Unit 2 – ASP Paradigms and Solvers

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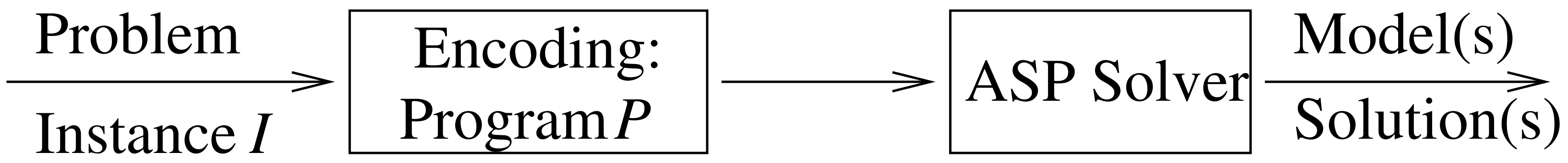
Unit Outline

ASP Paradigm

General idea: stable models are solutions!

Reduce solving a problem instance I to computing stable models of a LP



1. *Encode* I as a (non-monotonic) logic program P, such that solutions of I are represented by models of P
2. *Compute* some model M of P, using an ASP solver
3. *Extract* a solution for I from M.

Variant: Compute multiple models (for multiple / all solutions)

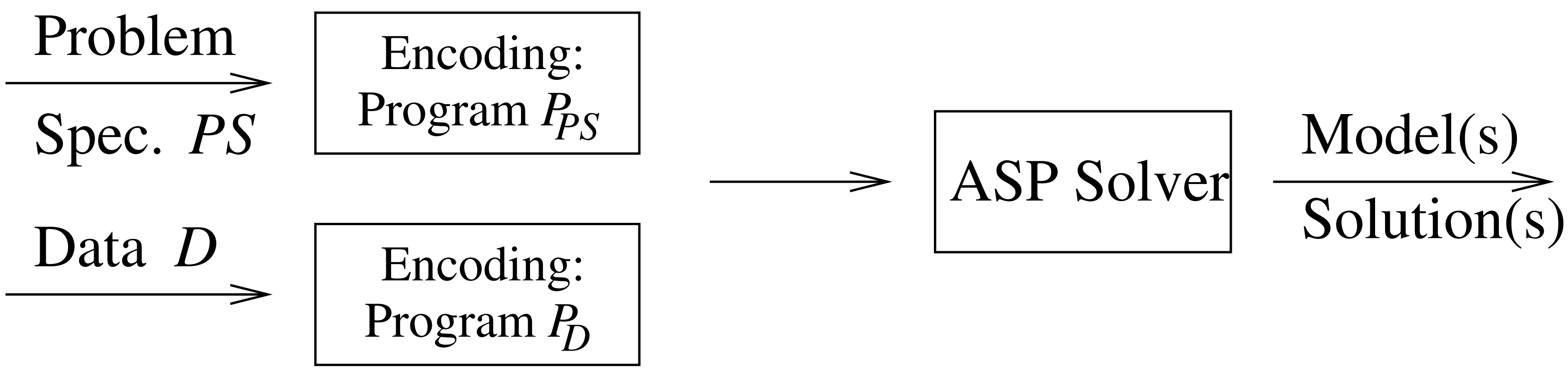
ASP Paradigm (ctd.)

Compared to SAT solving, ASP offers more features:

- transitive closure
- negation as failure
- predicates and variables

Generic problem solving by separating the

- **specification** of solutions ("logic" PS)
- **concrete instance** of the problem (data D)



Use of Double Negation

Defining a predicate p in terms of its negation $-p$

Example (Greatest Common Divisor — Euclid-style)

% base case

$gcd(X,X,X) \leftarrow int(X), X > 1.$

% subtract smaller from larger number

$gcd(D,X,Y) \leftarrow X < Y, gcd(D,X,Y_1), Y = Y_1 + X.$

$gcd(D,X,Y) \leftarrow X > Y, gcd(D,X_1,Y), X = X_1 + Y.$

This is not easy to come up with and needs more care in Prolog.

Use of Double Negation

Defining a predicate p in terms of its negation $-p$

Example (Greatest Common Divisor — ASP-style)

% Declare when D divides a number N.

$divisor(D,N) \leftarrow int(D), int(N), int(M), N = D * M.$

% Declare common divisors

$cd(T,N_1,N_2) \leftarrow divisor(T,N_1), divisor(T,N_2).$

% Single out non-maximal common divisors T

$-gcd(T,N_1,N_2) \leftarrow cd(T,N_1,N_2), cd(T_1,N_1,N_2), T < T_1.$

% Apply double negation: take non non-maximal divisor

$gcd(T,N_1,N_2) \leftarrow cd(T,N_1,N_2), not\ -gcd(T,N_1,N_2).$

The "Guess and Check" Methodology

Important element of ASP: Guess and Check methodology (or Generate-and-Test [Lifschitz, 2002]).

1. Guess: use unstratified negation or disjunctive heads to create candidate solutions to a problem (program part $\mathcal{G}$), and
2. Check: use further rules and/or constraints to test candidate solution if it is a proper solution for our problem (program part $\mathcal{C}$).

From another perspective:

- $\mathcal{G}$: defines the search space
- $\mathcal{C}$: prunes illegal branches.

Further discussion in [Eiter *et al.*, 2000], [Leone *et al.*, 2006] (+ additional component for computing optimal solutions).

Example: 3-Coloring

Problem specification PS: 3-Colorability condition

Problem specification P_{PS}

$$g(X) \vee r(X) \vee b(X) \leftarrow node(X) \quad \} \textbf{ Guess}$$

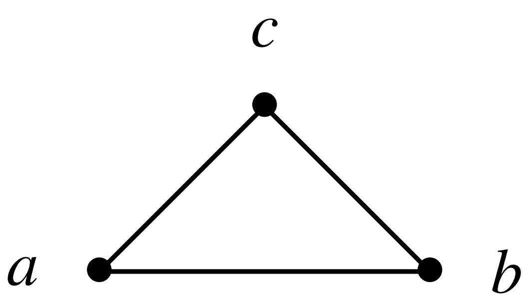
$$\left.\begin{array}{l} \leftarrow b(X), b(Y), edge(X,Y) \\ \leftarrow r(X), r(Y), edge(X,Y) \\ \leftarrow g(X), g(Y), edge(X,Y) \end{array}\right\} \textbf{ Check}$$

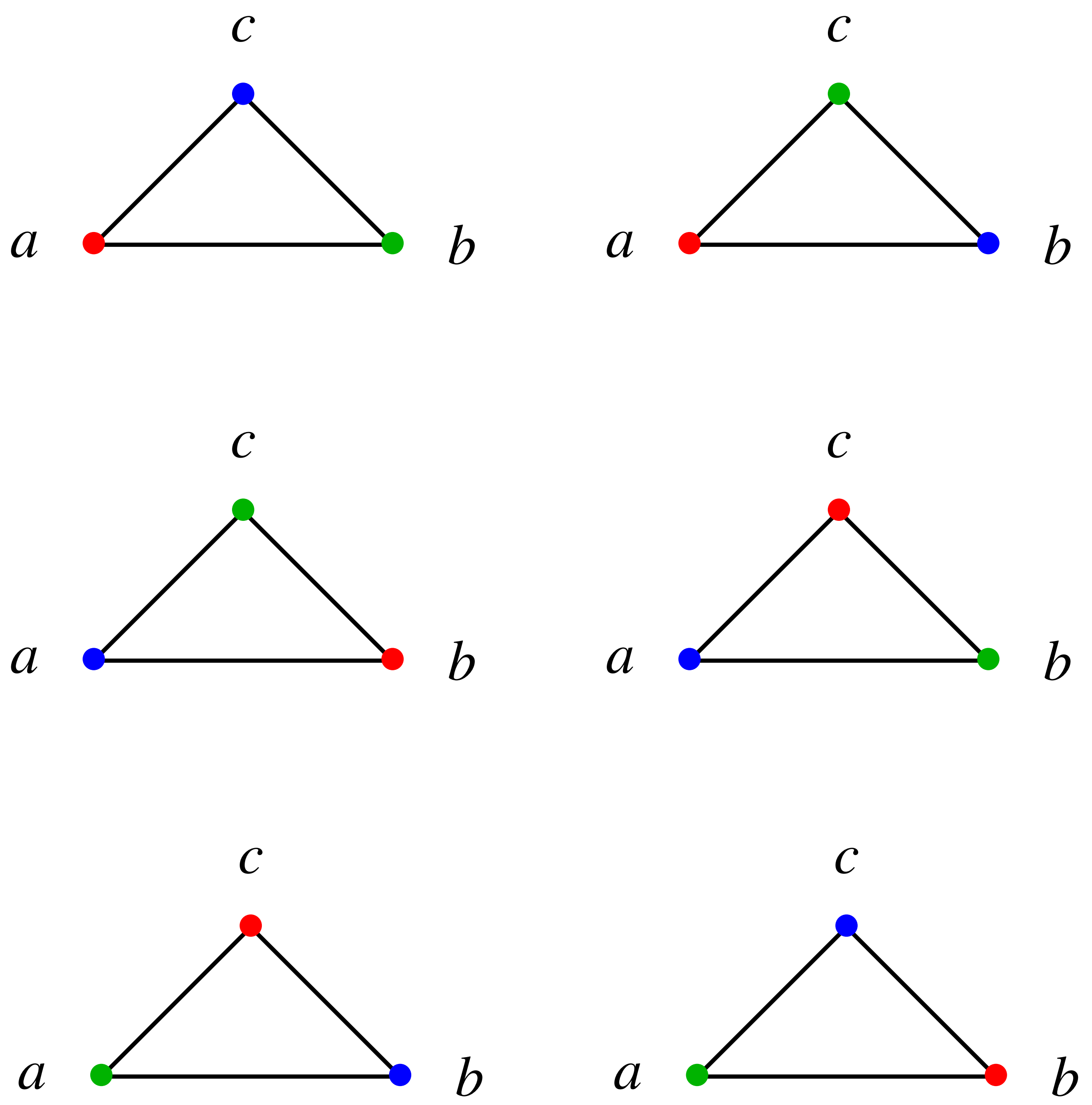
Data P_D: Graph $G = (V, E)$

$$P_D = \{node(v) \mid v \in V\} \cup \{edge(v,w) \mid (v,w) \in E\}.$$

Correspondence 3-colorings $\rightleftharpoons$ models:
$v \in V$ is colored with $c \in \{r, g, b\}$ iff $c(v)$ is in the model of $P_{PS} \cup P_D$.

Example: 3-Coloring (ctd.)





$$P_D = \{node(a), node(b), node(c), edge(a,b), edge(b,c), edge(a,c)\}$$

☞ Run example

Example: Hamiltonian Path/Cycle

Input: A directed graph represented by $node(\_)$ and $edge(\_,\_)$ and a starting node $start(\_)$.

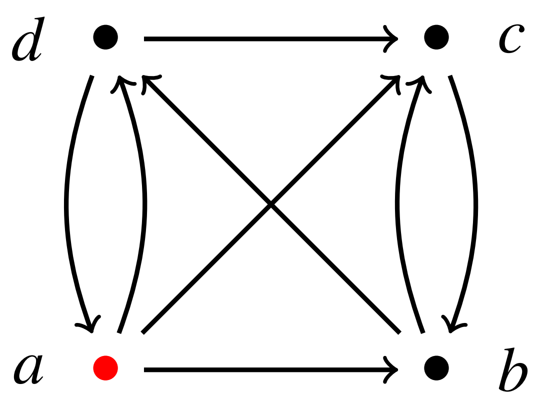
Problem: Find a path/cycle beginning at the starting node which contains all nodes of the graph.

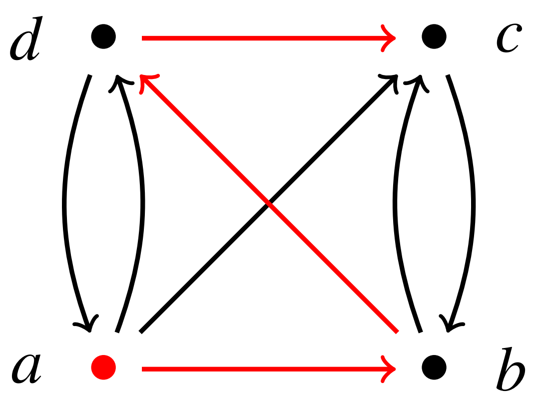
$$inPath(X,Y) \vee outPath(X,Y) \leftarrow edge(X,Y). \quad \} \textbf{ Guess}$$

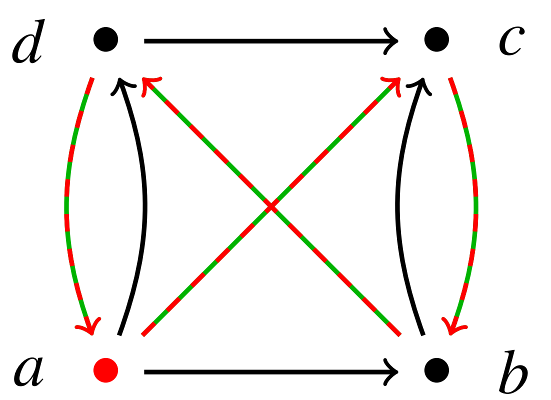
$$\left.\begin{array}{l}
\leftarrow inPath(X,Y),\ inPath(X,Y_1),\ Y \neq Y_1.\\
\leftarrow inPath(X,Y),\ inPath(X_1,Y),\ X \neq X_1.\\
\leftarrow node(X),\ not\ reached(X).\\
\leftarrow not\ start_reached.
\end{array}\right\} \textbf{ Check}$$

$$\left.\begin{array}{l}
reached(X) \leftarrow start(X).\\
reached(X) \leftarrow reached(Y),\ inPath(Y,X).\\
start_reached \leftarrow start(Y), inPath(X,Y).
\end{array}\right\} \textbf{ Auxiliary Predicate}$$

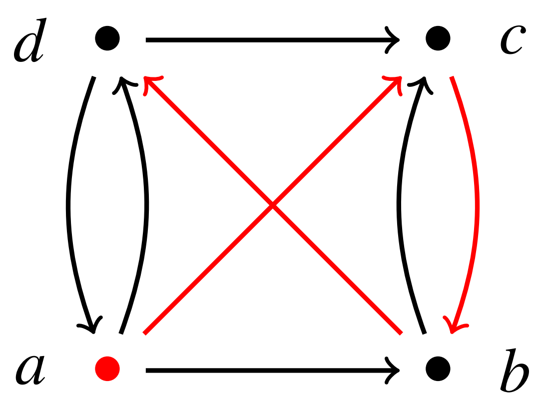
Example: Hamiltonian Path/Cycle (ctd.)

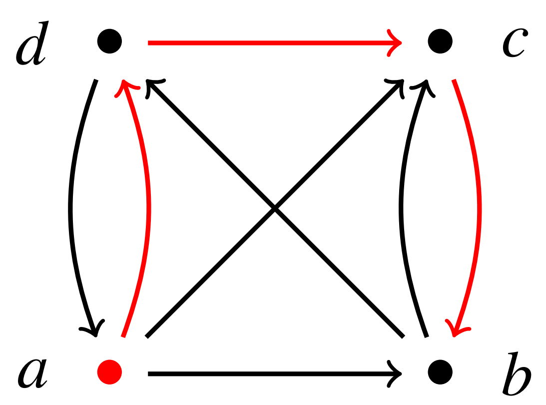






$$
\begin{aligned}
P_D = \{ & node(a), node(b), \\
& node(c), node(d), \\
& edge(a,b), edge(a,c) \\
& edge(c,b), edge(b,c) \\
& edge(b,d), edge(d,c) \\
& edge(d,a), edge(a,d) \\
& start(a) \}
\end{aligned}
$$





☞ Run Hamiltonian Path

☞ Run Hamiltonian Cycle

Example: Course Assignment

Information about members and courses of a computer science dept. cs:

$$
\begin{array}{lll}
member(sam, cs). & course(java, cs). & course(ai, cs). \\
member(bob, cs). & course(c, cs). & course(logic, cs). \\
member(tom, cs). & & \\
likes(sam, java). & likes(sam, c). & likes(tom, ai). \\
likes(bob, java). & likes(bob, ai). & likes(tom, logic).
\end{array}
$$

$$
\begin{array}{rl}
teach(X, Y) \leftarrow & member(X, cs), course(Y, cs), likes(X, Y), not - teach(X, Y). \\
-teach(X, Y) \leftarrow & member(X, cs), course(Y, cs), teach(X_1, Y), X_1 \neq X. \\
has\_course(X) \leftarrow & member(X, cs), teach(X, Y). \\
\leftarrow & member(X, cs), not\ has\_course(X). \\
\leftarrow & teach(X, Y_1), teach(X, Y_2), teach(X, Y_3), \\
& Y_1 \neq Y_2, Y_1 \neq Y_3, Y_2 \neq Y_3.
\end{array}
$$

☞ Run example

Saturation Technique

Saturation technique: check whether all possible guesses satisfy a certain property Pr, like not being a solution to a problem (e.g., 3-uncolorability: co-NP-hard)

To test a property Pr we

- design a program P and an answer set candidate M_{sat} such that M_{sat} is the single answer set of P if the property Pr holds, and
- P has other answer sets (excluding M_{sat}) otherwise.

The construction is such that

- any answer set of P is a subset of M_{sat}, and
- whenever the property is found to hold, any candidate answer set is "saturated" to M_{sat}.

Example: 3-uncolorability

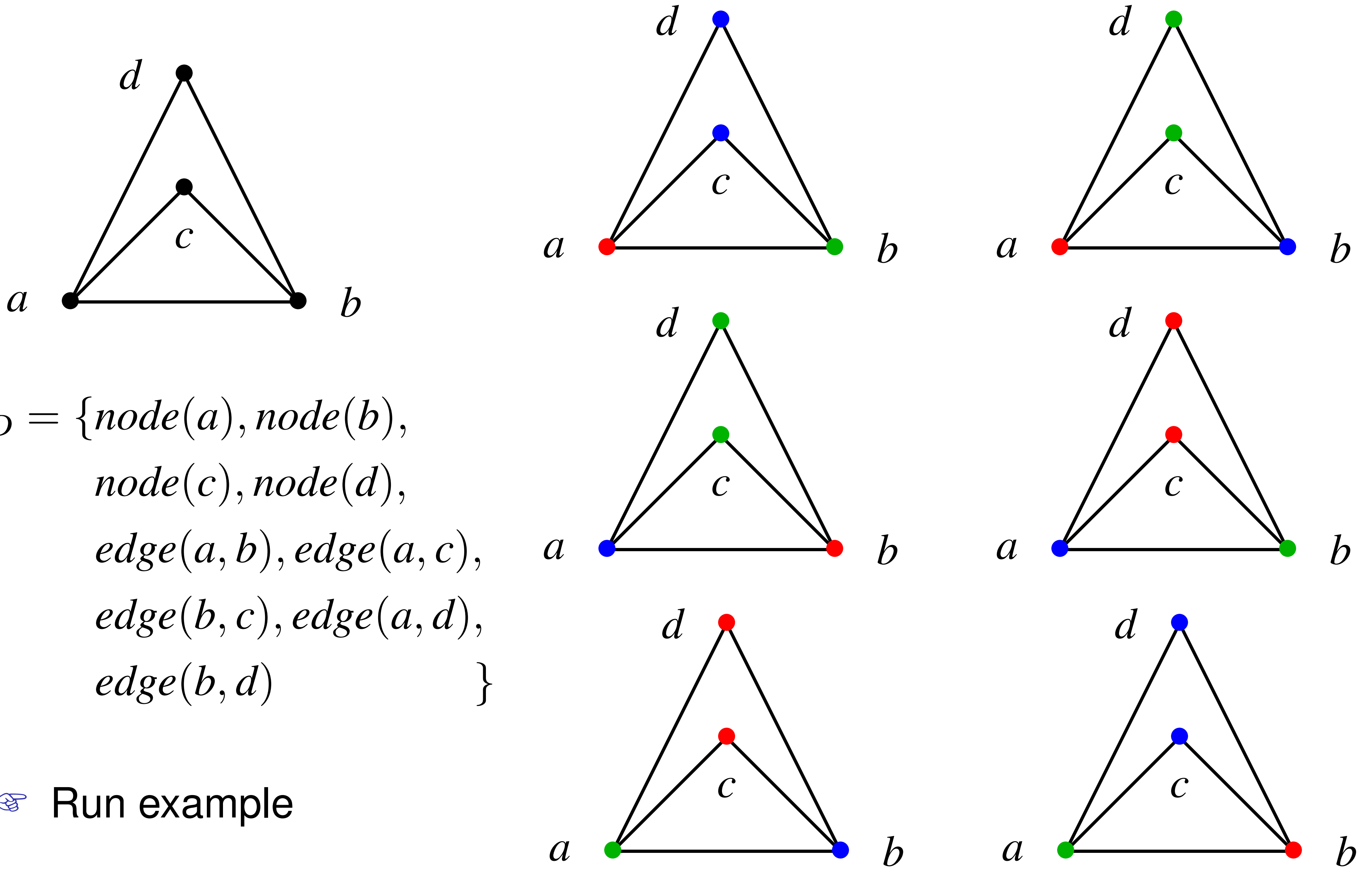
The constraints in the checking part of the 3-colorability program can be replaced by “saturation rules:”

$$\left.b(X) \vee r(X) \vee g(X) \leftarrow node(X).\ \right\} \textbf{Guess}$$

$$\left.\begin{array}{l} non\_col \leftarrow r(X), r(Y), edge(X,Y). \\ non\_col \leftarrow g(X), g(Y), edge(X,Y). \\ non\_col \leftarrow b(X), b(Y), edge(X,Y). \end{array}\right\} \textbf{Check}$$

$$\left.\begin{array}{l} r(X) \leftarrow non\_col, node(X). \\ g(X) \leftarrow non\_col, node(X). \\ b(X) \leftarrow non\_col, node(X). \end{array}\right\} \textbf{Saturize}$$

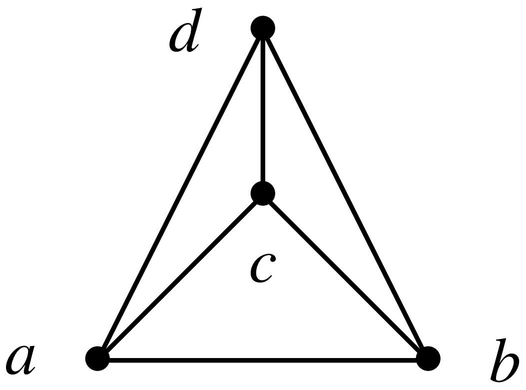
Example: 3-uncolorability (ctd.)



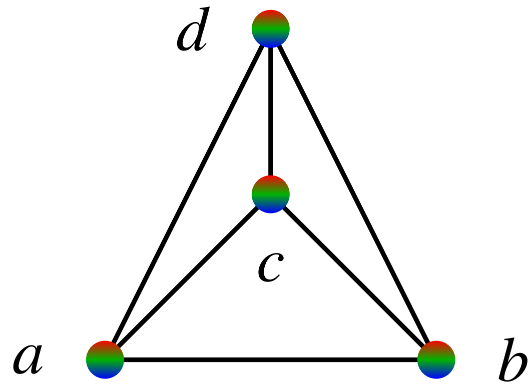
$$
\begin{aligned}
P_D = \{& node(a), node(b),\\
& node(c), node(d),\\
& edge(a,b), edge(a,c),\\
& edge(b,c), edge(a,d),\\
& edge(b,d) \qquad\qquad \}
\end{aligned}
$$

☞ Run example

Example: 3-uncolorability (ctd.)



$$\begin{aligned} P_D = \{ & node(a), node(b), \\ & node(c), node(d), \\ & edge(a,b), edge(a,c), \\ & edge(b,c), edge(a,d), \\ & edge(b,d), edge(c,d)\} \end{aligned}$$



☞ Run example

"Guess and Saturation Check" Paradigm

General design rule:
if we want to check that a property Pr holds for all guesses, we can

1. define the search space of guesses through a subprogram P_{guess}, using disjunctive rules, and
2. define a subprogram P_{check}, which checks Pr for a guess M_g.
3. If Pr holds for M_g, an appropriate set of saturation rules P_{sat} generates the special candidate answer set M_{sat}, otherwise
4. if Pr does not hold for M_g, an answer set results which is a strict subset of M_{sat} (thus preventing that M_{sat} is an answer set).

With additional guessing rules that are not involved in the saturation, we can express Σ_2^p-hard problems, like the strategic companies problem [Leone *et al.*, 2006], [Eiter *et al.*, 2000].

Iteration over a Set

Testing a property for all elements of a set without the use of negation.

This may be needed in some contexts:

- in combination with the saturation technique, or
- when the use of negation could lead to undesired behavior (e.g., in case of cyclic negation).

Example: Reachability for subgraphs

% Guess a subgraph for testing

$edge_1(X,Y) \vee edge_1(Y,X) \leftarrow edge(X,Y), edge(Y,X).$

$edge_1(X,Y) \leftarrow edge(X,Y), not\, edge(Y,X).$

% Compute all reachable nodes

$reached(X) \leftarrow start(X).$

$reached(X) \leftarrow reached(Y),\ edge_1(Y,X).$

% iterate to check if all nodes are reached

$all\_reached \leftarrow last(X), all\_reached\_upto(X).$

$all\_reached\_upto(X) \leftarrow all\_reached\_upto(Y), succ(Y,X), reached(X).$

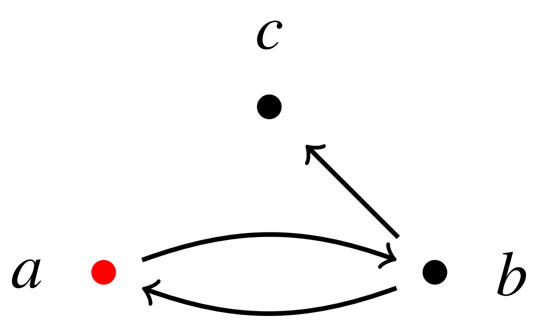
$all\_reached\_upto(X) \leftarrow first(X), reached(X).$

% Saturation rule

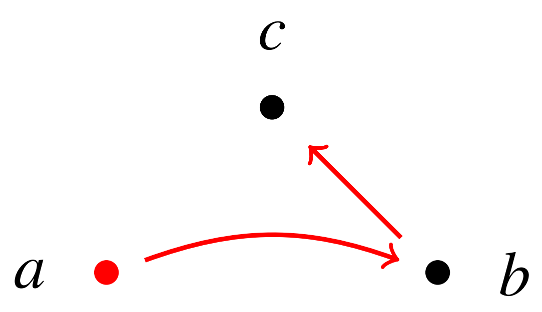
$edge_1(X,Y) \leftarrow all\_reached, edge(X,Y).$

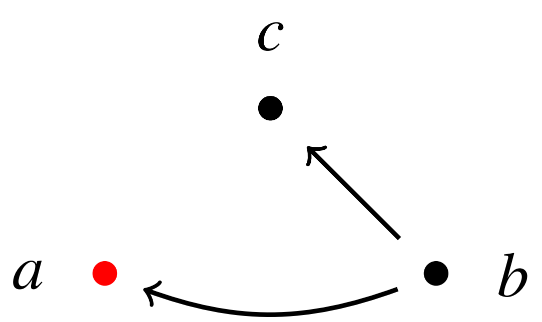
$succ$: (user-) defined predicate

Example: Reachability for subgraphs (ctd.)



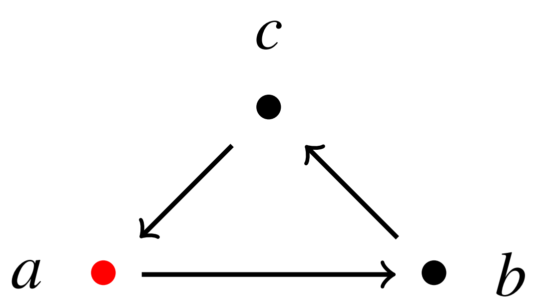
$$P_D = \{node(a), node(b), node(c), edge(a,b), edge(b,c), edge(b,a), start(a)\}$$



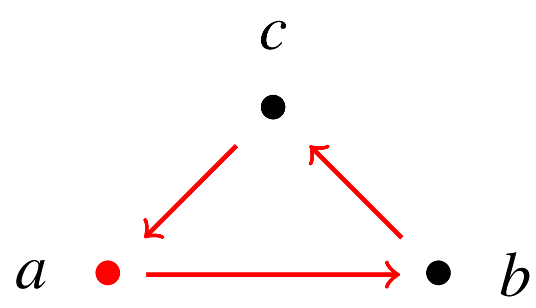


☞ Run example

Example: Reachability for subgraphs (ctd.)



$$P_D = \{node(a), node(b), node(c), edge(a,b), edge(b,c), edge(c,a), start(a)\}$$



☞ Run example

Answer Set Solvers

- NP-/Σ_2^p-completeness: Efficient answer set computation is not easy!
- Need to handle, for applications
  1. complex data (large data volumes)
  2. search
- Efforts to realize tractable fragments
- Many ASP solvers are available (function-free programs)

Approach

- Logic programming and deductive database techniques (for (1))
- SAT/Constraint Programming techniques for (2)

Different sophisticated algorithms have been developed
(like for SAT solving)

Answer Set Solvers

DLV[1]	`http://www.dbai.tuwien.ac.at/proj/dlv/`
Smodels[2]	`http://www.tcs.hut.fi/Software/smodels/`
GnT	`http://www.tcs.hut.fi/Software/gnt/`
Cmodels	`http://www.cs.utexas.edu/users/tag/cmodels/`
ASSAT	`http://assat.cs.ust.hk/`
NoMore(++)	`http://www.cs.uni-potsdam.de/~linke/nomore/`
Platypus	`http://www.cs.uni-potsdam.de/platypus/`
clasp	`http://www.cs.uni-potsdam.de/clasp/`
XASP	`http://xsb.sourceforge.net/`, distributed with XSB
aspps	`http://www.cs.engr.uky.edu/ai/aspps/`
ccalc	`http://www.cs.utexas.edu/users/tag/cc/`

- Several provide a number of extensions to the language described here.
- Answer Set Solver Implementation: see [Niemelä, 2004] tutorial
- ASP Solver competition
- ASPARAGUS Benchmark platform
  `http://asparagus.cs.uni-potsdam.de/`

[1] + many extensions, e.g., DLVEX, DLVHEX, DLV^{DB}, DLT, DLV-Complex
[2] + Smodels_cc

ASP Competition 2009

- ASP competition at the biannual LPNMR conference (started 2007)
- This year, the following systems ranked best:
  1. Potassco `http://potassco.sourceforge.net/`
  2. Claspfolio (Potassco + best options prediction)
  3. DLV `http://www.dbai.tuwien.ac.at/proj/dlv/`

`http://www.cs.kuleuven.be/~dtai/events/ASP-competition/`

SAT Competition 2009

- SAT competition at the annual SAT conference
  `http://www.satisfiability.org/`
- Clasp is an ASP solver from the Potassco suite and performed surprisingly well! `http://www.cs.uni-potsdam.de/clasp/`
- This year, the following systems ranked best in the *crafted instances* category (SAT+UNSAT instances):
  1. Clasp
  2. SATzilla2009_I
  3. SATzilla2009_C
- and for the *crafted instances* category (UNSAT instances):
  1. Clasp
  2. SATzilla2009_C
  3. IUT_BMB_SAT

`http://www.satcompetition.org/`

Architecture of ASP Solvers

Typically, a two level architecture

1. **Grounding Step**
   Given a program P with variables, generate a (subset) of its grounding which has the same models

2. **Model Search**
   More complicated than in SAT/CSP Solving:
   - Candidate generation (classical model)
   - model checking (stability!)
     - for SAT, model checking is in ALOGTIME
     - for normal propositional programs, model checking is P-complete
     - for disjunctive propositional programs, model checking is co-NP-complete

Grounding Step

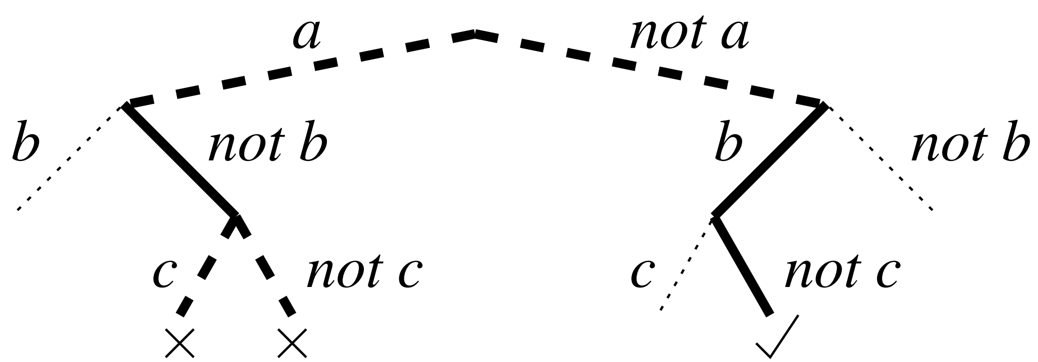
- Efficient grounding is at the heart of current systems
- Sophisticated techniques
  - DLV's grounder (built-in);
  - lparse (Smodels), gringo (clasp)
  - XASP, aspps
- Special techniques used:
  - "*Safe rules*" (DLV): every variable in a rule must occur in an unnegated atom in the body, whose predicate is not "$=$" or any another built-in predicate
  - *domain-restriction* (Smodels)
- Problem: Grounding bottleneck [Eiter *et al.*, 2007]

  Research on nonground evaluation (e.g., [Brüning and Schaub, 1999], [Leone *et al.*, 2006], [Calimeri *et al.*, 2008], [Lin and You, 2008], [Gebser *et al.*, 2008], [Palù *et al.*, 2008]);
  XASP (XSB Extensions)

Model search

- Applied for ground programs.
- Different Techniques:
  - Translations to SAT (e.g. Cmodels, ASSAT)
  - tailored search procedures (Smodels, DLV, NoMore, aspps, clasp)

$a{:-}\ not\ b.$
$b{:-}\ not\ a.$
$c{:-}\ not\ c,\ a.$



  - Backtracking procedures for assigning truth value to atoms
  - Similar to DPPL algorithm for SAT
  - Important: Heuristics (which atom/rule to consider next); involved

- Stability check: unfounded sets, reductions to UNSAT

The DLV System

`http://www.dbai.tuwien.ac.at/proj/dlv/`

- DLV is a state-of-the-art disjunctive answer set solver
- Developed at TU Wien / University of Calabria (since 1996)
- Possesses richer syntax than normal logic programs, resulting in **higher expressiveness**!
- Offers front-ends for specific KR-tasks (diagnosis, planning, etc.).

Features of DLV

- Language: **disjunctive extended logic programs**, no function symbols
- Additionally:
  - bounded integer arithmetic, and comparison built-ins
  - integrity constraints
  - weak constraints
  - aggregates
  - most recently: function symbols (DLV-Complex, r.e.-complete)
- Support for
  - answer set generation
  - *brave* and *cautious* reasoning
  - many extensions: DLVEX, DLVHEX, DLV^{DB}, DLT, DLV-Complex

DLV Syntax

- **Rules**

$$\mathtt{a}_1 \ \mathtt{v} \cdots \mathtt{v} \ \mathtt{a}_n \ \mathtt{:-} \ \mathtt{b}_1, \ldots, \mathtt{b}_k, \ \mathtt{not} \ \mathtt{b}_{k+1}, \ldots, \ \mathtt{not} \ \mathtt{b}_m.$$

where $\mathtt{n} \geq 1, \ \mathtt{m} \geq 0$ and all $\mathtt{a}_i, \mathtt{b}_j$ are atoms or strongly negated atoms (e.g. $-\mathtt{a}$); no function symbols.

- **Integrity Constraints**

$$\mathtt{:-} \ \mathtt{b}_1, \ldots, \mathtt{b}_k, \ \mathtt{not} \ \mathtt{b}_{k+1}, \ldots, \ \mathtt{not} \ \mathtt{b}_m.$$

can be regarded as rules with an empty (false) head.

- **Queries**

$$\mathtt{b}_1, \ldots, \mathtt{b}_k, \ \mathtt{not} \ \mathtt{b}_{k+1}, \ldots, \ \mathtt{not} \ \mathtt{b}_m?$$

Built-in Predicates

- Comparison Predicates:

$$\texttt{<},\ \texttt{>},\ \texttt{<=},\ \texttt{>=},\ \texttt{==},\ \texttt{!=}$$

- Arithmetic Predicates:

$$\#\texttt{int},\ \#\texttt{succ},\ +,\ *$$

$\#\texttt{int}(\texttt{X})$:	$\texttt{X}$ is known integer $(\texttt{1} \leq \texttt{X} \leq \texttt{N})$.
$\#\texttt{succ}(\texttt{X}, \texttt{Y})$:	$\texttt{Y}$ is successor of $\texttt{X}$, i.e., $\texttt{Y} = \texttt{X} + \texttt{1}$.
$+(\texttt{X}, \texttt{Y}, \texttt{Z})$:	$\texttt{Z} = \texttt{X} + \texttt{Y}$.
$*(\texttt{X}, \texttt{Y}, \texttt{Z})$:	$\texttt{Z} = \texttt{X} * \texttt{Y}$.

N.B. An upper bound for integers has to be specified when `dlv` is invoked.

Safety

Each variable occurring in a rule (resp. constraint) r in either

- the head,
- a default literal `not` b, or
- a built-in comparsion predicate,

must occur in at least 1 non-comparison `not`-free literal in the body of r.

Safe rules

```
a(X) :- not b(X), c(X).
a(X) :- X > Y, node(X), node(Y).
```

Unsafe rules

```
a(X) v -a(X).
a(X) :- not b(X).
:- X <= Y, node(X).
```

Declarative Problem Solving in DLV

Solve problems using disjunction/negation.

Maximum

Input: Employees and their salaries, represented by `empl(_,_)`.

Problem: Determine maximum salary of employees.

Solve Problem using projection and double negation!

```
-max(S) :- empl(N,S), empl(N1,S1), S < S1.
 max(S) :- empl(N,S), not -max(S).
```

Front-ends

- Besides the answer set semantics core, DLV offers front-ends for particular KR tasks:
  - diagnosis
  - inheritance reasoning
  - knowledge-based planning ($\mathcal{K}$ language)
- Also:
  - built-in front-end to SQL3
- Many external front ends to DLV exist (e.g., updates, preferences, plan diagnosis, execution monitoring, etc.)

Using DLV

- DLV is command-line oriented . . .
- . . . but there is also a simple GUI.

- Input is read from files whose names are passed on the command-line.
- If the command-line option "`--`" has been specified, input is also read from standard input (stdin).
- Output is printed to standard output (stdout), one line per model / answer set.

- Detailed documentation is at `http://www.dbai.tuwien.ac.at/proj/dlv/`

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