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On Equivalence and Inconsistency of Answer Set Programs with External Sources

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1. Answer Set Programs and HEX-Programs

An **Answer Set (ASP)-Program** is a set of rules of kind

$$a_1 \vee \cdots \vee a_k \leftarrow b_1, \dots, b_m, \mathbf{not}\, b_{m+1}, \dots, \mathbf{not}\, b_n,$$

where a_i for $1 \leq i \leq k$ and b_i for $1 \leq i \leq n$ are classical atoms.
An answer set of such a program P is an interpretation I (a set of atoms), which is a subset-minimal model of the GL-reduct P^I.

HEX-programs extend ASP by **external sources**:

- Rule bodies may contain external atoms of the form
  $$\&p[q_1, \dots, q_k](t_1, \dots, t_l),$$
  where
  p ... external predicate name,
  q_i ... predicate names or constants: $\tau(\&p, i) \in \{\mathbf{pred}, \mathbf{const}\}$,
  t_j ... terms.

  Semantics:
  $1 + k + l$-ary Boolean oracle function $f_{\&p}$:
  $\&p[q_1, \dots, q_k](t_1, \dots, t_l)$ is true under assignment A iff $f_{\&p}(A, q_1, \dots, q_k, t_1, \dots, t_l) = \mathbf{T}$.
  Answer sets are defined similarly as for ordinary ASP, but using the FLP-reduct fP^I [Faber et al., 2011] instead of the GL-reduct P^I.

Example: Set Partitioning

$$P = \left\{ \begin{array}{l} d(a_1). \dots d(a_n). \\ r_1 : p(X) \leftarrow d(X), \&diff[d, q](X). \\ r_2 : q(X) \leftarrow d(X), \&diff[d, p](X). \end{array} \right\}$$

2. Motivation

Equivalence of ASP-programs:

- Deciding equivalence of ASP-programs under program extensions received attention in the past.
- Possible application: program transformations and optimizations.

Existing equivalence notions: programs P and Q are called

- strongly equivalent [Lifschitz et al., 2001]
  if $P \cup R$ and $Q \cup R$ have the same answer sets for any program R;
- uniformly equivalent [Eiter and Fink, 2003]
  if $P \cup R$ and $Q \cup R$ have the same answer sets for any set of facts R;
- $\langle \mathcal{H}, \mathcal{B} \rangle$-equivalent [Woltran, 2007]
  if $P \cup R$ and $Q \cup R$ have the same answer sets for all programs $R \in \mathcal{P}_{\langle \mathcal{H}, \mathcal{B} \rangle}$ whose head resp. body atoms come only from $\mathcal{H}$ resp. $\mathcal{B}$.

(The latter subsumes the former ones.)

Question 1: How do these notions generalize to HEX-programs?
Question 2: What can be said about inconsistency of HEX-programs?

Challenge: The support for external atoms and the use of the FLP- instead of the GL-reduct make the extension non-trivial.

Contributions:

- A generalization of the notion of $\langle \mathcal{H}, \mathcal{B} \rangle$-equivalence to HEX-programs, i.e., a formal criterion for deciding if two HEX-programs are $\langle \mathcal{H}, \mathcal{B} \rangle$-equivalent.
- This subsumes strong and uniform equivalence.
- A related criterion for deciding inconsistency of a HEX-program.
- Notably, the notion is also applicable to special cases of HEX-programs, such as well-known ASP extensions, e.g., aggregates, DL-programs and constraint ASP.

3. The Equivalence Criterion

The following result is a generalization of the one by Woltran:

Definition

Given sets $\mathcal{H}, \mathcal{B}$ of atoms, a pair (X, Y) of interpretations is an $\langle \mathcal{H}, \mathcal{B} \rangle$-model of a program P if

(i) $Y \models P$ and for each $Y' \subsetneq Y$ with $Y' \models fP^Y$ we have $Y'|_{\mathcal{H}} \subsetneq Y|_{\mathcal{H}}$; and
(ii) if $X \subsetneq Y$ then there exists an $X' \subsetneq Y$ with $X'|_{\mathcal{H} \cup \mathcal{B}} = X$ such that (X', Y) is $\leq^{\mathcal{B}}_{\mathcal{H}}$-maximal for P.

We denote the set of all $\langle \mathcal{H}, \mathcal{B} \rangle$-models of a program P by $\sigma_{\langle \mathcal{H}, \mathcal{B} \rangle}(P)$.

Theorem (Equivalence of HEX-Programs)

For sets $\mathcal{H}$ and $\mathcal{B}$ of atoms and HEX-programs P and Q, we have $P \equiv_{\langle \mathcal{H}, \mathcal{B} \rangle} Q$ iff $\sigma_{\langle \mathcal{H}, \mathcal{B} \rangle}(P) = \sigma_{\langle \mathcal{H}, \mathcal{B} \rangle}(Q)$.

Proof idea: A technique for external source inlining [Redl, 2017] can be exploited to apply proof ideas by Woltran.

4. The Inconsistency Criteria

We provide two criteria based on models of the reduct and unfounded sets (UFSs) [Faber, 2005], respectively. Let P be a HEX-program. Then:

Theorem (Inconsistency of a Program based on its Reduct)

Program $P \cup R$ is inconsistent for all $R \in \mathcal{P}^e_{\langle \mathcal{H}, \mathcal{B} \rangle}$ iff for each model Y of P there is an $Y' \subsetneq Y$ such that $Y' \models fP^Y$ and $Y'|_{\mathcal{H}} = Y|_{\mathcal{H}}$.

Theorem (Inconsistency of a Program based on Unfounded Sets)

Program $P \cup R$ is inconsistent for all $R \in \mathcal{P}^e_{\langle \mathcal{H}, \mathcal{B} \rangle}$ iff for each model Y of P there is a UFS $U \neq \emptyset$ of P wrt. Y s.t. $U \cap Y \neq \emptyset$ and $U \cap \mathcal{H} = \emptyset$.

The latter theorem is especially useful for solver development since implementations do usually not explicitly construct the reduct.

5. Conclusion and Outlook

Main results:

- Decision criteria for
  (1) equivalence and
  (2) inconsistency of HEX-programs.

Future work:

- Extension of the results to non-ground programs.
- Applications: program transformations for solver optimizations.

6. References

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