

## Outline

- 1 Motivation
- 2 The Saturation Technique and its Restrictions
- 3 Deciding Inconsistency of Normal Programs in Disjunctive ASP
- 4 Query Answering over Subprograms
- 5 Discussion
- 6 Conclusion

## Answer Set Programs with Queries over Subprograms

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An ASP program consists of rules of form

$$a_1 \vee \dots \vee a_n \leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_p,$$

An interpretation  $I$  is a set of ground atoms;

it is an answer set of a ground program  $P$ , if  $I$  is a  $\subseteq$ -minimal model of the reduct  $P^I = \{H(r) \leftarrow B^+(r) \mid r \in \Pi, I \not\models b \text{ for all } b \in B^-(r)\}$ .

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### Two (Related) Restrictions

- **Meta-reasoning** about the answer sets of a (sub)program within another (meta)program not inherently supported.

- Despite  $\Sigma_1^2$ -completeness of disjunctive ASP, solving problems from the first level of the polynomial hierarchy is sometimes tricky.

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## Basic idea

- Exploits **disjunctions with head-cycles** to solve **comp-hard problems within ASP**.

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Check if a graph is not 3-colorable.

Consider  $P_{\text{not3col}} = F \cup P_{\text{guess}} \cup P_{\text{check}} \cup P_{\text{sat}}$  where

$$P_{\text{guess}} = \{r(X) \vee g(X) \vee b(X) \mid X \leftarrow \text{node}(X)\}$$

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Is has the answer set  $I_{\text{sat}} = A(P_{\text{not3col}})$  iff the graph  $G$  is not 3-colorable. Otherwise its answer sets are proper subsets of  $I_{\text{sat}}$  and represent 3-colorings.

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⇒ Checks which involve default-negations must be rewritten.

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## Example

Check if a graph has **no** vertex cover  $S$  with size  $|S| \leq k$  for some integer  $k$ .

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This encoding does not work as desired because model  $P_{\text{no}} = A(P_{\text{no}})$  is **unstable**.

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Decide if a ground normal ASP program  $P$  is inconsistent.

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- (3)  $\cup \{\text{headMod}(a) \leftarrow \text{inMod}(a), \text{headMod}(b) \mid b \in B^{-1}(a)\} \mid r \in P, a \in H(r)$
- (4)  $\cup \{\text{noKS} \leftarrow \text{false}(a), \text{headMod}(a) \mid r \in A(K(a))\}$
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However, the comparison of the least model of the reduct to the original guess in rule uses **default-negation**.

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## A Meta-Program for Propositional Programs

## Basic idea

- Use a **meta-program**  $M$  to simulate an ASP solver in disjunctive ASP.
- Encode the ASP program  $P$  to check for inconsistency as facts  $M_{P'}^a$ .
- Evaluate  $M \sqcup M_{P'}^a$  to identify inconsistency or the answer sets of  $P$ .
- (Alternative encodings exist, see e.g. [Eiter and Polleres, 2006]).

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## A Meta-Program for Propositional Programs

## Basic idea

- Use a **meta-program**  $M$  to simulate an ASP solver in disjunctive ASP.
- Encode the ASP program  $P$  to check for inconsistency as facts  $M_{P'}^a$ .
- Evaluate  $M \sqcup M_{P'}^a$  to identify inconsistency or the answer sets of  $P$ .
- (Alternative encodings exist, see e.g. [Eiter and Polleres, 2006]).

## Proposition

For any ground normal logic program  $P$ , we have that

and

- (1) If  $P$  is inconsistent,  $M \sqcup M_{P'}^a$  has exactly one answer set which contains  $\text{noKS}$ ;
- (2) If  $P$  is consistent,  $M \sqcup M_{P'}^a$  has at least one answer set and none of the answer sets of  $M_{P'}^a$  contains  $\text{noKS}$ .

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## A Meta-Program for Propositional Programs

## Definition

We define the **meta-program**  $M = M_{\text{error}} \cup M_{\text{facts}} \cup M_{\text{check}} \cup M_{\text{init}}$ , where:

- (8)  $M_{\text{error}} = \{\text{atom}(X) \leftarrow \text{head}(R, X), \text{atom}(X) \leftarrow \text{body}(R, X), \text{atom}(X) \leftarrow \text{body}(R, X, X)\}$
- (9)  $\cup \{\text{init}(R) \leftarrow \text{head}(R, X), \text{init}(R) \leftarrow \text{body}(R, X), \text{init}(R) \leftarrow \text{body}(R, X, X)\}$
- (10)  $M_{\text{facts}} = \{\text{true}(X) \vee \text{false}(X) \leftarrow \text{atom}(X)\}$
- (11)  $M_{\text{check}} = \{\text{inconsistent}(R) \leftarrow \text{init}(R), \text{false}(X) \leftarrow \text{body}(N(R, X))\}$
- (12)  $\cup \{\text{inconsistent}(R) \leftarrow \text{init}(R), \text{body}(R, X), \text{true}(X)\}$
- (13)  $\cup \{\text{true}(X, I) \vee \text{init}(X, I) \leftarrow \text{true}(X), \text{init}(I), \text{init}(X, I) \leftarrow \text{false}(X), \text{init}(I)\}$
- (14)  $\cup \{\text{body}(R, I) \leftarrow \text{inconsistent}(R)\}$
- (15)  $\cup \{\text{body}(R, I) \leftarrow \text{inconsistent}(R), \text{body}(R, X), \text{false}(X)\}$
- (16)  $\cup \{\text{body}(R, I) \leftarrow \text{head}(R, X), \text{body}(R, X, X), \text{iter}(X, A), \text{iter}(X, L), L \geq I\}$
- (17)  $\cup \{\text{noKS} \leftarrow \text{true}(X), \text{noKS}(R, X) \leftarrow \text{head}(R, X, X)\}$
- (18)  $\cup \{\text{noKS} \leftarrow \text{inconsistent}(R), \text{head}(R, X), \text{false}(X), \text{true}(Y) \leftarrow \text{body}(R, Y)\}$
- (19)  $\cup \{\text{noKS} \leftarrow \text{true}(X), \text{noKS}(X, I) \leftarrow \text{init}(I)\}$
- (20)  $\cup \{\text{noKS} \leftarrow \text{iter}(X, I), \text{iter}(X, I), I \neq b\}$
- (21)  $\cup \{\text{iter}(X, I) \leftarrow \text{false}(X), \text{init}(I), \text{iter}(X, I), \text{iter}(X, I), \text{iter}(X, I), b > I\}$
- (22)  $\cup \{\text{body}(R, I) \leftarrow \text{head}(R, X), \text{iter}(X, I) \leftarrow \text{body}(R, X, I)\}$
- (23)  $M_{\text{init}} = \{\text{true}(X) \leftarrow \text{atom}(X), \text{noKS}, \text{false}(X) \leftarrow \text{atom}(X), \text{noKS}\}$
- (24)  $\cup \{\text{iter}(X, I) \leftarrow \text{atom}(X), \text{init}(I), \text{noKS}, \text{init}(X, I) \leftarrow \text{atom}(X), \text{init}(I), \text{noKS}\}$
- (25)  $\cup \{\text{inconsistent}(R) \leftarrow \text{init}(R), \text{noKS}, \text{inconsistent}(R) \leftarrow \text{init}(R), \text{noKS}\}$
- (26)

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## A Meta-Program for Propositional Programs

To lift the idea to non-ground programs, we exploit function symbols: predicates in the input program become function symbols in the meta-program.

## Definition

For a ground normal logic program  $P$ , we let:

$$M_{\text{reg}}^P = (\text{int}(c) \mid 0 \leq c < |A(P)|) \cup \{\text{head}(r, b) \mid r \in P, b \in H(c)\} \\ \cup \{\text{body}(r, b) \mid r \in P, b \in B^+(c)\} \cup \{\text{body}(N(r, b)) \mid r \in P, b \in B^-(c)\}$$

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## A Meta-Program for Non-Ground Programs

To lift the idea to non-ground programs, we exploit function symbols: predicates in the input program become function symbols in the meta-program.

## Definition

For a (ground or non-ground) normal logic program  $P$ , we let:

$$M_{\text{reg}}^P = (\text{int}(c) \mid 0 \leq c < |A(P)|) \cup \{\text{head}(r, b) \mid r \in P, b \in H(c)\} \\ \cup \{\text{body}(P(c, \vec{v}_i), b) \mid r \in P, b \in B^+(c)\} \\ \cup \{\text{body}(N(c, \vec{v}_i), b) \mid r \in P, b \in B^-(c)\}$$

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## Proposition

For any normal logic program  $P$ , we have that

- (1) If  $P$  is inconsistent,  $M \cup M_{\text{reg}}^P$  has exactly one answer set which contains *noAS*;  
and
- (2) If  $P$  is consistent,  $M \cup M_{\text{reg}}^P$  has at least one answer set and none of the answer sets of  $M^*$  contains *noAS*.

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## A Meta-Program for Non-Ground Programs

## Example

Let  $P = \{f : d(a); r_1 : q(X) \leftarrow d(X), \text{not } p(X); r_2 : p(X) \leftarrow d(X), \text{not } q(X)\}$ .

We have:

$$M_{\text{reg}}^P = \{\text{head}(f, d(a)) \leftarrow \text{head}(r_1(X), q(X)) \leftarrow \text{head}(r_2, d(X)) \\ \cup \{\text{body}(P(r_1(X), d(X)) \leftarrow \text{head}(r_2, d(X)); \text{body}(N(r_1(X), p(X)) \leftarrow \text{head}(r_2, d(X)) \\ \cup \{\text{body}(N(r_2(X), p(X)) \leftarrow \text{head}(r_1, d(X)); \text{body}(P(r_2(X), d(X)) \leftarrow \text{head}(r_1, d(X))\}$$

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## A Meta-Program for Non-Ground Programs

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Let  $P = \{f : d(a); r_1 : q(X) \leftarrow d(X), \text{not } p(X); r_2 : p(X) \leftarrow d(X), \text{not } q(X)\}$ .

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## Outline

- 1 Motivation
- 2 The Saturation Technique and its Restrictions
- 3 Deciding Inconsistency of Normal Programs in Disjunctive ASP
- 4 Query Answering over Subprograms
- 5 Discussion
- 6 Conclusion

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## Query Answering over Subprograms

We reduce brave and cautious queries over subprograms to inconsistency checking.

Observation:

For a normal logic program  $P$  and a query  $q$  we have that

- (1)  $P \models_b q$  iff  $P \cup \{\leftarrow I \mid I \in q\}$  is consistent; and
- (2)  $P \models_c q$  iff  $P \cup \{\leftarrow q\}$  is inconsistent.

Reduction:

Proposition

For a normal logic program  $P$  and query  $q$  we have that

- (1)  $M \cup M_{\text{log}}^{\text{na}}(\leftarrow I \in q)$  is consistent and each answer set contains *noAS* iff  $P \models_b q$ ;
- and
- (2)  $M \cup M_{\text{log}}^{\text{na}}(\leftarrow q)$  is consistent and each answer set contains *noAS* iff  $P \models_c q$ .

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## Query Answering: Language Extension

Now we extend ASP with queries over subprograms:

Definition

A ground query atom is of form  $S \uparrow_r q$ , where  $r \in \{b, c\}$  determines the type of the query,  $S$  is a normal logic (sub)program, and  $q$  is a query over  $S$ .

Query atoms may occur in bodies of ASP programs in place of ordinary atoms.

Intuition:

$S \uparrow_b q$  resp.  $S \uparrow_c q$  is true (wrt. all interpretations  $I$  of  $P$ ) if  $S \models_b q$  resp.  $S \models_c q$ .

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## Query Answering: Language Extension

Now we extend ASP with queries over subprograms:

## Definition

A ground query atom is of form  $S \vdash_r q$ , where  $r \in \{h, c\}$  determines the type of the query,  $S$  is a normal logic (sub)program, and  $q$  is a query over  $S$ .

Query atoms may occur in bodies of ASP programs in place of ordinary atoms.

**Intuition:**  $S \vdash_h q$  resp.  $S \vdash_c q$  is true (wrt. all interpretations  $I$  of  $P$ ) if  $S \models_h q$  resp.  $S \models_c q$ .

**Formally** the semantics of such a program  $P$  uses the following translation:

$$P \mapsto P_{\text{SP}=q} \cup \bigcup_{\text{SP}=q \text{ in } P} (M \cup M_{\text{RG}}^{\text{SP}}(\neg \text{I}(q))) \bigcup_{\text{SP}=q \text{ in } P} (M \cup M_{\text{RG}}^{\text{SP}}(\text{I}(q)))$$

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## Query Answering: Language Extension

## Proposition

For a logic program  $P$  with query atoms, the answer sets of  $P$  and  $P_I$ , projected to the atoms in  $P$ , coincide.

## Example

Suppose  $P_{\text{guess}}$  guesses all edge selections in a graph and  $P_{\text{check}}$  derives *invalid* if the current selection is *not* a Hamiltonian cycle.

## Query Answering: Language Extension

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## Example

Suppose  $P_{\text{guess}}$  guesses all edge selections in a graph and  $P_{\text{check}}$  derives *invalid* if the current selection is *not* a Hamiltonian cycle.

Then

$$P = \{\text{notHamiltonian} \leftarrow P_{\text{guess}} \cup P_{\text{check}} \vdash_c \text{invalid}\}$$

(and thus  $P$ ) has an answer set containing *notHamiltonian* if and only if the graph at hand does not contain a Hamiltonian cycle.

Otherwise the program has at least one answer set but none of the answer sets contains atom *notHamiltonian*.

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## Query Answering:

## Checking Conditions with Default-Negation

In general:

## Steps

- Let program  $P_{\text{guess}}$  span a search space of all objects to check.
- Let  $P_{\text{check}}$  check if the current guess satisfies the criteria and derive atom *ok* in this case.
- Instead manually saturating whenever *ok* is true, just checks if *ok* is cautiously entailed by  $P_{\text{guess}} \cup P_{\text{check}}$ .

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## Outline

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## Discussion

## Alternative Approaches

- Nested HEX-programs** [Eiter et al., 2013]: Based on the ASP-extension of HEX-programs (with access to external sources) rather than plain ASP.

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## Discussion

## Alternative Approaches

- **Nested HEX-programs** [Eiter et al., 2013]: Based on the ASP-extension of HEX-programs (with access to external sources) rather than plain ASP.
- **Modular ASP** [Tartu et al., 2005]: Programs consisting of components but no compilation into a single program.

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- **Manifold programs** [Faber and Woltran, 2011]: Compilation-based using weak constraints.

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- **Stable-unstable semantics** [Bogaerts et al., 2016]: Similar goal (isolating the oracle program) and generalize to the PH, but no dedicated query atoms.

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- **Stable-unstable semantics** [Bogaerts et al., 2016]: Similar goal (isolating the oracle program) and generalize to the PH, but no dedicated query atoms.
- Encoding is also related to **debugging approaches** (e.g. Gabser et al., 2008; Oetsch et al., 2010): Rather than explaining inconsistency we exploit it for query answering.

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## Conclusion

## Two Restrictions of Answer Set Programming

- No meta-reasoning over collections of answer sets.
- Limitations of the saturation technique: difficult to use for ASP laymen and restricted use of default-negation.

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## Conclusion

### Two Restriction of Answer Set Programming

- No meta-reasoning over collections of answer sets.
- Limitations of the saturation technique: difficult to use for ASP laymen and restricted use of default-negation.

### Contribution and Solution

- Encoding for deciding inconsistency of a normal program.
- Encoding for query answering over a normal program.
- A language extension of ASP program with dedicated query atoms.
- More user-friendly alternative to saturation.

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## Conclusion

### Two Restriction of Answer Set Programming

- No meta-reasoning over collections of answer sets.
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### Contribution and Solution

- Encoding for deciding inconsistency of a normal program.
- Encoding for query answering over a normal program.
- A language extension of ASP program with dedicated query atoms.
- More user-friendly alternative to saturation.

### Future Work

- Extension to non-ground queries.
- Implementation and application for nested HEX-program evaluation.

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