(reality)	$\subseteq$	(overapprox.)

Conclusions

- We do not fully know the system M

  $\Rightarrow$ we cannot evaluate equilibria of M

- We know partial behavior f_i of unknown context C_i in M
- We show how to evaluate

  $\Rightarrow$ a lower and an upper bound on the real equilibria!

References

- Gerhard Brewka and Thomas Eiter. Equilibria in heterogeneous nonmonotonic multi-context systems. In *AAAI*, pg 385–390, 2007.
- Giunchiglia, F., and Serafini, L. Multilanguage hierarchical logics, or: How we can do without modal logics. *Artificial Intelligence* 65(1):29–70, 1994.
- Thomas Eiter, Michael Fink, Peter Schüller, and Antonius Weinzierl. Finding explanations of inconsistency in nonmonotonic multi-context systems. In *KR*, 2010. to appear.