

Efficient Evaluation of Answer Set Programs with External Sources Based on External Source Inlining

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1. Motivation

HEX-programs extend ASP by **external sources**:

- Rule bodies may contain **external atoms** of the form

$$\wp[q_1, \dots, q_k](t_1, \dots, t_l),$$

where

p ... external predicate name,

q_i ... predicate names or constants: $\tau(\wp, i) \in \{\text{pred, const}\}$,

t_j ... terms.

Semantics:

$1 + k + l$ -ary Boolean **oracle function** f_{\wp} :

$\wp[q_1, \dots, q_k](t_1, \dots, t_l)$ is true under assignment A

iff $f_{\wp}(A, q_1, \dots, q_k, t_1, \dots, t_l) = \mathbf{T}$.

Example: Set Partitioning

$$P = \left\{ \begin{array}{l} d(a_1) \dots d(a_n). \\ r_1: p(X) \leftarrow d(X), \&diff[d, q](X). \\ r_2: q(X) \leftarrow d(X), \&diff[d, p](X). \end{array} \right\}$$

Problem:

- Calling external sources during solving is **expensive**.
- This is in particular the case for **cyclic external sources**.
- Reasons** include both **algorithmic** and **technical overhead** (e.g. caching effects).

Our solution:

- Compile the HEX-program to an ordinary ASP-program by **inlining external sources**.
- To this end, we employ **support sets** (i.e., sets of input atoms which make the external atom true).
- Although inlining leads to an exponential blowup in the **worst case**, it is known that for **certain types of external sources** this is **not the case!**

2. Support Sets

- Let $e = \wp[\vec{y}](\vec{x})$ be an external atom in a program P .

Intuition:

A **positive** (resp. **negative**) support set is a set of **positive or negated input atoms of e** , whose satisfaction implies that e is true (resp. false).

Formally:

A **support set** for e is a consistent set $S_\sigma = S_\sigma^+ \cup S_\sigma^-$ with $\sigma \in \{\mathbf{T}, \mathbf{F}\}$, $S_\sigma^+ \subseteq \mathbf{HB}_c(P)$, and $S_\sigma^- \subseteq \neg\mathbf{HB}_c(P)$ s.t. $A \supseteq S_\sigma^+$ and $A \cap \neg S_\sigma^- = \emptyset$ implies $A \models e$ if $\sigma = \mathbf{T}$ and $A \not\models e$ if $\sigma = \mathbf{F}$ for all assignments A .

Example: Set Partitioning (cont'd)

A positive support set of $\&diff[d, q](b)$ in P is $S_{\mathbf{T}} = \{\mathbf{T}d(b), \mathbf{F}q(b)\}$ since for all A : $A \models d(b)$ and $A \not\models q(b)$ implies $A \models \&diff[d, q](b)$.

- Important concept: complete families of support sets:**

A family (=set) of support sets \mathcal{S}_σ for external atom e is **complete**, if it contains **all possibilities** how to satisfy resp. falsify the external atom. Formally:

A **positive resp. negative family of support sets** \mathcal{S}_σ with $\sigma \in \{\mathbf{T}, \mathbf{F}\}$ for external atom e is a set of positive resp. negative support sets of e ; \mathcal{S}_σ is **complete** if for each assignment A with $A \models e$ resp. $A \not\models e$ there is an $S_\sigma \in \mathcal{S}_\sigma$ s.t. $A \supseteq S_\sigma^+$ and $A \cap \neg S_\sigma^- = \emptyset$.

3. Inlining of External Atoms – Our Encoding

- Due to **cyclic** and **nonmonotonic external atoms**, inlining is **not trivial** (the formalism is on the **second level of the polynomial hierarchy**).
- Our encoding is based on the **saturation technique**.

A positive external atom e in a program P with a complete family of positive support sets $\mathcal{S}_{\mathbf{T}}$ is inlined as follows (negative ones are handled similarly):

$$P|_e = \{x_e \leftarrow S_{\mathbf{T}}^+ \cup \{\bar{a} \mid \neg a \in S_{\mathbf{T}}^-\} \mid S_{\mathbf{T}} \in \mathcal{S}_{\mathbf{T}}\} \quad (1)$$

$$\cup \left\{ \begin{array}{l} \bar{a} \leftarrow \text{not } a; \bar{a} \leftarrow x_e \\ a \vee \bar{a} \leftarrow \text{not } \bar{x}_e \end{array} \mid a \in I(e, P) \right\} \quad (2)$$

$$\cup \{ \bar{x}_e \leftarrow \text{not } x_e \} \quad (3)$$

$$\cup P|_{e \rightarrow x_e} \quad (4)$$

where \bar{a} is a new atom for each a , x_e and \bar{x}_e are new atoms for e , and $P|_{e \rightarrow x_e} = \bigcup_{r \in P} r|_{e \rightarrow x_e}$ where $r|_{e \rightarrow x_e}$ denotes r with e replaced by x_e .

5. Implementation and Experiments

We implemented our novel **inlining** approach in the **DLVHEX** solver and compared it to two previous evaluation approaches for HEX-programs:

- traditional**: Respect external atoms in the core algorithms.
- sup.sets**: Use support sets only for external atom verification.

We considered several **benchmark problems**, including:

- House problem** (abstraction of configuration problems):

n	all answer sets			first answer set		
	traditional	sup.sets	inlining	traditional	sup.sets	inlining
7	251.68 (81)	83.24 (3)	22.21 (2)	22.25 (2)	3.19 (0)	1.53 (0)
8	266.22 (85)	183.48 (43)	59.54 (11)	61.33 (10)	22.42 (1)	3.10 (0)
9	272.70 (85)	263.01 (85)	86.07 (13)	76.74 (12)	56.57 (12)	6.18 (0)
10	278.26 (83)	275.47 (83)	121.39 (16)	102.86 (12)	98.96 (12)	11.97 (0)
11	292.05 (85)	300.00 (100)	167.00 (45)	158.73 (41)	176.44 (49)	22.52 (0)
12	300.00 (100)	300.00 (100)	180.43 (41)	159.64 (47)	210.52 (51)	40.43 (0)

- DL-programs** (integration of ASP with description logics):

n	all answer sets			first answer set		
	traditional	sup.sets	inlining	traditional	sup.sets	inlining
20	1.08 (0)	0.34 (0)	0.31 (0)	0.34 (0)	0.34 (0)	0.31 (0)
30	27.73 (3)	0.98 (0)	0.34 (0)	5.66 (0)	0.98 (0)	0.34 (0)
40	145.06 (35)	16.68 (2)	0.40 (0)	84.73 (14)	16.74 (2)	0.40 (0)
50	249.78 (76)	80.69 (15)	0.48 (0)	213.45 (60)	80.61 (15)	0.47 (0)
60	285.70 (90)	184.25 (47)	0.57 (0)	265.61 (85)	184.23 (47)	0.57 (0)
70	298.13 (99)	254.00 (74)	0.72 (0)	297.17 (99)	254.06 (73)	0.72 (0)

6. Conclusion and Outlook

Main results:

- Novel evaluation algorithm for HEX-programs and an implementation.
- Experiments show a significant (up to exponential) **speedup**.

Future work:

- Refinements and optimizations of the rewriting.
- Heuristics for deciding when to rewrite.

8. References

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